

We are going to be able to:

- ~ Build Linear Models from Data
- ~ Build Linear Models using Direct Variation
- ~ Straight Line Depreciation

REVIEW: What are our steps for problem solving?

1. *Identify* the question being asked.
2. *Define* variables.
3. *Translate* into a mathematical equation.
4. *Solve* the equation and check the reasonableness of the answer.
5. *Answer* the question in a sentence form.

## BUILDING MODELS FROM VERBAL

### DESCRIPTIONS:

If we are talking about linear functions, slope is an important part of the equation. Keep in mind that we can assign a dependent variable ( $Y$ ) and an independent variable ( $X$ ), and look at the changes in  $x$  and  $y$ . This average rate of change of the dependent variable given a constant change in the independent variable describes our slope.

## Lesson 3.5: Building Linear Models

For example, if your cell phone company charges you \$ .05 a minute to talk, the slope of the function would be  $m = .05/1$

## Lesson 3.5: Building Linear Models

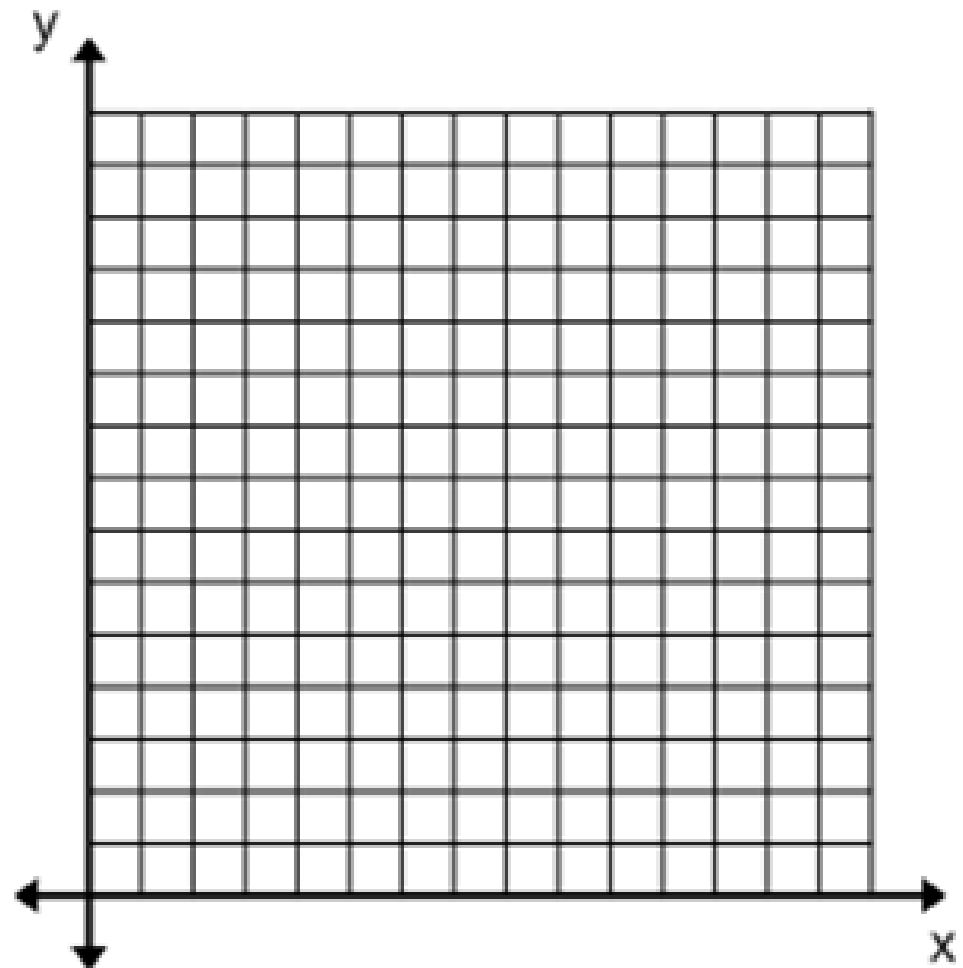
**EXAMPLE:**      Cost function: A simple cost function can be described as  $C(x) = ax + b$ , where  $b$  represents the fixed costs of a business, and  $a$  represents the variable cost (like the cost for each item manufactured). Suppose a small bicycle factory has daily fixed costs of \$2000, and each bicycle costs \$80 to manufacture.

A) Write a linear function that expresses the cost of manufacturing  $x$  bicycles in a day.

Lesson 3.5: Building Linear Models

B) Graph the linear function.

(Label the horizontal axis as number of bikes, the vertical axis as cost).



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C) What is the cost of manufacturing 12 bikes in one day? (Plug 12 in for  $x$ )

D) How many bicycles can be manufactured for \$3520? (Solve for  $x$ )

## Depreciation:

Depreciation is a reduction in value over time on a specific item, like cars. Many companies use a linear model to model depreciation of assets.



### Lesson 3.5: Building Linear Models

EXAMPLE 2: Suppose that a publishing company purchased a new fleet of cars for their sales people, and each car cost \$29,400. The company will depreciate the cars over 7 years using a straight-line model, so that each car depreciates by  $\$29,400/7 = \$4200$  per year.

A) Write the linear function that expresses the book value  $V$  of each car as a function of its age,  $x$ .

## Lesson 3.5: Building Linear Models

*B) What is the implied domain of the function?*

*C) What is the book value of each car after 3 years?*

*D) When will the book value of the car be \$21,000?*

## Buildin Linear Models Involving Direct Variation

Variation refers to how one quantity varies in relation to some other quantity.

Supposed  $X$  and  $Y$  represent two quantities. We say that  $Y$  varies **DIRECTLY** with  $X$ , or  $Y$  is directly proportional to  $X$ , if there is some number,  $k$ , such that

$$y = kx$$

The number  $k$  is called the constant of Proportionality.

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The number  $k$  is called the constant of Proportionality.

Think about what this means.

- If we're looking for a constant proportion in the rate of change, this means  $k$  is the slope of the line, and there is no  $y$ -intercept.

So to find  $k$ , find the rate of change as a proportion of  $y/x$ .

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EXAMPLE 3: Car payments: Suppose that Jeff just bought a used car for \$10,000. He decides to put \$1000 down on the car and borrow the rest. The bank lends Jeff the \$9000 he needs at 4.9% interest for 48 months. His payments are \$206.86. The monthly payment  $p$  on a car always varies directly with the amount borrowed ( $b$ ).

- A) Find a function that relates the monthly payment ( $p$ ) to the amount borrowed ( $b$ ) for any car loan with the same terms.

### Lesson 3.5: Building Linear Models

A) Find a function that relates the monthly payment ( $p$ ) to the amount borrowed ( $b$ ) for any car loan with the same terms.

B) If Jeff put down \$2000, what would his payment be?

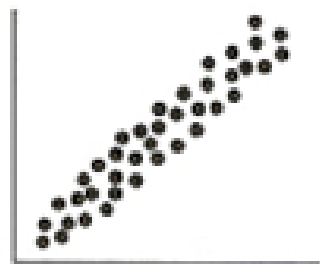
## BUILDING LINEAR MODELS FROM DATA

We often try to determine if data can be predicted using a linear model. One way to do this is to make a scatter diagram or scatter plot. We do this by plotting data points based on the value of the  $x$  and its  $y$ .

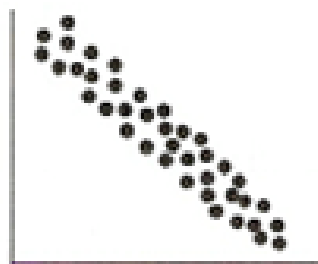
Looking at the pattern of data gives us an idea whether we can use a linear function as a model.

## Lesson 3.5: Building Linear Models

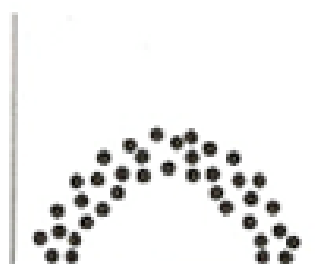
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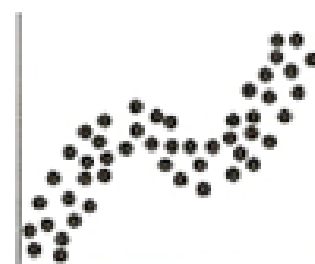
(a) Linear  
 $y = mx + b, m > 0$



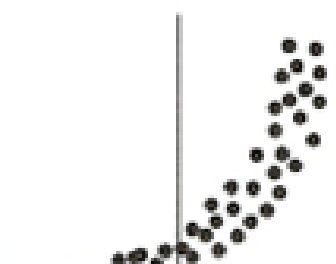
(b) Linear  
 $y = mx + b, m < 0$



(c) Nonlinear



(d) Nonlinear

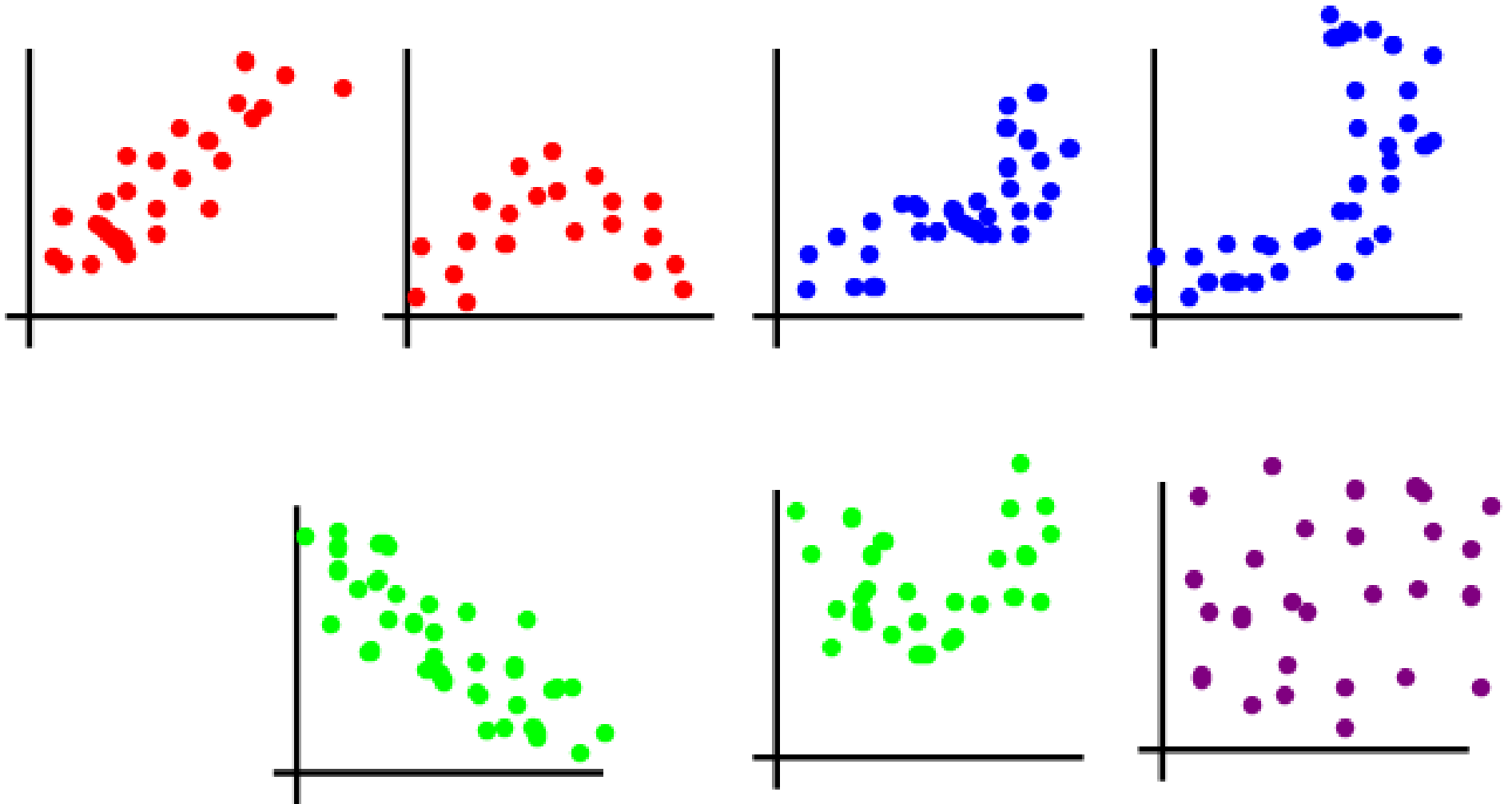


(e) Nonlinear



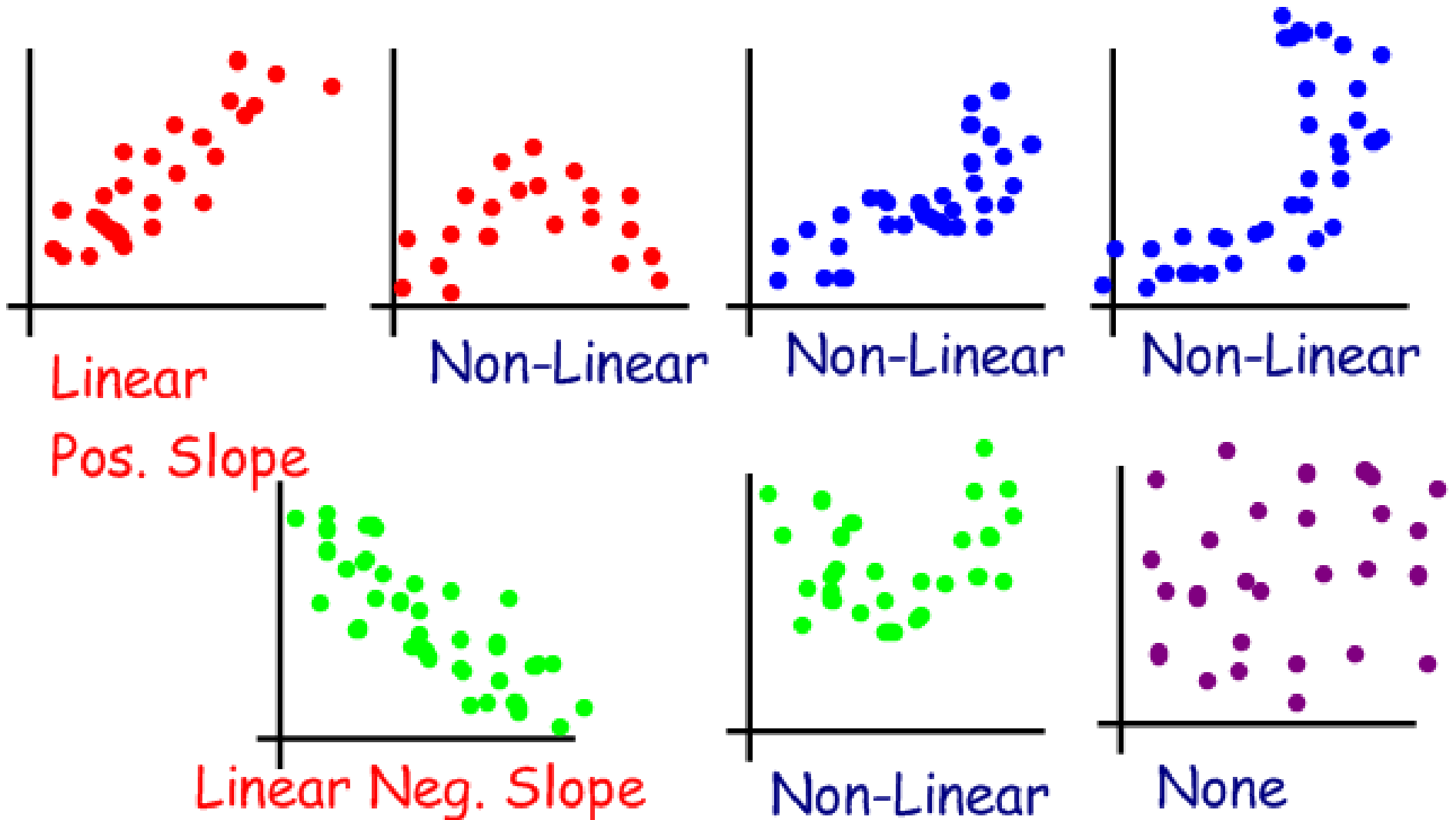
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The following are Scatter plots. What kind of relationship can you see?



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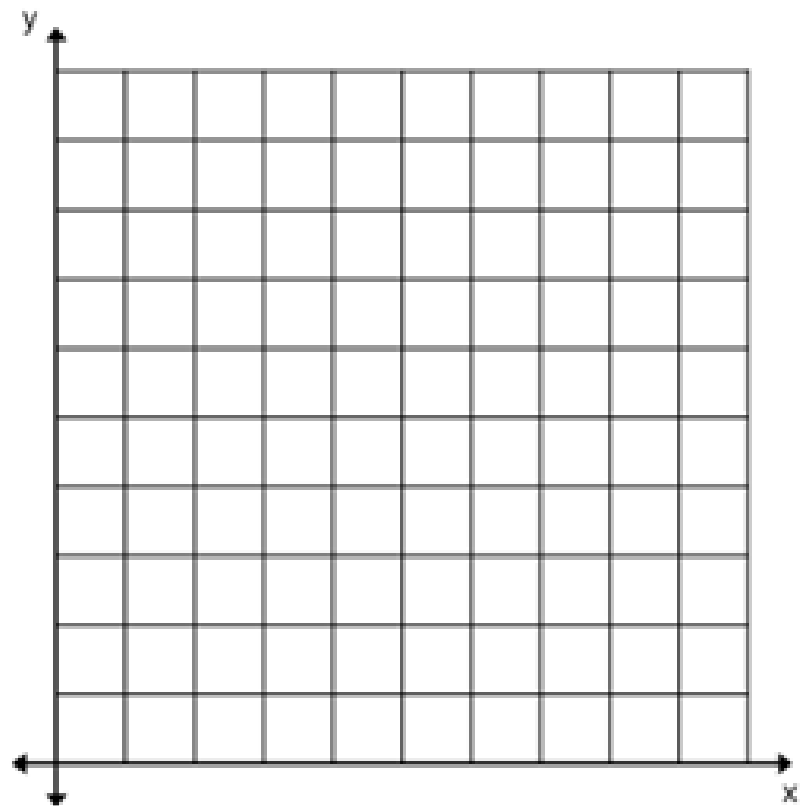


### Lesson 3.5: Building Linear Models

**EXAMPLE 4:** A company looked at the age of their employees ( $x$ ) and the number of sick days ( $y$ ) they took in a year to see if there was a relationship.

Age	20	28	40	47	50
Sick days	15	10	9	4	3

A) Draw the Scatter Diagram using age as the independent variable



## FITTING A LINE TO THE DATA:

(Finding a line of Best Fit)

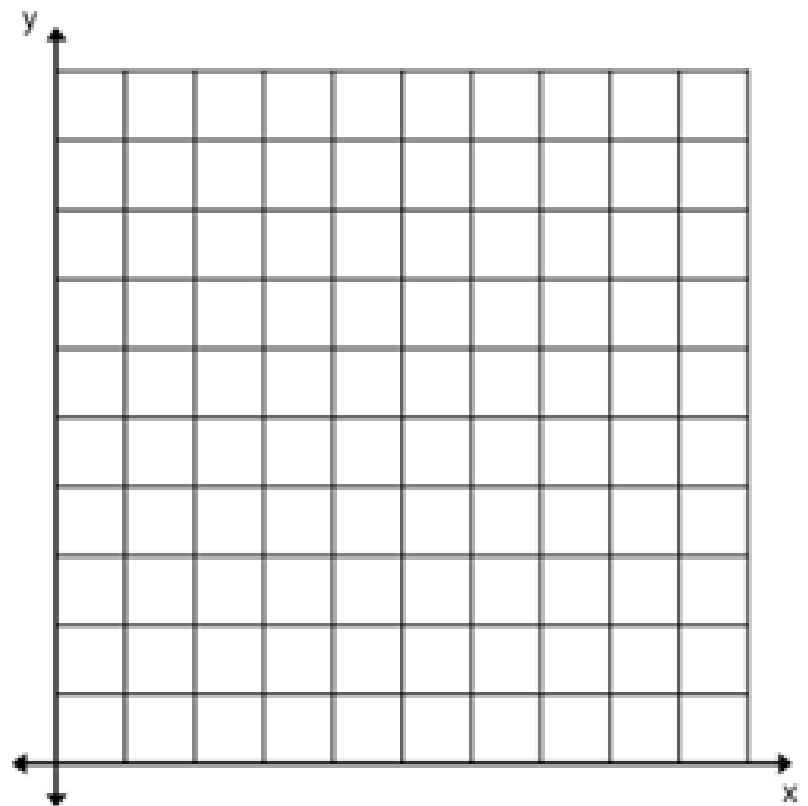
- Step 1:** Choose 2 points that look fairly central, and find the equation of the line containing the points (use the point-slope form).
- Step 2:** Graph the line on your scatter diagram.
- Step 3:** Use the graph to predict data based on your line of best fit (or prediction line).
- Step 4:** Interpret your slope. Does the y-intercept make sense?

### Lesson 3.5: Building Linear Models

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## Lesson 3.5: Building Linear Models

- B) Find the equation for a line of best fit.
- C) Predict the number of sick days for someone 45 years old.

## Lesson 3.5: Building Linear Models

B) Find the equation for a line of best fit.

Choose (28, 10) and (47, 4)  $m = \frac{10-4}{28-47} = \frac{6}{-19} = -.3158$

$$y - 10 = -.3158(x - 28) \rightarrow y - 10 = -.3158x + 8.8424 \rightarrow y = -.3158 + 18.8424$$

C) Predict the number of sick days for someone 45 years old.

Predict the number of sick days for someone 45 years old.

- Three ways:
1. See if the value is in the original data set.
  2. Graph the line and find  $y$  when  $x=45$
  3. plug in  $x=45$  and solve.

$$y = -.3158(45) + 18.8424 = 13.4738 \text{ or between 13 and 14 days}$$

## NOTE:

Everyone's line of prediction will be different depending on which two points they choose, or how they round values.

When looking at a test with multiple answers, choose the one that is closest to yours.



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- ~ Build Linear Models using Direct Variation
- ~ Straight Line Depreciation

Can you?

# Homework:

Pg. 239: #'s 1-6 all, 7, 9, 13,  
15, 21, 23, 25, 29

\*Remember "Average Rate of  
Change" is Slope.