Compound Inequalities!

Before we learn about them, we are going to have a quick refresher on SETs!

REVIEW

Set Notation

A *Set* is a collection of "well-defined" objects.

"well-defined means that there is a rule for determining whether or not the object is in the set.

Elements are the objects in a set. $\in$

We use curly braces { } to encolse the elements. If we have set D that includes elements 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, then we would write it like:

D={0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

When we list the elements out like this, we are representing the set using the *Roster Method*.

a.) Use the Roster Method to represent the set of all even digits.

Set Notation Cont.

Set-Builder Notation is a way to denote a set.

For Example: The numbers in set D={0, 1, 2, 3, 4, 5, 6, 7, 8, 9} are called Digits. Set-Builder Notaion would be D={x|x is a digit}.

We name sets by using capital letters.

Ex: We could name the set of even numbers E.
So, E={x|x is an even number}

When we talk about rules for sets, we usually use the sets A and B.

Most of our definitions will have sets A and B.

END

of

REVIEW

Sets

Consider the table:

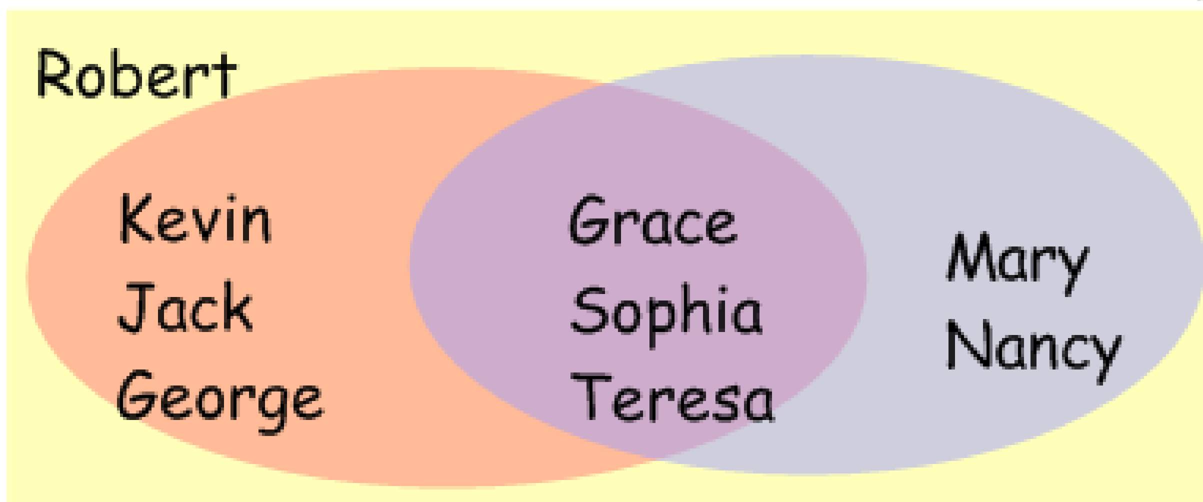
Let set A be the set of all students whose age is less than 25.

Let set B be the set of all students who are female.

Student	Age	Gender
Grace	19	F
Sophia	23	F
Kevin	20	M
Robert	32	M
Jack	19	M
Mary	35	F
Nancy	40	F
George	22	M
Teresa	20	F

A= {Grace, Sophia, Kevin, Jack, George, Teresa}

B= {Grace, Sophia, Mary, Nancy, Teresa}



Sets

Example 1:

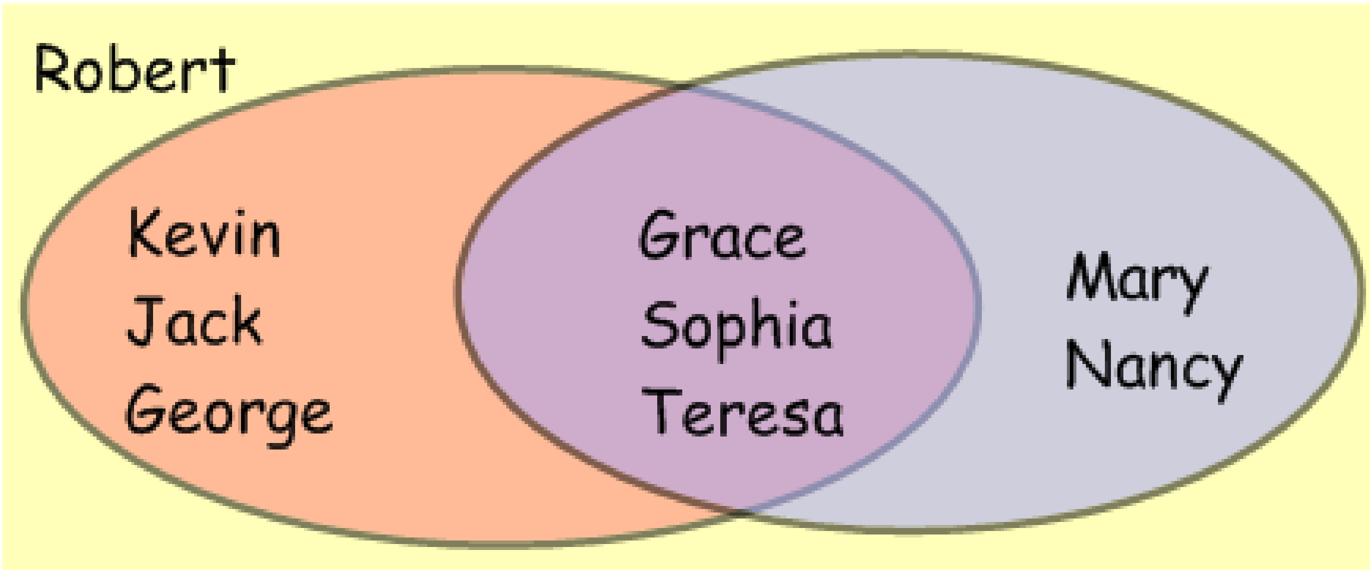
a.) List all the students that are in set A or set B.

{Grace, Sophia, Teresa, Mary, Nancy, Kevin, Jack, George}

This is called the **Union** of the sets.

Writen as:

$$A \cup B$$



Sets

Example 1 (cont.):

b.) List all the students that are in A **and** B.

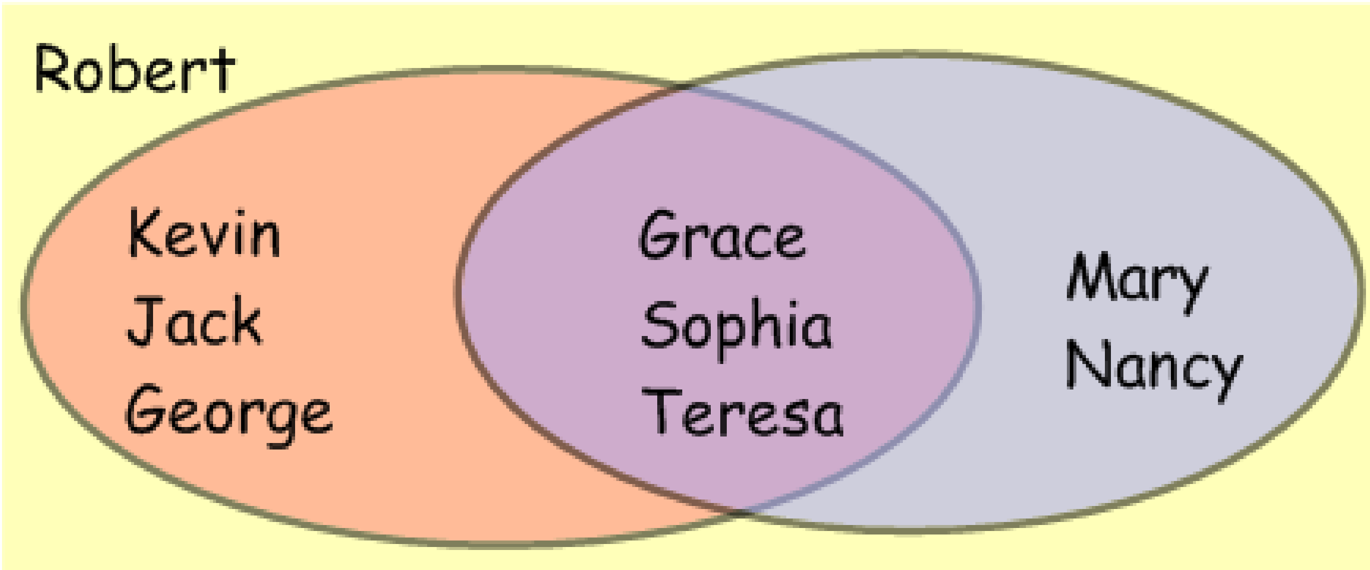
{ Grace, Sophia, Teresa }

This is called the **Intersection** of the sets.

Writen as:

$$A \cap B$$

A



Example 2:

Find the Intersection and the Union of the sets.

$A = \{1, 3, 5, 7, 9\}$ $B = \{1, 2, 3, 4, 5\}$

Intersection: $A \cap B = \{1, 3, 5\}$

Union: $A \cup B = \{1, 2, 3, 4, 5, 7, 9\}$

Example 3:

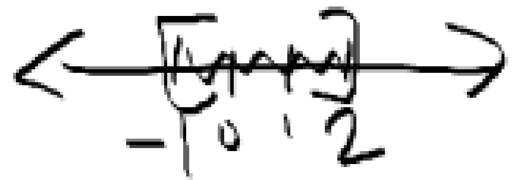
Find the Intersection and the Union of the sets.

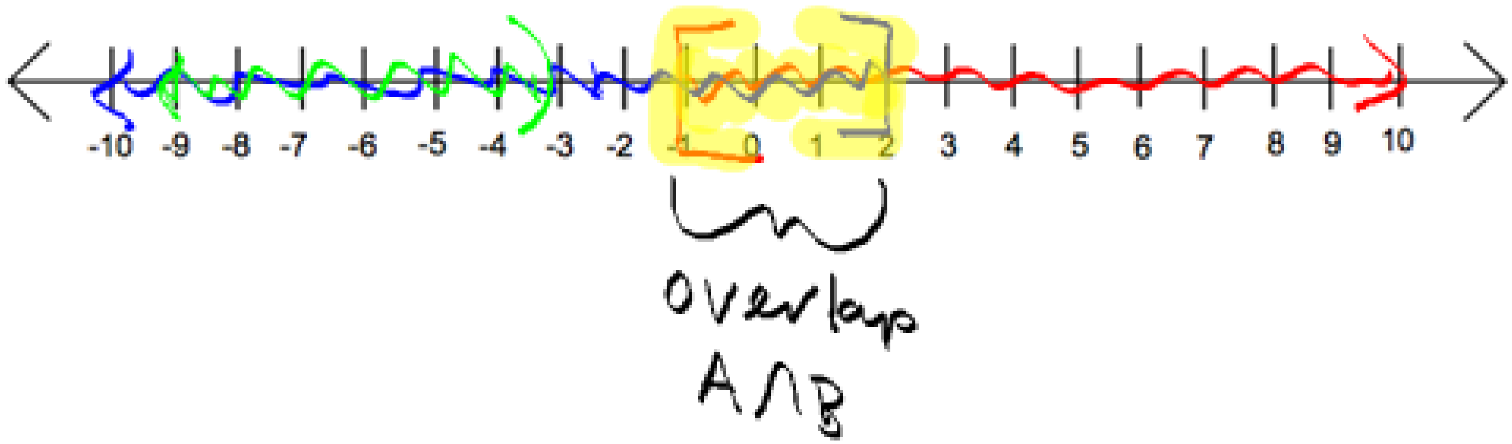
$A = \{x | x \le 2\}$, $B = \{x | x \ge -1\}$, $C = \{x | x < -3\}$

a.) Determine $A \cap B$. Graph the set and write in set builder notation and interval notation.

$A \cap B = [-1, 2]$

$A \cap B = \{x | -1 \le x \le 2\}$





Example 3:

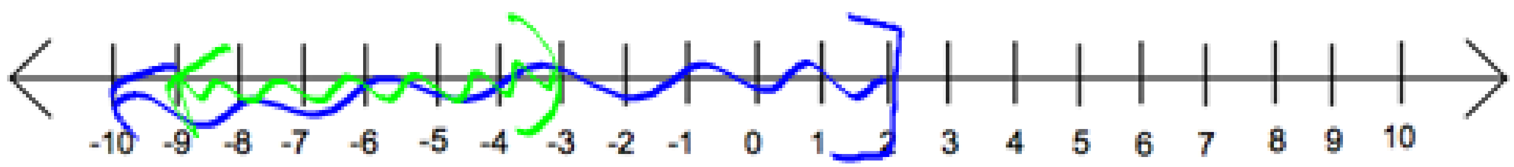
Find the Intersection and the Union of the sets.

$A = \{x \mid x \leq 2\}$, $B = \{x \mid x \geq -1\}$, $C = \{x \mid x < -3\}$

b.) Determine $A \cup C$. Graph the set and write in set builder notation and interval notation.

$A \cup C = \{x \mid x \leq 2\}$

$A \cup C = (-\infty, 2]$



Compound Inequalities:

Compound inequalities are just two regular inequalities smashed into one using "and" or "or".

For Example:

Two regular inequalities are $3x + 1 > 4$, $2x - 3 < 7$.
If we put an "and" or an "or" in between, then we make a compound inequality.

$$3x + 1 > 4 \text{ and } 2x - 3 < 7$$

Example 4: Inequalities involving "AND"

Write in Set + interval notation

Solve $3x + 2 > -7$ and $4x + 1 \leq 9$. Graph the solution set.

Steps to solve a compound inequalitiy involving "and":

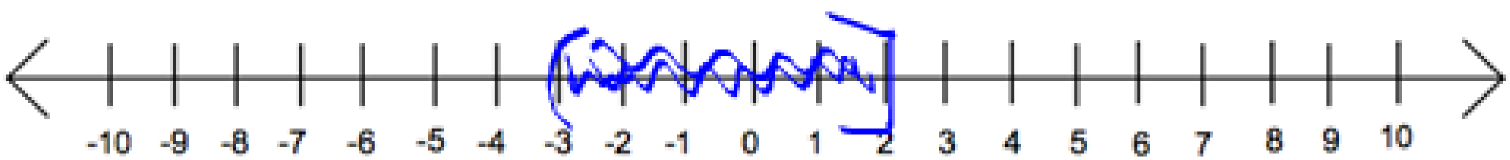
Step 1: Solve each inequality separately.

Step 2: Find the INTERSECTION of the solution sets.

$$3x + 2 > -7$$
$$-2 \quad -2$$
$$3x > -9$$
$$\frac{3x}{3} > \frac{-9}{3}$$
$$x > -3$$

$$4x + 1 \leq 9$$
$$-1 \quad -1$$
$$4x \leq 8$$
$$\frac{4x}{4} \leq \frac{8}{4}$$
$$x \leq 2$$

$$\boxed{(-3, 2] \qquad \{x \mid -3 < x \leq 2\}}$$



Lesson 1.5: Compound Inequalities

We can write inequalities involving "and" a little more compactly.

If we have $a < b$ and our answers are $x > a$ and $x < b$, we can write them like this:

$$a < x < b$$

For example:

If we have $x > -2$ and $x < 5$, we can write them like this:

$$-2 < x-3 < 5$$

$+3 \quad +3 \quad +3$

Example 5:

Write in set & interval not.

Solve $-3 < -4x + 1 < 13$. Graph the solution set.

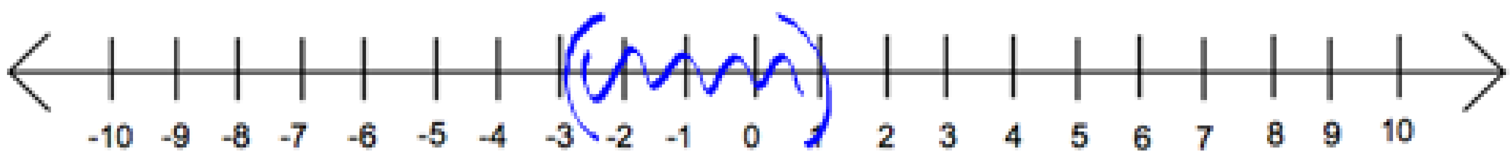
$$\begin{array}{ccccc} -3 & < & -4x + 1 & < & 13 \\ -1 & & -1 & & -1 \\ \hline -4 & < & -4x & < & 12 \\ \overline{-4} & & \overline{-4} & & \overline{-4} \end{array}$$

$$1 > x > -3$$

$$-3 < x < 1$$

$$\boxed{\begin{array}{c} \{x \mid -3 < x < 1\} \\ (-3, 1) \end{array}}$$

*always write smaller # FIRST!



Example 6: Inequalities involving "OR"

Solve $\frac{1}{2}x - 1 < 1 \; or \; \frac{2x-1}{3} \geq -1$

Graph the solution set.

Steps to solve a compound inequalitiy involving "or":

Step 1: Solve each inequality separately.

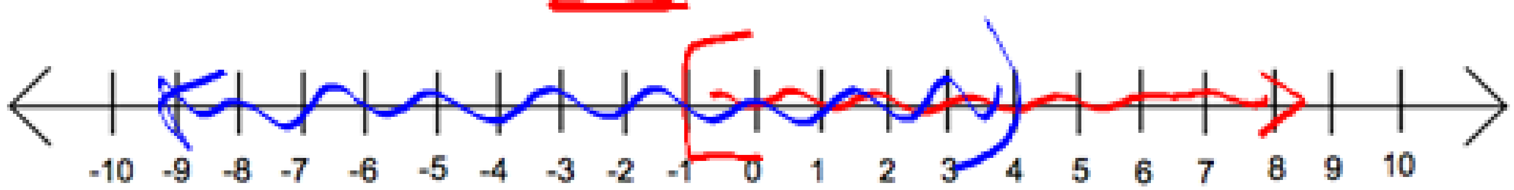
Step 2: Find the UNION of the solution sets.

$$\frac{1}{2}x - 1 < 1$$
$$\quad +1 \quad +1$$
$$2\left(\frac{1}{2}x\right) < (2)2$$
$$x < 4$$

$$3\left(\frac{2x-1}{3}\right) \geq (-1)3$$
$$2x - 1 \geq -3$$
$$\quad +1 \quad +1$$
$$\frac{2x}{2} \geq \frac{-2}{2}$$
$$x \geq -1$$

$\mathbb{R}$ or {all real #}

$(-\infty, \infty)$

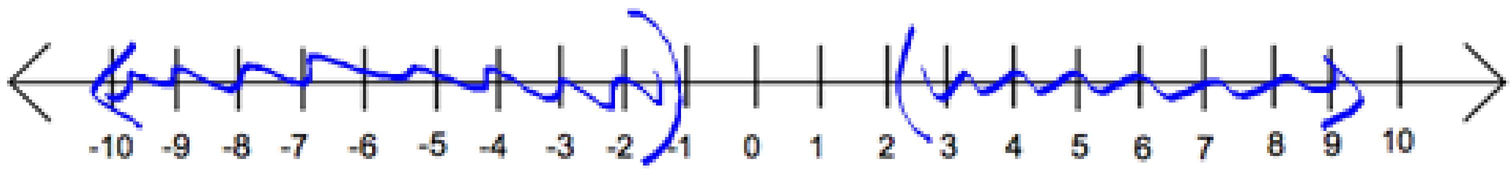


Example 7: Solve and graph.

$$5(x + 2) > 20 \text{ or } 4(x - 4) < -20$$

$$5x + 10 > 20$$
$$-10 \quad -10$$
$$\frac{5x}{5} > \frac{10}{5}$$
$$x > 2$$

$$4x - 16 < -20$$
$$+16 \quad +16$$
$$\frac{4x}{4} < \frac{-4}{4}$$
$$x < -1$$



$(-\infty, -1) \cup (2, \infty)$

$\{x \mid x < -1 \text{ or } x > 2\}$

Lesson 1.5: Compound Inequalities

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Ex 8: In 2005, a married couple filing a joint federal tax return whose income places them in the 25% tax bracket will pay federal income taxes between \$8180 and \$23,317.50, inclusive. The couple must pay federal income taxes equal to \$8180 plus 25% of the amount over \$59,400. Find the range of taxable income the couple makes in order for them to be in the 25% tax bracket.

Step 1: Identify

We need to find the range of taxable income for a married couple in the 25% tax bracket. This is a direct translation problem involving an inequality.

Step 2: Name

Let's have ***t*** represent the taxable income.

Lesson 1.5: Compound Inequalities

Step 3: Translate

Find the range of taxable income.

The federal tax bill equals \$8180 plus 25% of the taxable income over \$59,400. Because the tax bill is between \$8180 and \$23,317.50, we have:

$$8180 \leq 8180 + 0.25(t - 59{,}400) \leq 23{,}317.50$$

Step 4 : Solve

$$8180 \leq 8180 + .25t - 14850 \leq 23{,}317.50$$

$$8180 \leq .25t - 6670 \leq 23{,}317.50$$

$$+6670 \qquad +6670 \qquad +6670$$

$$\frac{14850}{.25} \leq \frac{.25t}{.25} \leq \frac{29987.50}{.25}$$

$$59{,}400 \leq t \leq 119{,}950$$

Step 5: Check Find the range of taxable income.

Step 6: Answer the Question

The range of taxable income is between \$59,400 and \$119,950.

Homework:

Pg 109-112: #'s 2, 4, 5, 7, 9, 11-19 all,
21-43 odds, 79, 80, 81, 85
(30 problems)

On #'s 17 - 43, please give the intersection/union in set builder notation, interval notation, and graph.