Lesson 1.1: Linear Equations and Inequalities

&

Lesson 1.2: Problem Solving (Part 1)

Lesson 1.1: Linear Equations and Inequalities

Objectives:

- Determine if a number is a solution to an equation.
- Solve Linear Equations
- Determine whether an Equation is a Conditional Equation (1 solution), a Contradiction (no solutions), or an Identity (infinite solutions)

Lesson 1.2: Problem Solving (Part 1)

Objectives:

- Translate English sentences into mathematical statements.
- Solving problems using mathematical models.
- Review percentage problems

Lesson 1.1: Linear Equations and Inequalities

DEFINITIONS:

Linear equation is an equation that has one variable which is written to the first power. $ax + b = 0$ (a and b are real numbers and $a \neq 0$.

A Solution is any value of the variable that results in a true statement. The value *satisfies* the equation.

DETERMINING IF A NUMBER IS A SOLUTION:

Substitute the value of the variable into the equation and simplify. If both sides are =, the value is a solution.

Example:

Determine if $x = 5$, and $x = 3$ are solutions to $3(x-1)=-2x+12$.

$3(5-1)=-2(5)+12$

$3(4)=-10+12$

$12 \neq 2$

$X=5$ does not work

$X=3$

$3(3-1)=-2(3)+12$

$3(2)=-6+12$

$6=6$ ✓

$X=3$ works

Lesson 1.1: Linear Equations and Inequalities

SOLVING LINEAR EQUATIONS means to find all of the solutions of the equation. (Find the *Solution Set*). If two or more equations have the same solution set, they are called *equivalent equations*.

Tools for Solving:

- **ADDITION PROPERTY OF EQUALITY:** Says that we can add the same number to both sides of the equation and it will remain equal. This covers subtraction, too, because subtracting is just adding a negative number.
- **MULTIPLICATION PROPERTY OF EQUALITY:** Says that you can multiply (or divide) both sides of the equation by the same value and it remains equal. NOTE: when you utilize the multiplication property, you must be sure to multiply every term on each side of the equation.
- **DISTRIBUTIVE PROPERTY:** Helps you remove parentheses at the beginning (before you use the addition or multiplication properties).

SUMMARY OF STEPS FOR SOLVING:

1. Remove any parentheses using the distributive property.
2. Combine like terms on each side of the equation (simplify each side).
3. Use the Addition Property to move all the variables to one side and all the constants to the other. undo +/−
4. Use the Multiplication Property to get the coefficient of the variable to equal 1. undo */÷
5. CHECK YOUR SOLUTION!!!!

Lesson 1.1: Linear Equations and Inequalities

Examples: Solve

a.) $2(x+4) = x - 4(x - 5)$

$2x + 8 = x - 4x + 20$

$2x + 8 = -3x + 20$

$+3x \quad -8 \qquad +3x \quad -8$

$5x = 12$

$\frac{5x}{5} = \frac{12}{5}$

$x = \frac{12}{5}$

Lesson 1.1: Linear Equations and Inequalities

Examples: Solve

b.) $6\left(\frac{x+5}{2}-4\right)=\left(\frac{2x-1}{3}\right)6$

$\frac{6}{1}\cdot\frac{3(x+5)}{2_1}-6(4)=\frac{6^2}{1}\cdot\frac{(2x-1)}{3_1}$

$3x+15-24=4x-2$

$3x-9=4x-2$

$-3x\quad+2\qquad-3x+2$

$-7=x$

$\boxed{x=-7}$

Lesson 1.1: Linear Equations and Inequalities

CATEGORIZING LINEAR EQUATIONS:

Conditional: Means it's true for some values of x but not all (usually 1 solution) Example: $x + 7 = 10$ is only true for $x = 3$.

Contradiction: The equation is false for all values (no solutions or $\emptyset$). Example: $3x + 8 = 3x + 6$. If you subtract $3x$ from both sides you get $8 = 6$, which doesn't work.

Identity: The equation is true for every possible values (all real numbers or $\mathbb{R}$) Example: $3x + 11 = 3x + 11$. When you simplify, you end up with the exact same thing on both sides of the equations.

Examples: Solve and Classify

c.) $5(2x+1) - x = 4(2 - x) + 13x$

$10x + 5 - x = 8 - 4x + 13x$

$9x + 5 = 9x + 8$

no solution, $\emptyset$

Contradiction

Examples: Solve and Classify

d.) $2(3x - 2) - (x - 6) = -3(6 - x) + 2(x + 10)$

$6x - 4 - x + 6 = -18 + 3x + 2x + 20$

$5x + 2 = 5x + 2$

$\mathbb{R}$

Identity

Solving Linear Equations

Example: $20\left(\frac{y+1}{4}+\frac{y-2}{10}\right)=\left(\frac{y+7}{20}\right)20$

Find the LCD = 20

$\frac{20}{1}\cdot\frac{(y+1)}{4}+\frac{20}{1}\cdot\frac{(y-2)}{10}=\frac{20}{1}\cdot\frac{(y+7)}{20}$

$5y+5+2y-4=y+7$

$7y+1=y+7$

$-y \quad -1 \quad -y \quad -1$

$6y=6$

$\frac{6y}{6}=\frac{6}{6}$ → $y=1$ Conditional

Lesson 1.2: Problem Solving (Part 1)

Mathematical operations in verbal expressions

Addition	Subtraction
plus more than the sum of increased by added to	minus less than the difference of decreased by subtracted from
Multiplication	**Division**
the product of multiplied by times twice	the quotient of divided by the ratio of half
Exponents	
squared cubed to the _____ power	

Words that mean "Equal"

- ~ is
- ~ was
- ~ is equivalent to
- ★ ~ yields
- ★ ~ gives
- ~ equals
- ~ are
- ★ ~ results in
- ~ is equal to

Lesson 1.2: Problem Solving (Part 1)

Translate each English Statement into a mathematical statement.

Ex 1: The product of 3 and y is equal to 21.

$3y = 21$

Ex 2: Two times the sum of 3 and X is equivalent to the product of 5 and X.

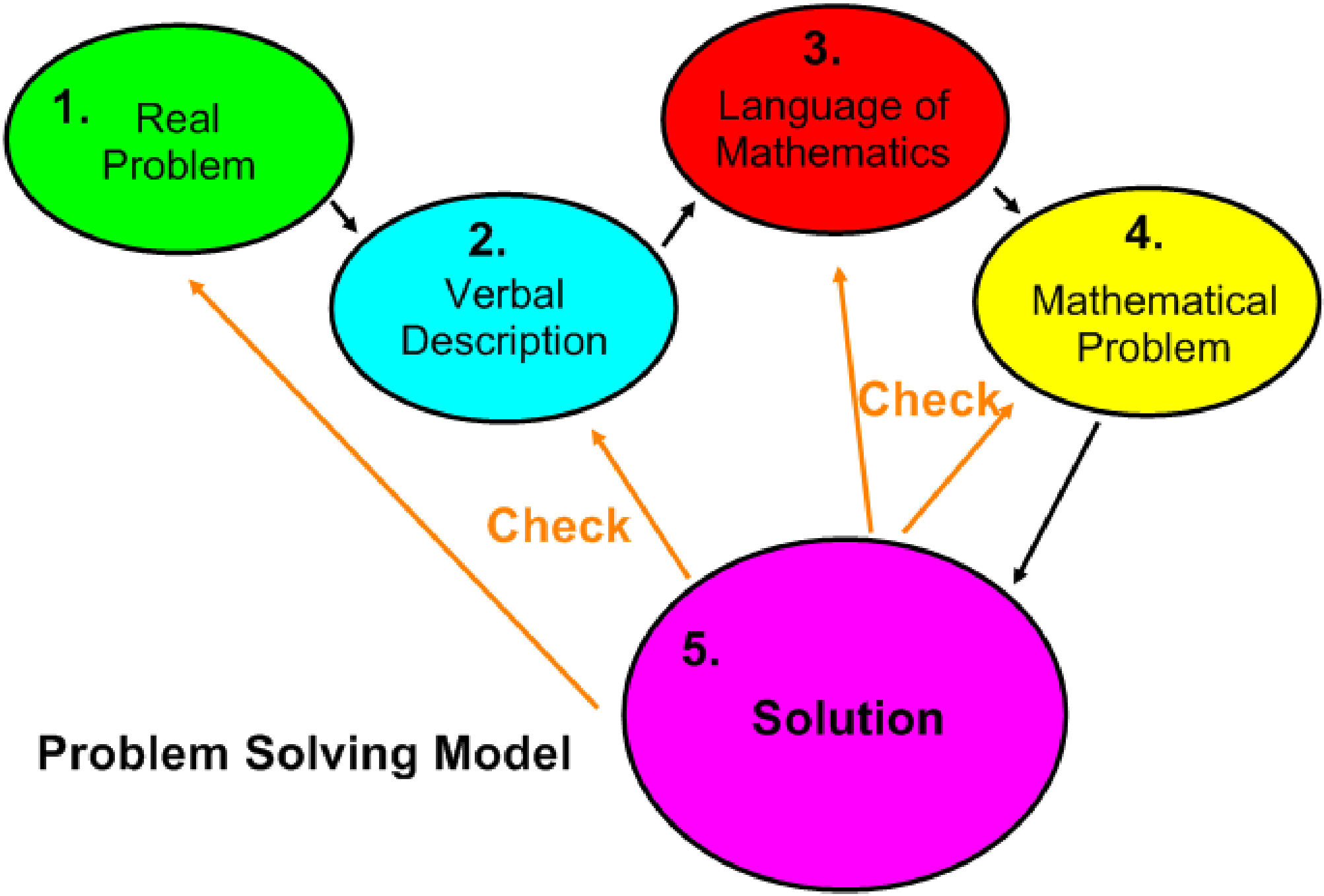
$2(3+x) = 5x$

Ex 3: The difference of x and 10 equals the quotient of x and 2.

$X - 10 = \frac{X}{2}$

$X - 10 = X \div 2$

Lesson 1.2: Problem Solving (Part 1)



Problem Solving Model

Lesson 1.2: Problem Solving (Part 1)

5 Categories of Problems

1. **Direct Translation** - problems where we must translate from English into Mathematics by using key words in the verbal description.

2. **Mixture** - problems where two or more quantities are combined in some fashion.

3. **Geometry** - problems where the unkown quantities are related through geometric formulas.

4. **Uniform Motion** - problems where an object travels at a constant speed.

5. **Work Problems** - problems where two or more entities join forces to complete a job.

Steps for Solving Problems with Mathematical Models

Step 1: Identify what you are looking for.

Step 2: Give Names to the Unknowns.

variables

Step 3: Translate the Problem into the Language of Mathematics.

Step 4: Solve the Equation(s) Found in Step 3.

Step 5: Check the Reasonableness of your Answer.

Step 6: Answer the Question (in a complete sentence).

Ex 4: The sum of three consecutive odd integers results in 45. Find the integers.

Step 1: 3 odd integers

Step 2: $n, (n+2), (n+4)$

Step 3: $n + (n+2) + (n+4) = 45$

Step 4:
$$3n + 6 = 45$$
$$-6 \quad -6$$
$$\frac{3n}{3} = \frac{39}{3}$$
$$n = 13$$

Step 6: The odd integers are 13, 15, 17.

Ex 5: Before Taxes, Mandy earned $725 one week after working 52 hours. Her employer pays time-and-a-half for all hours worked in excess of 40 hours. What is Mandy's hourly wage?

1.5x

Step 1: hourly wage

Step 2: x = hourly wage

$1.5x$ = time and a half wage

how many hrs does she get paid 1.5?

$52 - 40 = 12$

Step 3: $40(x) + 12(1.5x) = 725$

Step 4: $40x + 18x = 725$

$\frac{58x}{58} = \frac{725}{58}$

$x = \$12.5$

Step 6: Mandy gets paid $12.50 per hour.

Ex 6: MCI has a long-distance phone plan that charges $2.00 a month plus $0.09 per minute of usage. Sprint has a long-distance phone plan that charges $3.50 a month plus $0.07 per minute of usage. For how many minutes of long-distance calls will the costs for the two plans be the same?

Step 1: how many min until cost is the same?

Step 2: m = minutes used

Step 3: (cost) MCI: $2.00 + .09m$ cost be the same? → equal
Sprint: $3.50 + .07m$

$$2 + .09m = 3.50 + .07m$$

Step 4:

Ex 6: cont.

Step 4:

$2 + .09m = 3.50 + .07m$

$-2 \quad -.07m \quad -2.00 - .07m$

$\frac{.02m}{.02} = \frac{1.50}{.02}$

$m = 75 \text{ min}$

Step 6: You need to use 75 min for the cost of the two plans to be the same.

Ex 7: Suppose that you have just entered your favorite clothing store and find that everything is marked at a discount of 40% off. If the sale price of a coat is $144, what was the original price?

Step 1: original price of coat?

Step 2: X = original price

Step 3: $X - .40X = 144$

original — discount 40% of original price

Step 4: $\frac{.60X}{.60} = \frac{144}{.60}$

$X = 240$

Ex 7: cont.

Step 6: The coat originally cost $240.

(but w/ the 40% discount, it cost $144.)

Lesson 1.1: Linear Equations and Inequalities

Objectives:

- Determine if a number is a solution to an equation.
- Solve Linear Equations
- Determine whether an Equation is a Conditional Equation (1 solution), a Contradiction (no solutions), or an Identity (infinite solutions)

Lesson 1.2: Problem Solving (Part 1)

Objectives:

- Translate English sentences into mathematical statements.
- Solving problems using mathematical models.
- Review percentage problems

Can you?

Homework:

Pg. 61: 15-37 odds (13 problems)

&

Homework:

Pg. 75-76: 9-13 odds,19-23 odds,
25-31 odds, 32, 33, 35, 43
(14 problems)