By the end of the lesson, we will be able to:

~Determine if an ordered pair is a solution to a system of equations.

~Solve a system of two linear equations by Graphing.

~Solve a system of two linear equations by Substitution.

~Solve a system of two linear equations by Elimination.

~Identify Inconsistent Systems.

~Express the solutions of a System of Dependent Equations.

A **System of Linear Equations** is a grouping of two or more linear equations - each of which contains one or more variables.

Examples:

$$\begin{cases} 2x + y = 5 \\ x - 5y = -10 \end{cases}$$

$$\begin{cases} x + 3y + z = 8 \\ 3x - y + 6z = 12 \\ -4y - y + 2z = -1 \end{cases}$$

* We are only going to work with equations with two variables today.

A **Solution** of a system of equations consists of values for the variables that are solutions of each equation of the system.

When solving systems of two linear equations containing two unknowns, we represent the solution as an ordered pair, (x, y).

Remember that the ***Solution* is the value of the variable(s) that make ***each*** equation a true statement.**

Determine if the ordered pair is a solution to the system of linear equations.

$$\begin{cases} 2x + 3y = 9 \\ -5x - 3y = 0 \end{cases}$$

a. (6, -1)

$2 \cdot 6 + 3 \cdot -1$

$12 + -3$

$3 = 9$ ✓

$-5 \cdot 6 - 3 \cdot -1$

$-30 + 3 = 0$ ✗

NO

b. (-3, 5)

$2(-3) + 3 \cdot 5 = 9$

$-6 + 15 = 9$

$9 = 9$

$-5(-3) - 3(5) = 0$

$15 - 15 = 0$

$0 = 0$

yes

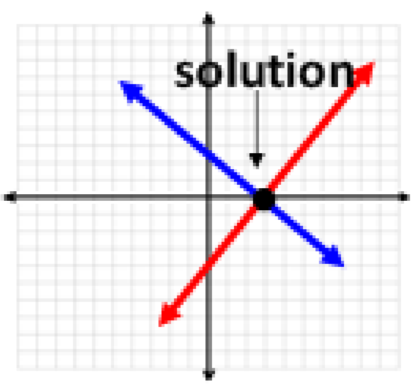
Lesson 4.1: Systems of Linear Equations in Two Variables

Visualizing the Solutions in a System of Two Linear Equations Containing Two Unknowns (Variables)

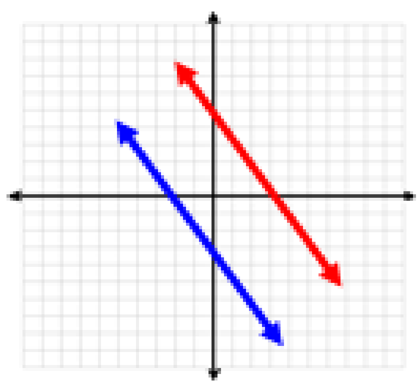
We can view the problem of solving a system of equations visually in three different ways - since they are all lines.

1. **INTERSECT:** If the lines intersect, then the system of equations has one solution given by the point of intersection. We say that the system is **consistent** and the equations are **independent**.
2. **PARALLEL:** If the lines are parallel, then the system of equations has NO solutions because the lines never intersect. In this circumstance, we say that the system is **inconsistent**.
3. **COINCIDENT:** If the lines lie on top of each other (are coincident), then the system of equations has infinitely many solutions. The solution set is the set of all points on the line. The system is **consistent** and the equations are **dependent**.

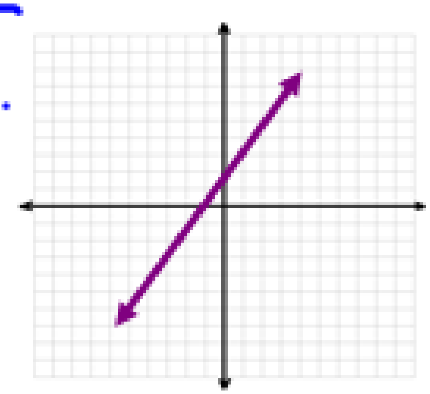
1.



2.



3.



Solving by Graphing: We can graph the two lines and find where they intersect - this is the solution.

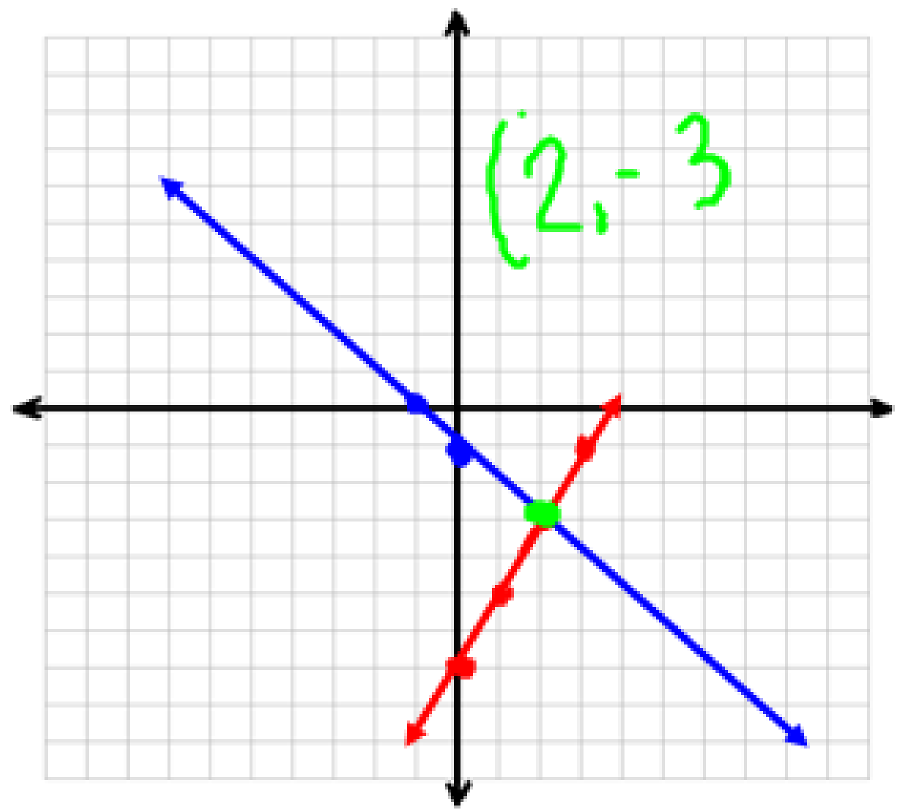
Example 1:

$$\begin{cases} x + y = -1 \\ -2x + y = -7 \end{cases}$$

x-int (-1, 0) y-int (0, -1)

$y = 2x - 7$

$(2, -3)$



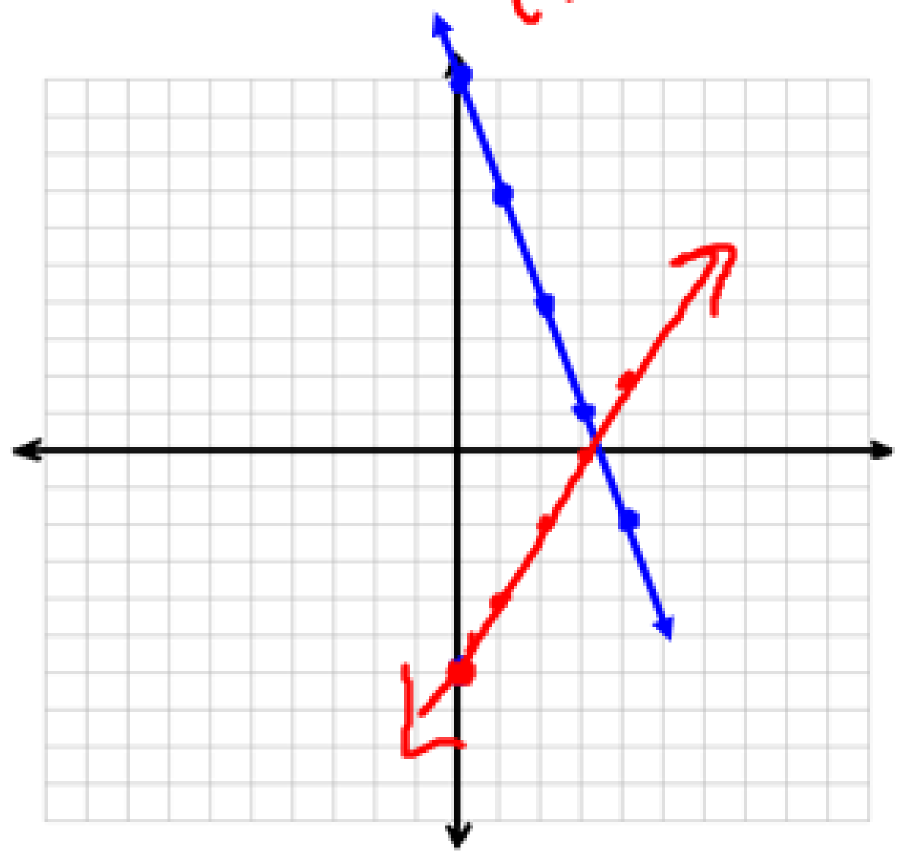
Solving by Graphing: We can graph the two lines and find where they intersect - this is the solution.

Example 2:

$$\begin{cases} y = -3x + 10 \\ y = 2x - 6 \end{cases}$$

Hard to find do algebraically

Calc
(3.2, .4)



It's not always easy to graph, so we have some other ways to solve systems of equations. These are algebraic ways.

~Substitution~

~Elimination~

Substitution:

Step 1:	Solve one of the equations for one variable. Solve for y in this example.	System: $-x + y = 1$ $3x + 2y = -3$ Step 1: $y = x + 1$ $(first\ equation)$
Step 2:	SUBSTITUTE the expression found in Step 1 into the **OTHER** (second) equation.	Step 2: $3x + 2(x + 1) = -3$
Step 3:	Solve the linear equation in one variable found in Step 2.	Step 3: $3x + 2x + 2 = -3$ $5x + 2 = -3$ $5x = -5$ $x = -1$
Step 4:	Substitute the value of the variable into the expression found in Step 1 to find the value of the other variable.	Step 4: $y = (-1) + 1$ $y = 0$
Step 5:	Write answer in point form!!! Check your answer.	Step 5: $(-1, 0)$ Check: (You plug -1 in for x and 0 in for y.... make sure the equations are EQUAL!)

Substitution:

$$\begin{cases} 2x + y = 4 \\ 3x + 2y = 1 \end{cases}$$

$2x + y = 4 \rightarrow y = -2x + 4$

$3x + 2(-2x + 4) = 1$

$3x - 4x + 8 = 1$

$\quad\quad\quad\; -8 \quad -8$

$\frac{-x}{-1} = \frac{-7}{-1}$

$x = 7$

$y = -2(7) + 4$

$y = -14 + 4$

$y = -10$

$(7, -10)$

Lesson 4.1: Systems of Linear Equations in Two Variables

Elimination:

Step 0:	Align the equations with like terms in columns (if necessary).	System: $-3x + y = 1$ $2x + 2y = 10$
Step 1:	Multiply both sides of one or both equations by a number so that there are opposite coefficients for one of the variables. (Ex.: 3x and -3x have opposite coefficients- they equal zero when added.)	Step 1: $-3x + y = 1$ $(Mult. by -2)$ $2x + 2y = 10$ $\mathbf{6x - 2y = -2}$ $+\ \underline{2x + 2y = 10}$
Step 2:	Add equations together (like terms) to eliminate the variable.	Step 2: $6x - 2y = -2$ $+\ \underline{2x + 2y = 10}$ $8x \quad = 8$
Step 3:	Solve the resulting equation for the variable that is left.	Step 3: $8x = 8$ $x = 1$
Step 4:	Substitute the value of the variable found in Step 3 into one of the ORIGINAL equations to find the value of the remaining variable.	Step 4: $-3(1) + y = 1$ $-3 + y = 1$ $y = 4$
Step 5:	Write answer in point form!!! Check your answer.	Step 5: $(1,4)$ Check: (You plug 1 in for x and 4 in for y.... make sure the equations are EQUAL!)

Elimination:

$$\begin{cases} (3x - 2y = -3)\cdot -1 \rightarrow -3x + 2y = 3 \\ 3x + y = 3 \longrightarrow 3x + y = 3 \end{cases}$$

$$3y = 6$$

$$\frac{3y}{3} = \frac{6}{3}$$

$$y = 2$$

$$3x + 2 = 3$$

$$-2 \quad -2$$

$$3x = 1$$

$$\frac{3x}{3} = \frac{1}{3}$$

$$x = 1/3$$

$$\left(\frac{1}{3}, 2\right)$$

When should I use *Graphing*, *Substitution*, or Elimination?

Method	Advantages/Disadvantages	When Should I Use It?
Graphical	Allows us to "see" the answer but if the solutions are not integers, it can be difficult to determine the solution.	When a visual solution is required.
Substitution	Method gives exact solutions. The algebra can be easy provided one of the variables has a coefficient of 1. If none of the coefficients are one, the algebra can get messy.	If one of the coefficients of the variables is 1 or one of the variables is already solved for (as in "x = " or "y = ").
Elimination	Method gives exact solutions. It is easy to use when none of the variables has a coefficient of 1.	If both equations are in standard form ($Ax + By = C$).

Examples: Which method would be best?

$$\begin{cases} x + 3y = 8 \\ \frac{1}{3}x + y = 9 \end{cases}$$

Substitution

$$\begin{cases} 4p + 5q = 7 \\ 3p - 2q = 34 \end{cases}$$

elimination

Solve with elimination:

$$\begin{cases}(2x + 3y = 4)2 \rightarrow & 4x + 6y = 8 \\ -4x - 6y = 1 \rightarrow & -4x - 6y = 1\end{cases}$$

$0 = 9$ False

$0 \neq 9$

No Solution

Type: Parallel, Inconsistent

When we get a false statement (like 9=0), the system has no solution. The system is then considered INCONSISTENT.

The graph of the lines will be parallel.

Solve with Substitution:

$$\begin{cases} 3x + y = 1 \rightarrow y = -3x + 1 \\ -6x - 2y = -2 \end{cases}$$

$-6x - 2(-3x + 1) = -2$

$-6x + 6x - 2 = -2$

$-2 = -2$ True

Infinite Solutions

Type: Coincident, Consistent + dependent

When we get a true statement (like -2 = -2), the system has infinite solutions. The system is then considered CONSISTENT and DEPENDENT.

The graph of the lines will be the same line.

Lesson 4.1: Systems of Linear Equations in Two Variables

Homework:

2-6 all 11, 15, 17, 19

Pg. 266: #'s ~~1-10 all~~, ~~11-19 odds~~, 27, 29, 39, 45, 57, ~~59~~

AND Pg. 254: #'s 3, 6, 14, 15, 17, 21