We are going to be able to:

~ Build Linear Models from Data
~ Build Linear Models using Direct Variation
~ Straight Line Depreciation

REVIEW: What are our steps for problem solving?

1. *Identify* the question being asked.
2. *Define* variables. $x =$ $y =$
3. *Translate* into a mathematical equation.
4. *Solve* the equation and check the reasonableness of the answer.
5. *Answer* the question in a sentence form.

BUILDING MODELS FROM VERBAL DESCRIPTIONS:

If we are talking about linear functions, slope is an important part of the equation. Keep in mind that we can assign a dependent variable (Y) and an independent variable (X), and look at the changes in x and y. This average rate of change * of the dependent variable given a constant change in the independent variable describes our slope.

For example, if your cell phone company charges you $ .05 a minute to talk, the slope of the function would be $m=.05/1$

EXAMPLE: Cost function: A simple cost function can be described as $C(x) = ax + b$, where b represents the fixed costs of a business, and a represents the variable cost (like the cost for each item manufactured). Suppose a small bicycle factory has daily fixed costs of $2000, and each bicycle costs $80 to manufacture.

A) Write a linear function that expresses the cost of manufacturing x bicycles in a day.

$$C(x) = 80x + 2000$$

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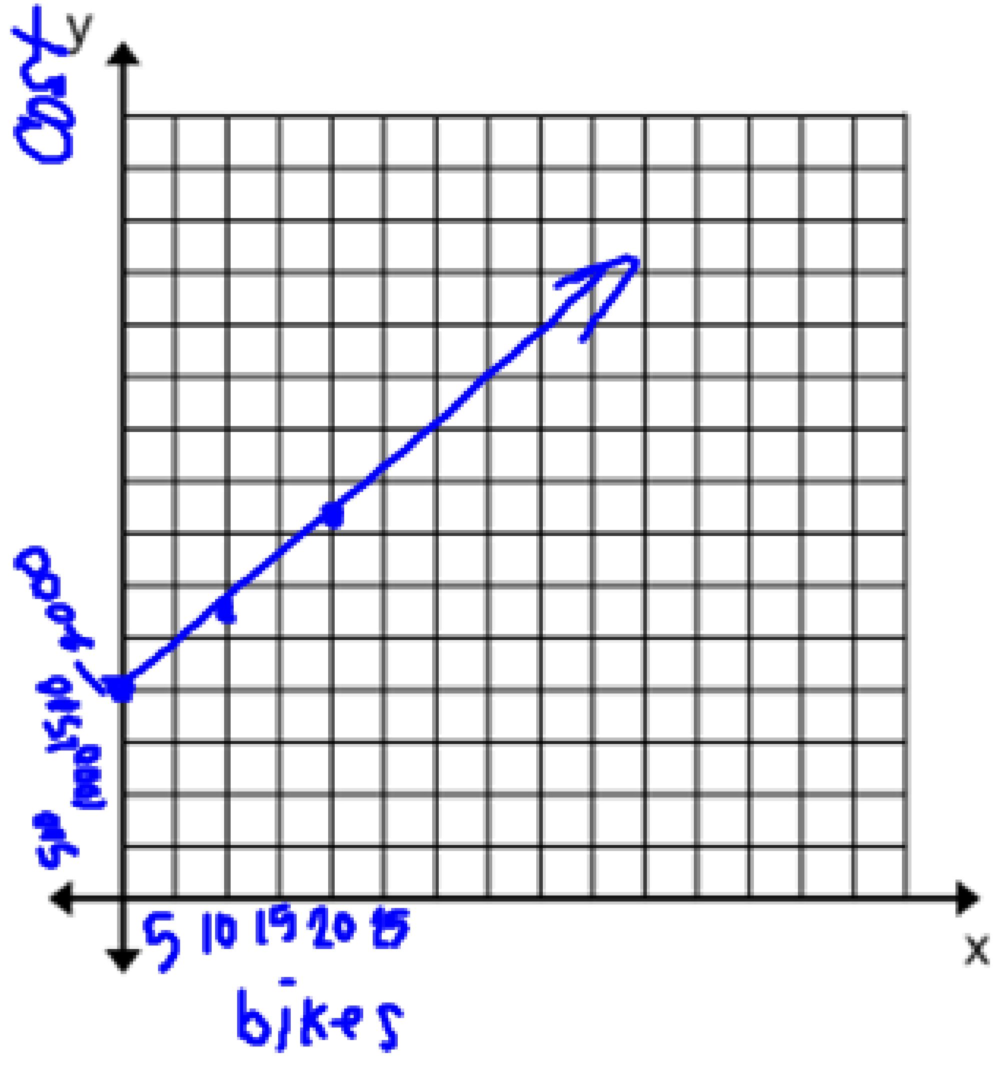
B) Graph the linear function.

(Label the horizontal axis as number of bikes, the vertical axis as cost).

$m = \frac{80}{1} = \frac{800}{10}$

or

x	y
10	2800
20	3600



C) What is the cost of manufacturing 12 bikes in one day? (Plug 12 in for x)

$C(12) = 80(12) + 2000$

$C(12) = 2960$

The cost is \$2960.

D) How many bicycles can be manufactured for \$3520? (Solve for x)

$3520 = 80x + 2000$

$-2000 \qquad -2000$

$\frac{1520}{80} = \frac{80x}{80} \rightarrow x = 19$

We can manufacture 19 bikes.

Depreciation:

Depreciation is a reduction in value over time on a specific item, like cars. Many companies use a linear model to model depreciation of assets.

EXAMPLE 2: Suppose that a publishing company purchased a new fleet of cars for their sales people, and each car cost $29,400. The company will depreciate the cars over 7 years using a straight-line model, so that each car depreciates by $29,400/7 = $4200 per year.

A) Write the linear function that expresses the book value V of each car as a function of its age, x.

$V(x) = -4200x + 29{,}400$

B) What is the implied domain of the function?

$\{x \mid x \geq 0\}$ or $[0, \infty)$

C) What is the book value of each car after 3 years?

$V(3) = -4200(3) + 29400$

$V(3) = \$16{,}800$

The cars are worth \$16,800 after 3 yrs.

D) When will the book value of the car be $21,000?

$21{,}000 = -4200x + 29400$

$-29{,}400 \qquad -29400$

$\frac{-8{,}400}{-4200} = \frac{-4200x}{-4200} \rightarrow x = 2$

After 2 yrs

Buildin Linear Models Involving

Direct Variation

Variation refers to how one quantity varies in relation to some other quantity.

Supposed X and Y represent two quantities. We say that Y varies DIRECTLY with X, or Y is directly proportional to X, if there is some number, k, such that

$$y = kx$$

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The number *k* is called the constant of Proportionality.

Think about what this means.

- If we're looking for a constant proportion in the rate of change, this means k is the slope of the line, and there is no y-intercept.

So to find k, find the rate of change as a proportion of y/x.

EXAMPLE 3: ***Car payments:*** Suppose that Jeff just bought a used car for \$10,000. He decides to put \$1000 down on the car and borrow the rest. The bank lends Jeff the \$9000 he needs at 4.9% interest for 48 months. His payments are \$206.86. The monthly payment p on a car always varies directly with the amount borrowed (b).

$p = kb$

A) Find a function that relates the monthly payment (p) to the amount borrowed (b) for any car loan with the same terms.

$$\frac{206.86}{9000} = \frac{k(9000)}{9000} \rightarrow k = .022984$$

A) Find a function that relates the monthly payment (p) to the amount borrowed (b) for any car loan with the same terms.

$$p = .022984\, b$$

B) If Jeff put down $2000, what would his payment be?

$b = 8000$

$$p = .022984(8000)$$

$$p = 183.88$$

payment is $183.88

BUILDING LINEAR MODELS FROM DATA

We often try to determine if data can be predicted using a linear model. One way to do this is to make a *scatter diagram* or *scatter plot*. We do this by plotting data points based on the value of the x and it's y.

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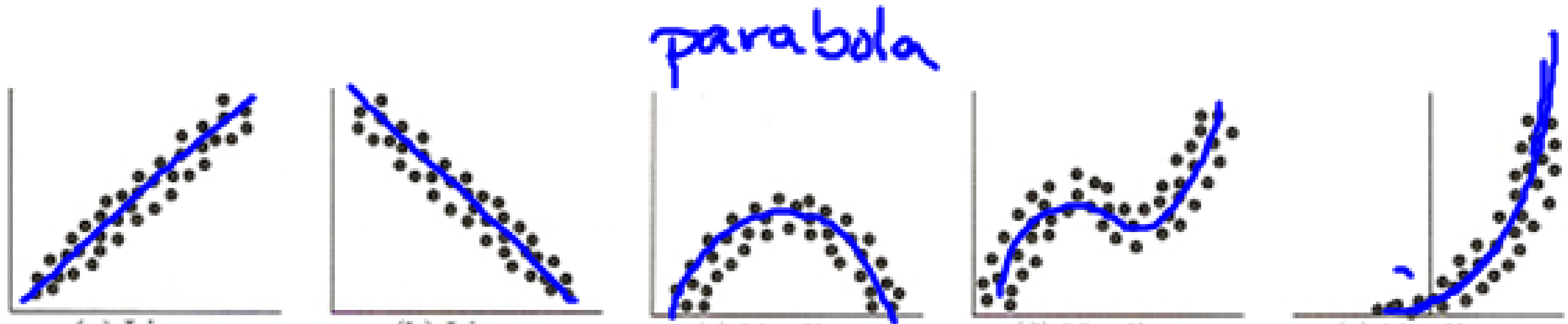


(a) Linear $y = mx + b, m > 0$

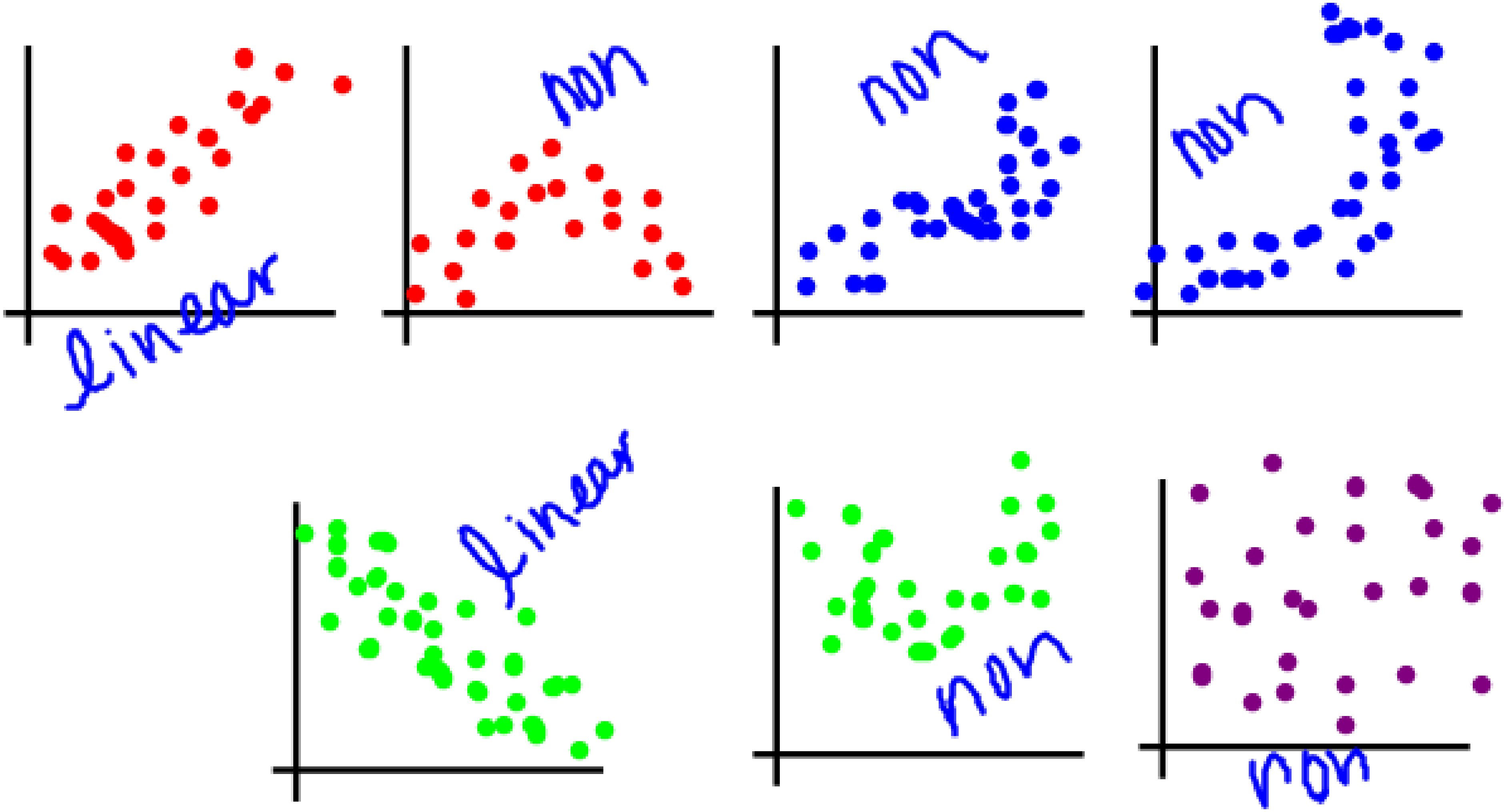
(b) Linear $y = mx + b, m < 0$

(c) Nonlinear

(d) Nonlinear

(e) Nonlinear

The following are Scatter plots. What kind of relationship can you see?



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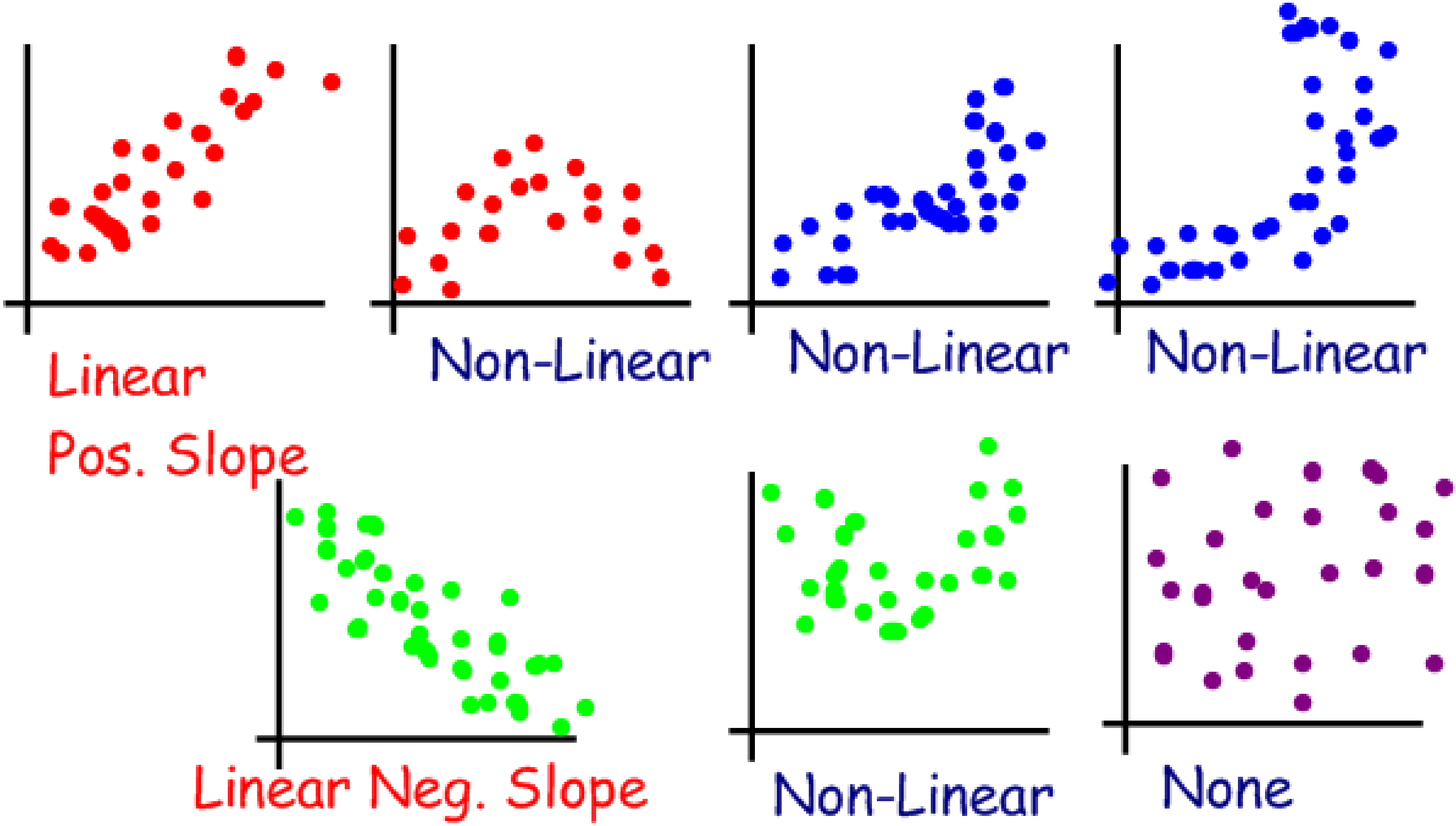
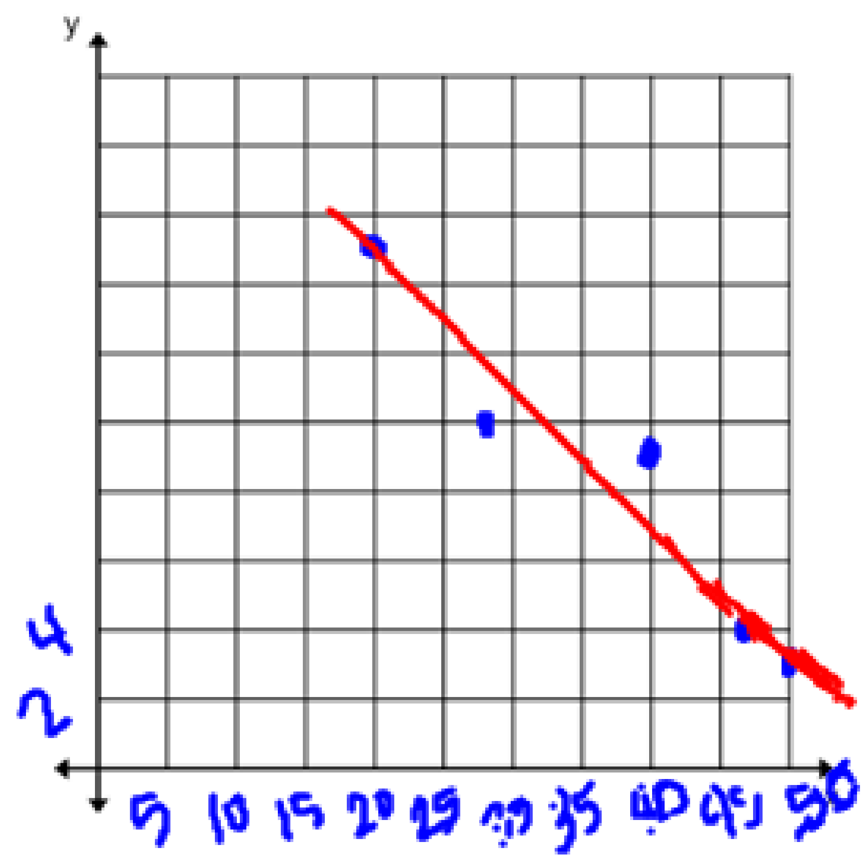


EXAMPLE 4: A company looked at the age of their employees (x) and the number of sick days (y) they took in a year to see if there was a relationship.

Age x	20	28	40	47	50
Sick days y	15	10	9	4	3

A) Draw the Scatter Diagram using age as the independent variable



FITTING A LINE TO THE DATA:

(Finding a line of Best Fit)

Step 1: Choose 2 points that look fairly central, and find the equation of the line containing the points (use the point-slope form).

Step 2: Graph the line on your scatter diagram.

Step 3: Use the graph to predict data based on your line of best fit (or prediction line).

Step 4: Interpret your slope. Does the y-intercept make sense?

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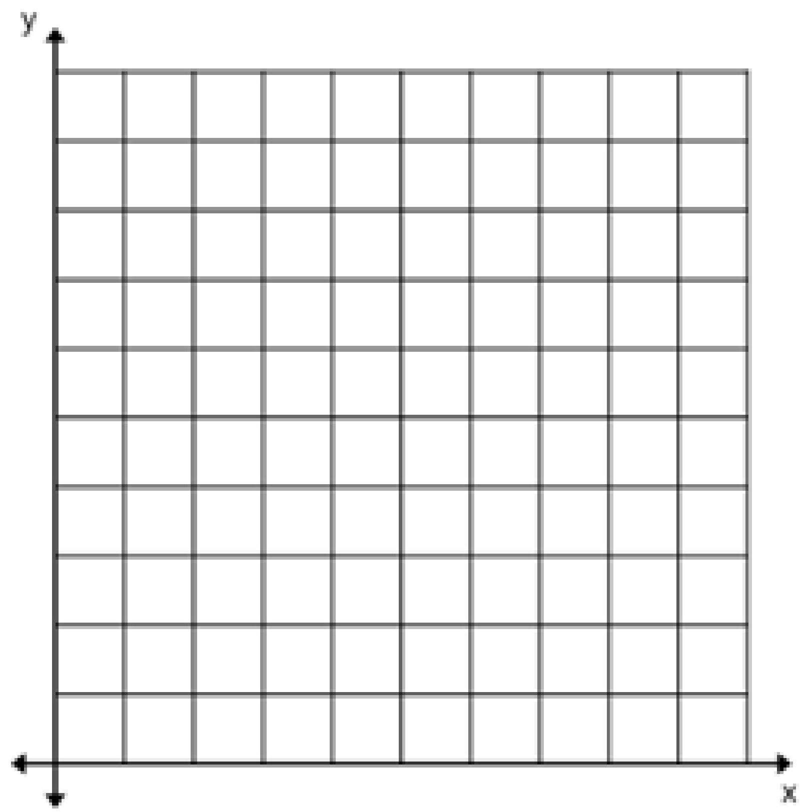

B) Find the equation for a line of best fit.

$(20, 15)$ $(47, 4)$ — x_1, y_1; x_2, y_2

$m = \frac{4-15}{47-20} = \frac{-11}{27} = -.41$

$y - 15 = -.41(x - 20)$

$y - 15 = -.41x + 8.2$

$+15 \qquad +15$

$y = -.41x + 23.2$

C) Predict the number of sick days for someone 45 years old.

$y = -.41(45) + 23.2$

$y = 4.75$

5 sick days

B) Find the equation for a line of best fit.

Choose (28, 10) and (47, 4) $m = \frac{10-4}{28-47} = \frac{6}{-19}$ = -.3158

$y - 10 = -.3158(x - 28) \rightarrow y - 10 = -.3158x + 8.8424 \rightarrow y = -.3158 + 18.8424$

C) Predict the number of sick days for someone 45 years old.

Predict the number of sick days for someone 45 years old.

Three ways:
1. See if the value is in the original data set.
2. Graph the line and find y when x=45
3. plug in x=45 and solve.

$y = -.3158(45) + 18.8424 = 13.4738$ *or between* 13 *and* 14 *days*

NOTE:

Everyone's line of prediction will be different depending on which two points they choose, or how they round values.

When looking at a test with multiple answers, choose the one that is closest to yours.

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Can you?

Homework:

Pg. 239: #'s 1-6 all, 7, 9, 13,
15, 21, 23, 25, 29

*Remember "Average Rate of Change" is Slope.