Lesson 1.6: Absolute Value Equations and Inequalities

$|x-25| = 17$

$|2x + 4| + 1 \geq 13$

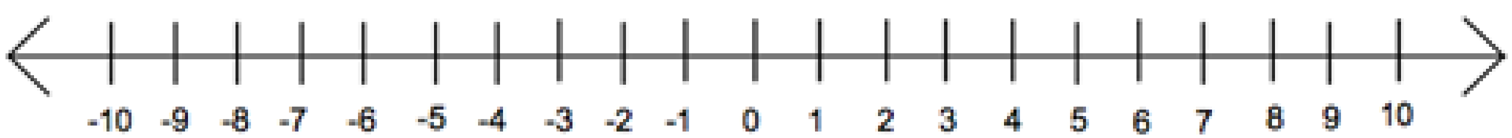
Lesson 1.6: Absolute Value Equations and Inequalities

What is an Absolute Value?

1. $|3| = 3$
2. $|-10| = 10$
3. $|-2| = 2$

Why do we make them positive?

The **Absolute Value** is the distance from Zero on the number line.



Lesson 1.6: Absolute Value Equations and Inequalities

Example 1:

Evaluate **$|3x-6| + 3.2$** if $x = -2$

$$|3(-2) - 6| + 3.2$$

$$= |-6 - 6| + 3.2$$

$$= |-12| + 3.2$$

$$= 12 + 3.2$$

$$= \boxed{15.2}$$

Lesson 1.6: Absolute Value Equations and Inequalities

Example 2 part A:

- For what values of x would this equation be true? $|x| = 3$

$x = 3$ $x = -3$

- For what values of x would this equation be true? $|x+2| = 5$

$(x+2) = 5$ $(x+2) = -5$

$-2 \quad -2$ $-2 \quad -2$

$x = 3$ $x = -7$

Lesson 1.6: Absolute Value Equations and Inequalities

Example 2 part B: The Evil Twin - *the negative side*

Solve $|x-25| = 17$

This means that we could be on the positive side and have

$$(x-25) = 17$$

or we could be on the negative side and have

$$|-(x-25)| = 17$$

We can take $-(x-25) = 17$ and divide by a negative on both sides. Then we would have

$$(x-25) = -17$$

"Okay, one time, but just remember who the evil twin in this family really is."

Example 2: The Evil Twin - *the negative side*

Solve $|x-25| = 17$

So we have two options our equation could be:

$(x-25) = 17$ or $(x-25) = -17$

$$\begin{array}{l|l} (x-25) = 17 & (x-25) = -17 \\ +25 \quad +25 & +25 \quad +25 \\ \hline x = 42 & x = 8 \end{array}$$

$\{x \mid x = 42, 8\}$ or

$\{x \mid x = 42, x = 8\}$

Example 2: The Evil Twin - *the negative side*

Solve $|x-25| = 17$

So we have two options our equation could be:

$(x-25) = 17$ or $(x-25) = -17$

$+25 \quad +25$ | $+25 \quad +25$

$x = 42$ | $x = 8$

Now write in set notation:

{42, 8}

Example 3:

Solve $|x+6| = 18$

So we have two options our equation could be:

$x+6 = 18$ **or** $x + 6 = -18$

$-6 \quad -6$ | $-6 \quad -6$

$x = 12$ | $x = -24$

$\{x \mid x = 12, -24\}$

or

$\{12, -24\}$

Example 4:

$$\frac{3|x+6|}{3} = \frac{36}{3}$$

Solve $3|x + 6| = 36$

$$|x+6| = 12$$

Note: we must ALWAYS get the absolute value alone before we do the evil twin.

So we have two options our equation could be:

$(x + 6) = 12$ **or** $(x + 6) = -12$

$$\begin{array}{cc} -6 \quad -6 & -6 \quad -6 \\ \hline x = 6 & x = -18 \end{array}$$

$\{x \mid x = 6, -18\}$ or

$\{6, -18\}$

Example 6:

Solve $|2x + 7| - 5 = 0$

$$\quad\quad\; +5 \;\; +5$$

$$|2x+7| = 5$$

$$2x+7=5 \qquad\qquad 2x+7=-5$$

$$-7 \;\; -7 \qquad\qquad -7 \;\; -7$$

$$\frac{2x}{2} = \frac{-2}{2} \qquad\qquad \frac{2x}{2} = \frac{-12}{2}$$

$$x=-1 \qquad\qquad x=-6$$

$$\{x \mid x=-1,-6\}$$

$$\{-1,-6\}$$

Lesson 1.6: Absolute Value Equations and Inequalities

Example 6.1:

Solve $|2x + 7| + 5 = 0$

What happens if an absolute value is supposedly equals a negative number? Can that happen?

$-5 \quad -5$

$|2x + 7| = -5$

No Solution

$\{\ \}$ or $\emptyset$

Example 7:

Solve $|x - 2| = 2x - 10$

So we have two options our equation could be:

$x - 2 = 2x - 10$ **or** $x - 2 = -(2x - 10)$

$x - 2 = -2x + 10$

$-2x + 2 \quad -2x \quad +2$

$\frac{-x}{-1} = \frac{-8}{-1}$

$x = 8$

$+2x + 2 \quad +2x \quad +2$

$\frac{3x}{3} = \frac{12}{3}$

$x = 4$

$\{x \mid x = 8, 4\}$

$\{8, 4\}$

Example 8:

Solve $|2x - 3| = |x + 6|$

Note: We treat this just like a regular Abs. Value equation. We let 2x-3 = x+6 AND 2x-3 = -(x+6). (refer to pg 115)

So we have two options our equation could be:

$2x-3 = x+6$ **or** $2x-3 = -(x+6)$

$$\begin{aligned} 2x-3 &= x+6 \\ -x+3 &\quad -x+3 \\ x &= 9 \end{aligned}$$

$$\begin{aligned} 2x-3 &= -x-6 \\ +x+3 &\quad +x+3 \\ \frac{3x}{3} &= \frac{-3}{3} \\ x &= -1 \end{aligned}$$

$\{x \mid x = 9, -1\}$ or $\{9, -1\}$

Example 9:

You MUST get the absolute value alone first before you do "evil twins"!!!

Solve $-2\,|2x-7| - 1 = -35$

$$\qquad\qquad +1 \qquad +1$$

$$\frac{-2\,|2x-7|}{-2} = \frac{-34}{-2}$$

$$|2x-7| = 17$$

$$2x+7=17 \qquad\qquad 2x-7=17$$

$$-7 \quad -7 \qquad\qquad +7 \quad +7$$

$$\frac{2x}{2} = \frac{10}{2} \qquad\qquad \frac{2x}{2} = \frac{24}{2}$$

$$x=5 \qquad\qquad x=12$$

$\{x \mid x=5, 12\}$ or $\{12, 5\}$

Lesson 1.6: Absolute Value Equations and Inequalities

Now we are going to look at Absolute Value Inequalities ($<$, $>$, $\leq$, $\geq$).

We will treat them the same way, except we need to remember to *flip the inequality sign when we divide by a negative (the evil twin).*

We will also graph our solution set and write in interval notation.

Example 10: $|2x + 4| + 1 \geq 13$

$$|2x+4| + 1 - 1 \geq 13 - 1$$

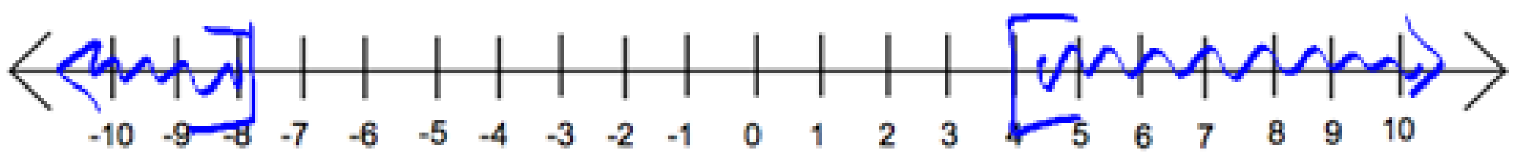
$$|2x+4| \geq 12$$

$$2x + 4 \geq 12 \qquad\qquad 2x + 4 \leq -12$$

$$2x + 4 - 4 \geq 12 - 4 \qquad\qquad 2x + 4 - 4 \leq -12 - 4$$

$$\frac{2x}{2} \geq \frac{8}{2} \qquad\qquad \frac{2x}{2} \leq \frac{-16}{2}$$

$$x \geq 4 \qquad\qquad x \leq -8$$



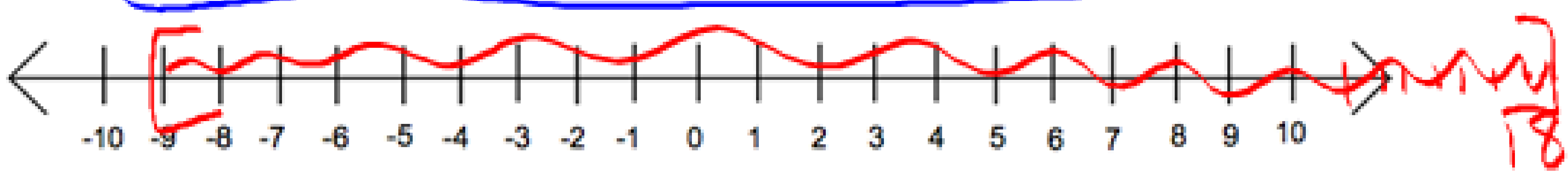
$(-\infty, -8] \cup [4, \infty)$ $\quad \{x \mid x \leq -8 \text{ or } x \geq 4\}$

Lesson 1.6: Absolute Value Equations and Inequalities

Example 11: $|2x - 9| \le 27$

$$2x - 9 \le 27 \qquad\qquad 2x - 9 \ge -27$$
$$+9 \quad +9 \qquad\qquad +9 \quad +9$$
$$\frac{2x}{2} \le \frac{36}{2} \qquad\qquad \frac{2x}{2} \ge \frac{-18}{2}$$
$$x \le 18 \qquad\qquad x \ge -9$$

$[-9, 18]$ $\{x \mid -9 \le x \le 18\}$



STOP AND THINK!!!!

Just like with Absolute values, we have some special cases (remember that absolute values can't equal a negative?).

Special Case #1:

An absolute value that is < or ≤ a negative number will never have a solution.

This is because an abs. val. is the distance from zero. Distance can never be less than zero (a negative).

Example: $|2x + 6| < -12$

No Solution { } or ∅

Solve and graph.

Ex: $|x - 4| + 6 \leq 1$

$$\begin{array}{r} |x-4| + 6 \leq 1 \\ -6 \quad -6 \\ \hline |x-4| \leq -5 \end{array}$$

No Solution

$\{\ \}$ or $\emptyset$

Special Case #2:

An absolute value that is > or ≥ a negative number will have a solution of all real numbers.

This is because an abs. val. is the distance from zero. Distance will always be greater than zero (a negative).

Example: $|x + 2| > -4$

$\mathbb{R}$ or {all real #}

$(-\infty, \infty)$

Lesson 1.6: Absolute Value Equations and Inequalities

Solve and graph.

Ex: $|5x - 2| + 15 \geq 10$

Homework:

Pg 122: #'s 55-74 all, 81, 82

- For Equations, give answers in set notation.
- For Inequalities, give answers in set notation, interval notation, and graph.