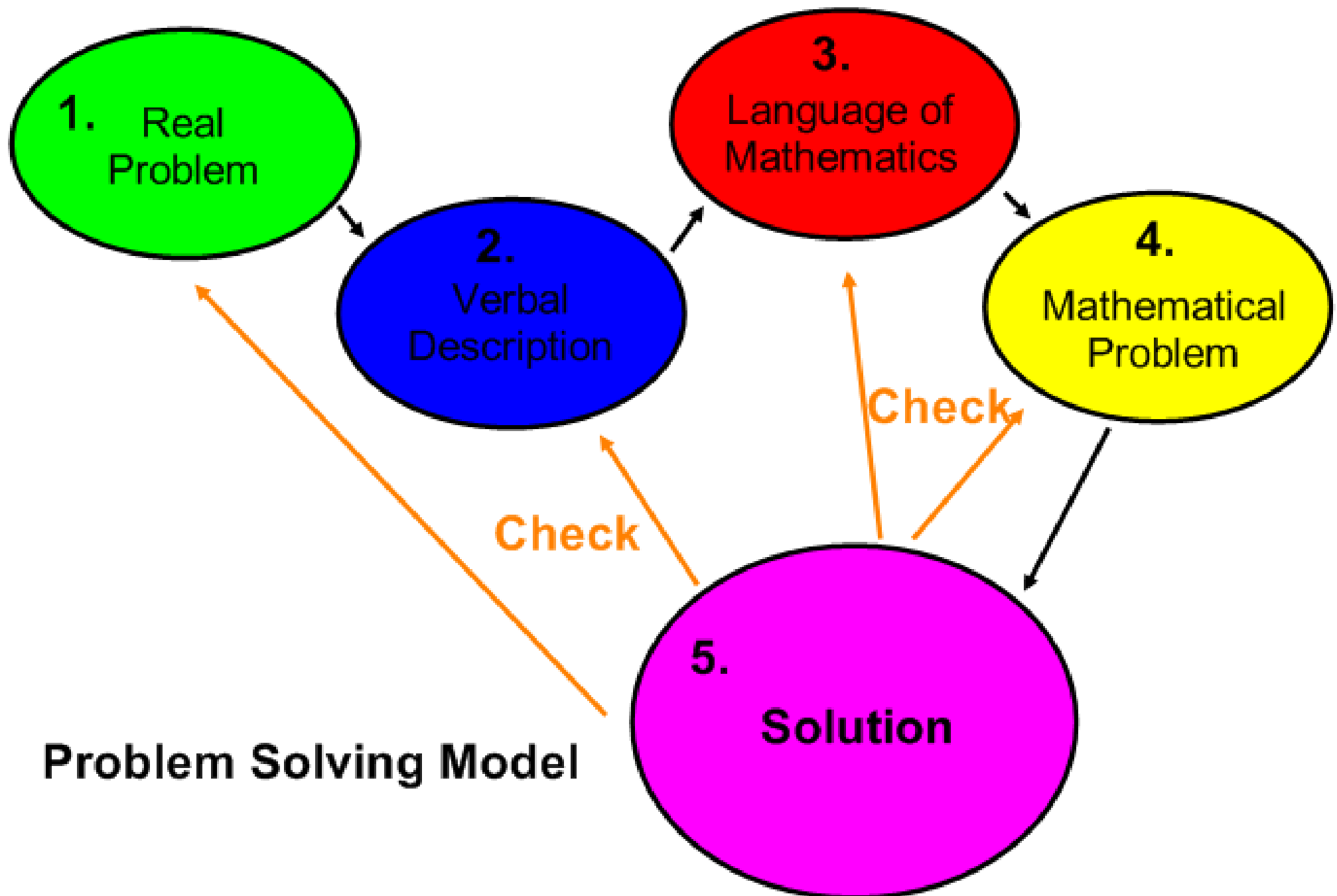


Lesson 1.2: Problem Solving (Part 2)

Lesson 1.2: Problem Solving (Part 2)



Problem Solving Model

Lesson 1.2: Problem Solving (Part 2)

5 Categories of Problems

- 1. Direct Translation** - problems where we must translate from English into Mathematics by using key words in the verbal description.
- 2. Mixture** - problems where two or more quantities are combined in some fashion.
- 3. Geometry** - problems where the unknown quantities are related through geometric formulas.
- 4. Uniform Motion** - problems where an object travels at a constant speed.
- 5. Work Problems** - problems where two or more entities join forces to complete a job.

Lesson 1.2: Problem Solving (Part 2)

Steps for Solving Problems with Mathematical Models

Step 1: Identify what you are looking for.

Step 2: Give Names to the Unknowns.

Step 3: Translate the Problem into the Language of Mathematics.

Step 4: Solve the Equation(s) Found in Step 3.

Step 5: Check the Reasonableness of your Answer.

Step 6: Answer the Question (in a complete sentence).

Lesson 1.2: Problem Solving (Part 2) Step 2:

Ex 1: $I = Prt$; where $I = \text{Interest}$ (simple interest),
 $P = \text{Principal}$, $r = \text{rate of interest}$, and
 $t = \text{time (in years)}$

Supposed that Kenzie has a credit card balance of \$2800. Each month, the credit card company charges 14% annual simple interest on any outstanding balances. What is the interest that Kenzie will be charged on this loan after one month? What is Kenzie's credit card balance after one month?

Step 3: $I = 2800 (.14) (\frac{1}{12})$

$I = ?$

$P = 2,800$

Step 4: $I = 32.67$

$r = .14$

$t = \frac{1}{12}$

Step 6: The interest is \$32.67 +
her balance will be
\$2832.67.

Lesson 1.2: Problem Solving (Part 2)

$$I = Prt$$

Ex 2: Suppose that Ethan has a car loan of \$6500. The bank charges 6% annual simple interest. What is the interest charge on Ethan's car loan after 1 month?

$$I = ?$$

$$P = 6500$$

$$r = .06$$

$$t = \left(\frac{1}{12}\right)$$

$$\text{Step 2: } I = (\text{Interest})$$

$$\text{Step 3: } I = 6500(.06)\left(\frac{1}{12}\right)$$

$$\text{Step 4: } I = 32.5$$

Step 6: The interest after 1 month is \$32.50

Lesson 1.2: Problem Solving (Part 2)

Mixing Problems

Mixture problems are problems in which two or more items are combined to form a third item. There are a number of different kinds of mixture problems, but they all follow the basic idea.

Portion from item A + Portion from item B = Whole or Total

Lesson 1.2: Problem Solving (Part 2)

Ex 3: Mitch has \$15,000 to invest. His goal is to obtain an overall annual rate of return of 9% or \$1350 annually. His financial advisor recommends that he invest some of the money in corporate bonds that pay 12% and the rest in government-backed Treasury bonds paying 4%. How much should be placed in each investment in order for Mitch to achieve his goal?

Step 1: Identify

Mitch needs to figure out how much to put in corporate bonds vs. Treasury bonds in order to get \$1350 in interest.

Step 2: Name

Let's have b represent the amount Mitch invests in corporate bonds. How can we represent the amount Mitch invests in Treasury bonds?

$$t \quad b + t = 15000 \rightarrow t = \underbrace{15000 - b}$$

Lesson 1.2: Problem Solving (Part 2)

$$t = 15000 - b$$

Ex 3: Remember: Mitch has \$15,000 to invest. His goal is to obtain an overall annual rate of return of 9% or \$1350 annually. Corporate bonds that pay 12% and Treasury bonds pay 4%.

Step 3: Translate

So now, we need a mathematical model.

	Principal (\$)	Rate (%)	Time (Yr)	Interest (\$)
Corporate Bond	b	0.12	1	$0.12b$
Treasury Bond	$15,000 - b$	0.04	1	$0.04(15,000 - b)$
Total	15,000	0.09	1	$0.09(15000) = \$1350$

Interest from Corporate bonds + Interest from Treasury bonds = \$1350

$$0.12b + 0.04(15000 - b) = 1350$$

Lesson 1.2: Problem Solving (Part 2)

Ex 3: Remember: Mitch has \$15,000 to invest. His goal is to obtain an overall annual rate of return of 9% or \$1350 annually. Corporate bonds that pay 12% and Treasury bonds pay 4%.

Step 4 : Solve

$$.12b + .04(15000 - b) = 1350$$

$$.12b + 600 - .04b = 1350$$

$$\begin{array}{r} .08b + 600 = 1350 \\ -600 \quad -600 \\ \hline \end{array}$$

$$\begin{array}{r} .08b = 750 \\ -08 \quad -08 \\ \hline \end{array}$$

$$b = 9375$$

$$\begin{array}{l} t = 15000 - 9375 \\ t = 5625 \end{array}$$

Lesson 1.2: Problem Solving (Part 2)

Ex 3: Remember: Mitch has \$15,000 to invest. His goal is to obtain an overall annual rate of return of 9% or \$1350 annually. Corporate bonds that pay 12% and Treasury bonds pay 4%.

Step 5: Check

Step 6: Answer the Question

Mitch invested \$9375 in corp.
bonds + \$5625 in treasury
bonds.

Lesson 1.2: Problem Solving (Part 2)

Ex 4: The manager of a coffee shop wishes to form a new blend of coffee. She wants to mix Sumarta Beans, known for their strong, distinctive taste, that sell for \$12/lb with milder Brazilian Beans that sell for \$8/lb to get 50 lbs of the new blend. The new blend will sell for \$9/lb and there will be NO difference in revenue from selling the new blend vs. selling the beans separately. How many lbs. of the Sumatra and Brazilian beans are required?

Step 1: Identify

This is a mixture problem because we want to know how many pounds of each (Sumatra and Brazilian) beans we need to make a new blend.

Step 2: Name

Let's have s represent the amount of Sumatra Beans that are required. *So what will the # of lbs be for the Brazilian beans?*

$$s + b = 50 \rightarrow b = 50 - s$$

Lesson 1.2: Problem Solving (Part 2)

Ex 4:

Step 3: Translate

	Price (\$/lb)	Number of lbs.	Revenue
Sumatra	12	s	$12s$
Brazilian	8	$50-s$	$8(50-s)$
Blend	9	50	$9(50) = \$450$

So now, we need a mathematical model.

Revenue from Sumatra + Revenue from Brazilian = Revenue of New Blend

$$12s + 8(50-s) = 450$$

Lesson 1.2: Problem Solving (Part 2)

Ex 4:

Step 4 : Solve

$$12s + 8(50 - s) = 450$$

$$12s + 400 - 8s = 450$$

$$\begin{array}{r} 4s + 400 = 450 \\ -400 \quad -400 \\ \hline \end{array}$$

$$\frac{4s}{4} = \frac{50}{4}$$

$$s = 12.5 \text{ lbs}$$

Brazilian
 $s + b = 50$

$$b = 50 - s$$

$$b = 50 - 12.5$$

$$b = 37.5 \text{ lbs}$$

Step 5: Check

Step 6: Answer the Question

She needs 12.5 lbs of Sumatra beans and 37.5 lbs of the Brazilian Beans.


Lesson 1.2: Problem Solving (Part 2)

Uniform Motion Problems

Objects that move at a constant velocity are said to be in uniform motion. When the average velocity of an object is known, it can be interpreted as its constant velocity.

For example, a car traveling at an average velocity of 40 miles/hour is in uniform motion.

If an object moves at an average speed r , the distance d covered in time t is given by:

$$d = rt$$


Lesson 1.2: Problem Solving (Part 2)

Ex 5: Austin and Dillon decide to have a 10-mile race. Austin can run at an average speed of 12 miles/hour while Dillon can run at an average speed of 10 miles/hour. To "even things up", Austin agrees to give Dillon a head start of 0.15 hour. When will Austin catch up to Dillon?

Step 1: Identify

when will they meet?
time

Step 2: Name

$t = \text{time}$

Lesson 1.2: Problem Solving (Part 2)

Ex 5: Austin and Dillon decide to have a 10-mile race. Austin can run at an average speed of 12 miles/hour while Dillon can run at an average speed of 10 miles/hour. To "even things up", Austin agrees to give Dillon a head start of 0.15 hour. When will Austin catch up to Dillon?

Step 3: Translate

	Rate (mph)	Time (hrs)	= Distance (Miles)
Austin	12	t	$12t$
Dillon	10	$t + 0.15$	$10(t + 0.15)$

$$12t = 10(t + 0.15)$$

Lesson 1.2: Problem Solving (Part 2)

Ex 5:

Step 3: Translate

So now, we need a mathematical model.

	Rate (mph)	Time (hrs)	Distance (Miles)
Austin	12	t	$12t$
Dillon	10	$t + 0.15$	$10(t + 0.15)$

Distance Austin runs = Distance Dillon runs

$$12t = 10(t + 0.15)$$

$$\begin{array}{r} 12t = 10t + 1.5 \\ -10t \quad -10t \\ \hline \end{array}$$

$$\frac{2t}{2} = \frac{1.5}{2}$$

$$t = 0.75 \text{ hrs.}$$

Lesson 1.2: Problem Solving (Part 2)

Ex 5:

Step 4 : Solve



They will meet up
after 0.75 hrs (45 min).

Step 5: Check

Step 6: Answer the Question

Lesson 1.2: Problem Solving (Part 2)

Homework:

Pg. 76:

30, 37, 40, 42, 43, 49-51 all,
53, 57, 59, 61, 63, 65, 68

(14 problems)