Lesson 10.1 & 10.2: Distance & Midpoint Formulas, and Circles

Objectives:

- Use the Distance Formula.
- Use the Midpoint Formula.
- Write the Standard Form of the Equation of a Circle.
- Graph a Circle.
- Find the Center and Radius of a Circle from an Equation in General Form.

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The Distance Formula

The distance $d(P_1,\ P_2)$ between two points $P_1 = (x_1,\ y_1)$ and $P_2 = (x_2,\ y_2)$, is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

This formula is found by graphing the points on a Cartesian coordinate plane, then making a right triangle and using the Pythagorean Theorem.

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Examples: Find the distance between the two points.

$x_1\ y_1$ $x_2\ y_2$

A) $P_1(-3,6)$ *and* $P_2(3,-2)$

$$d = \sqrt{(3+3)^2+(-2-6)^2}$$

$$= \sqrt{6^2+(-8)^2}$$

$$= \sqrt{36+64}$$

$$= \sqrt{100}$$

$$\boxed{d = 10}$$

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Examples: Find the distance between the two points.

B) $P_1(-4, -1)$ *and* $P_2(2,\ 2)$

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Examples: Find the distance between the two points.

C) Consider the three points $A(-2,5)$, $B(12,3)$, $C(10,-11)$ (10, -11)

a) Plot the points on a coordinate plane and make the triangle ABC.

A(-2, 5) B(12, 3)
$x_1\ y_1$ $x_2\ y_2$

$$\overline{AB} = d = \sqrt{(12+2)^2 + (3-5)^2}$$

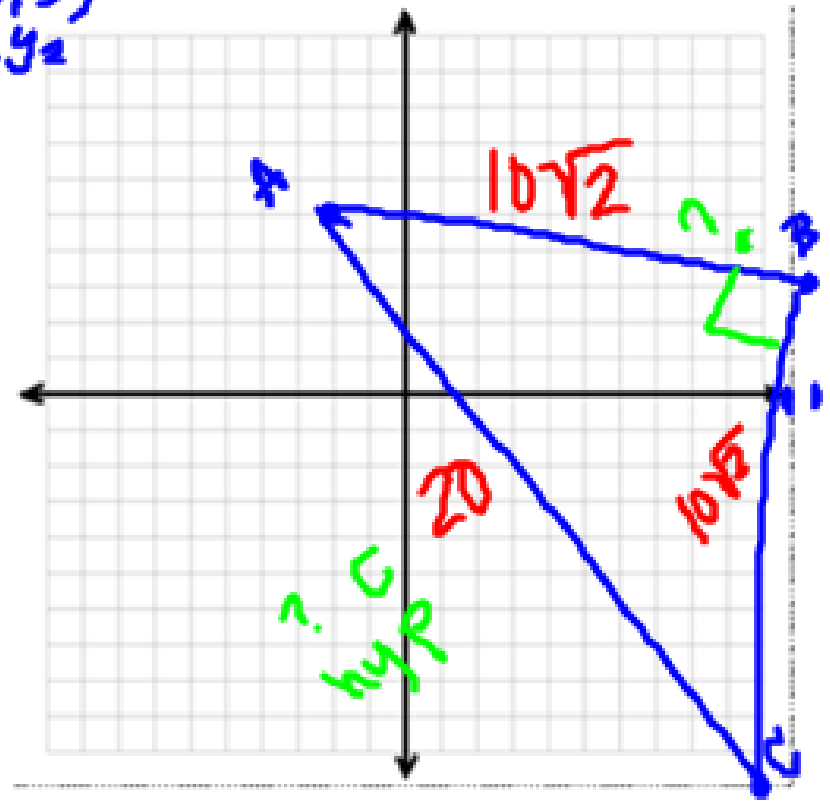
$$= \sqrt{14^2 + (-2)^2}$$

$$= \sqrt{196+4}$$

$$= \sqrt{200} = \sqrt{2 \cdot 100}$$

10 10

$$\overline{AB} = 10\sqrt{2}$$



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Examples: Find the distance between the two points.

C) Consider the three points $A(-2,5)$, $B(12,3)$, $C(10,-11)$

x_1 y_1 x_1 y_1 x_2 y_2 $(10,-11)$

b) Find the length of each side of the triangle. x_2 y_2

$\overline{AC} = d = \sqrt{(10+2)^2+(-11-5)^2}$

$= \sqrt{12^2+(-16)^2}$

$= \sqrt{144+256}$

$= \sqrt{400}$

$\overline{AC} = 20$

$\overline{BC} = d = \sqrt{(10-12)^2+(-11-3)^2}$

$= \sqrt{(-2)^2+(-14)^2}$

$= \sqrt{4+196}$

$= \sqrt{200}$

$\overline{BC} = 10\sqrt{2}$

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Examples: Find the distance between the two points.

C) Consider the three points $A(-2,5)$, $B(12,3)$, $C(10,-11)$

c) Verify the triangle is a right triangle. (Hint: use the Pythagorean Theorem)

$a^2 + b^2 = c^2$

$(10\sqrt{2})^2 + (10\sqrt{2})^2 = 20^2$

yes, right Δ

$100(2) + 100(2) = 400$

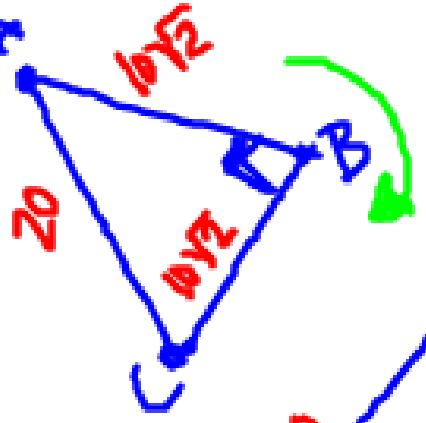
$200 + 200 = 400$

$400 = 400$ ✓

Examples: Find the distance between the two points.

C) Consider the three points $A(-2,5)$, $B(12,3)$, $C(10,-11)$

d) Find the area of the triangle.

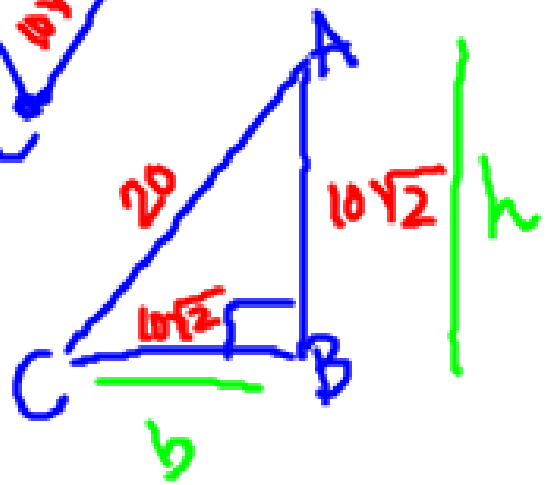


$$A = \frac{1}{2} b h$$

$$= \frac{1}{2} \cdot \frac{10\sqrt{2}}{1} \cdot \frac{10\sqrt{2}}{1}$$

$$= \frac{1}{2} \cdot \frac{100(2)}{1}$$

$$A = 100 \text{ units}^2$$



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The Midpoint Formula:

The midpoint $M = (x, y)$ of the line segment from $P_1 = (x_1, y_1)$ to $P_2 = (x_2, y_2)$ is

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

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Examples: Find the midpoint of the line segment joining the two points.

D) $P_1 = (5, -4)$ $P_2 = (3, 2)$

x_1 y_1 x_2 y_2

$M = \left(\frac{5+3}{2}, \frac{-4+2}{2}\right) \rightarrow \left(\frac{8}{2}, \frac{-2}{2}\right) \rightarrow (4, -1)$

$M: (4, -1)$

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CIRCLE Definitions:

A *circle* is the set of all points in the Cartesian plane that are a fixed distance r from a fixed point (h, k). r is called the *radius*. (h, k) is called the *center*.

The *standard form* of an equation of a circle is

$$(x-h)^2 + (y-k)^2 = r^2$$

$(x-3)^2 + (y+2)^2 = 4$ Center: $(3, -2)$ $r = 2$

Note that since both the h and the k are inside parentheses, we must change the signs of BOTH of them when finding the vertex point (h, k).

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Examples: Write the standard form of the equation of the circle with the following:

h , K

E) $r = 3, \quad C(-2, 5)$

$(x+2)^2 + (y-5)^2 = 9$

F) $r = \sqrt{5}, \quad C(-1, -3)$

$(x+1)^2 + (y+3)^2 = 5$

G) $r = \frac{1}{2}, \quad C(0, 3)$ $\left(\frac{1}{2}\right)^2 = \frac{1}{2} \cdot \frac{1}{2}$

$(x-0)^2 + (y-3)^2 = 1/4$

$x^2 + (y-3)^2 = 1/4$

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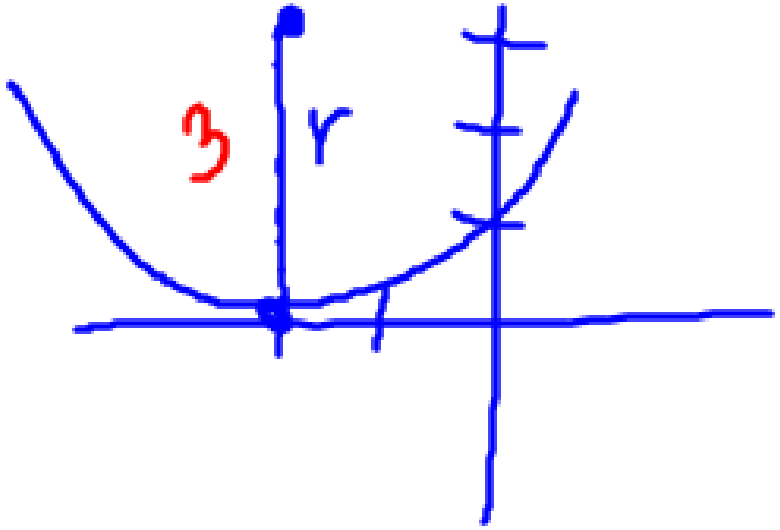
Examples: Write the standard form of the equation of the circle with the following:

H) Center at (-2, 3) and tangent to the x-axis.

$$(x+2)^2 + (y-3)^2 = 9$$

Note: "tangent to" means that the edge of the circle touches the x-axis.



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Examples: Write the standard form of the equation of the circle with the following:

1) with endpoints of a diameter at $(-5, -3)$ and $(7, 2)$

x_1 y_1 x_2 y_2

M: $\left(\frac{-5+7}{2}, \frac{-3+2}{2}\right)$

$\left(\frac{2}{2}, \frac{-1}{2}\right)$

M $\left(1, -\frac{1}{2}\right)$ Center

$d = \sqrt{(7+5)^2 + (2+3)^2}$

$= \sqrt{12^2 + 5^2}$

$= \sqrt{144+25}$

$= \sqrt{169}$

$d = 13$ diameter

$r = \frac{13}{2}$

Circle:

$(x-1)^2 + \left(y+\frac{1}{2}\right)^2 = \left(\frac{13}{2}\right)^2$

$(x-1)^2 + \left(y+\frac{1}{2}\right)^2 = \frac{169}{4}$

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Graphing a Circle:

Step 1: Identify the center (h, k) and the radius r in the equation.

Step 2: Plot the center point. From that point, go the number of units in the radius to the right and plot another point. Plot three more points beginning at the center and going up, down, and left. right

Step 3: Connect the 4 outside points with even arcs to form the circle.

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Examples: Graph the equations. Give Domain and Range.

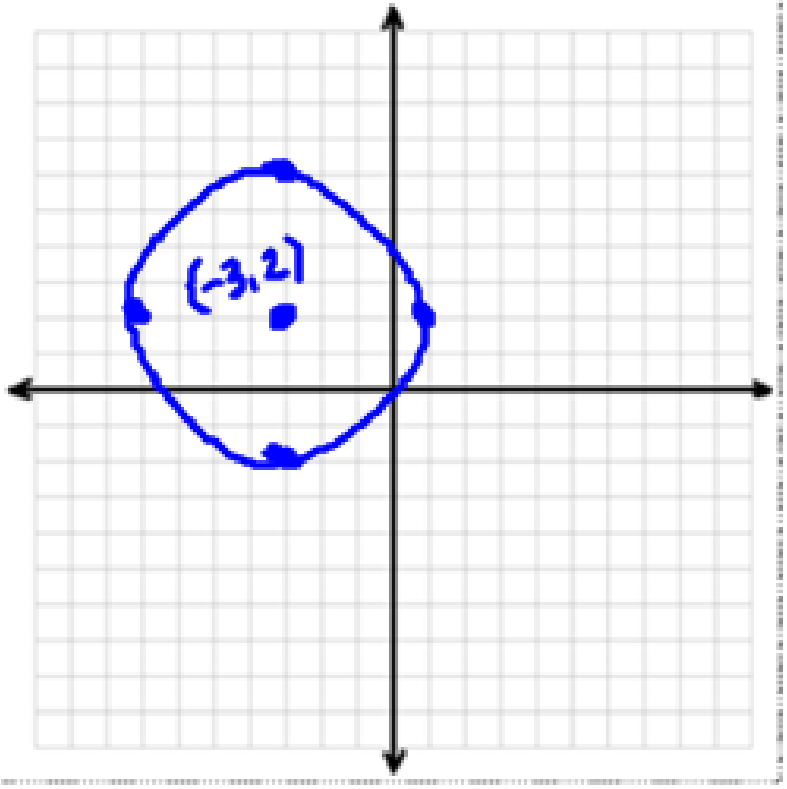
1) $(x+3)^2 + (y-2)^2 = 16$

center : $(-3, 2)$

$r = \sqrt{16} = 4$

D: $[-7, 1]$ or $\{x \mid -7 \le x \le 1\}$

R: $[-2, 6]$ or $\{y \mid -2 \le y \le 6\}$



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Examples: Graph the equations. Give Domain and Range.

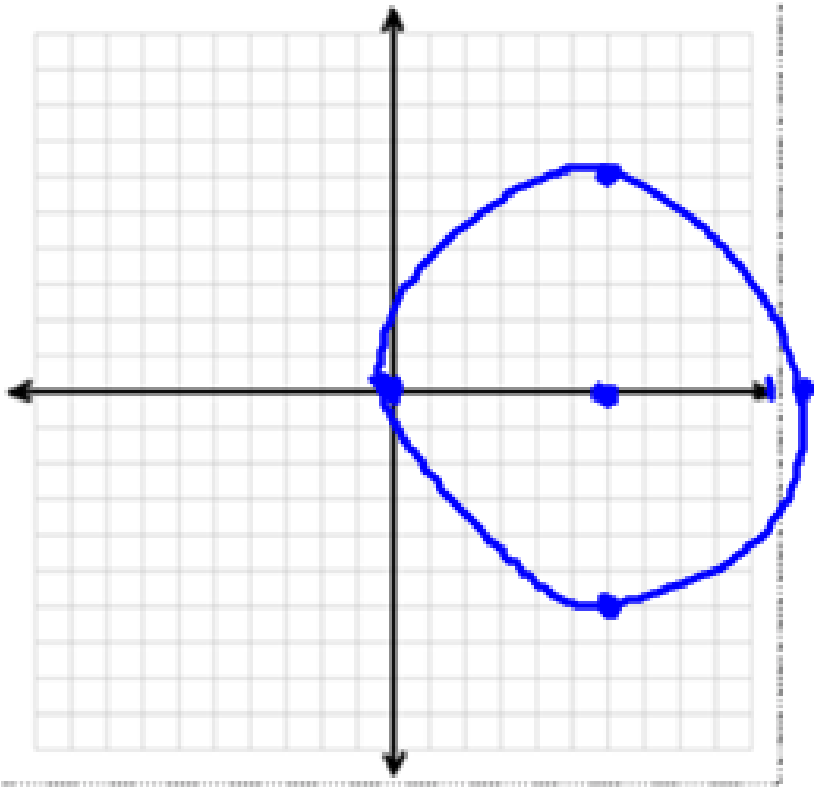
K) $(x-6)^2 + y^2 = 36$

Center: (6, 0)

r = 6

D: [0, 12] or $\{x \mid 0 \le x \le 12\}$

R: [-6, 6] or $\{y \mid -6 \le y \le 6\}$



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Can you?

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