By the end of the lesson, we will be able to:

~Solve systems of Three Linear Equations containing three variables.

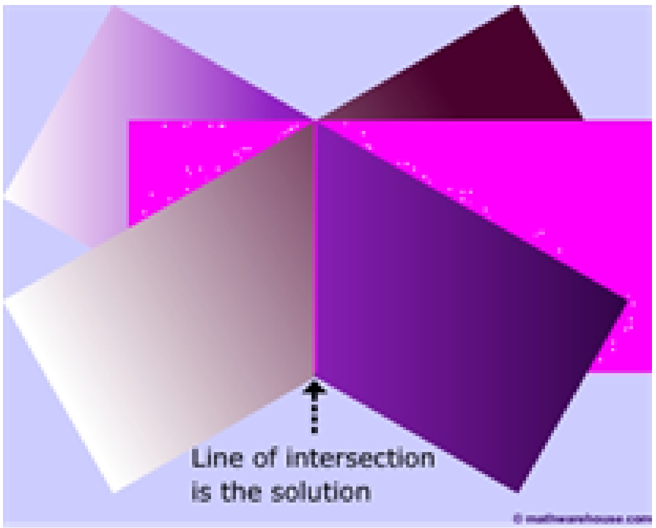
~Solve Application Problems with systems of three equations.

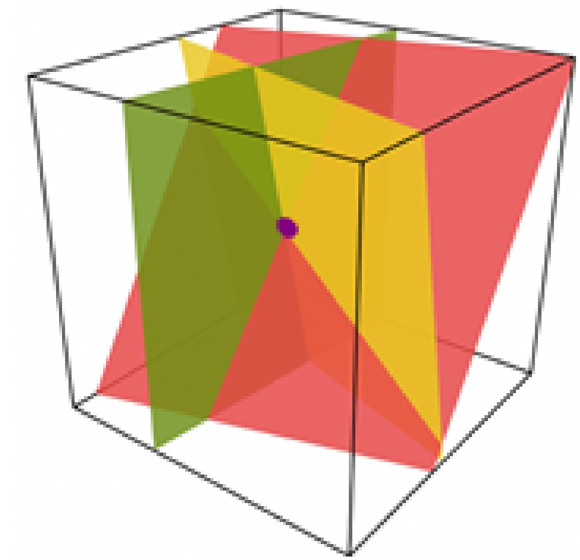
Systems of three linear equations containing three variables have the same possible solutions as a system of two linear equations containing two variables:

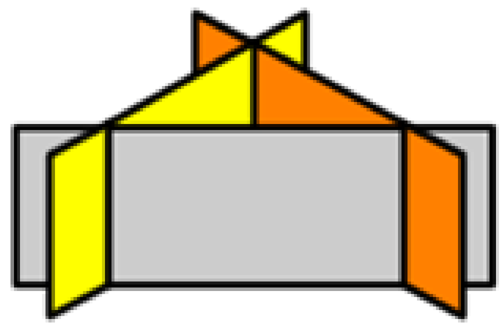
1. **Exactly one solution** - *A* consistent system with independent equations
2. **No solution** - *An* inconsistent system
3. **Infinitely many solutions** - *A* consistent system with dependent equations

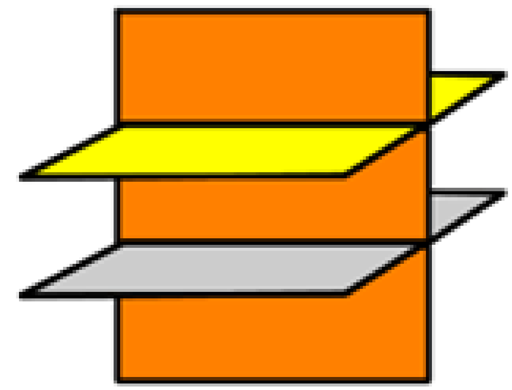
Lesson 4.3: Systems of Linear Equations in Three Variables

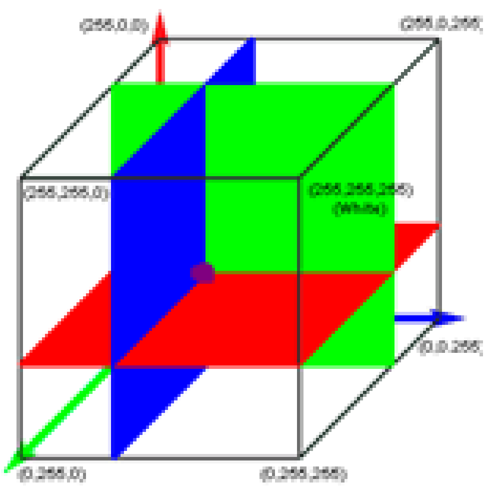
Which is which kind of solution?

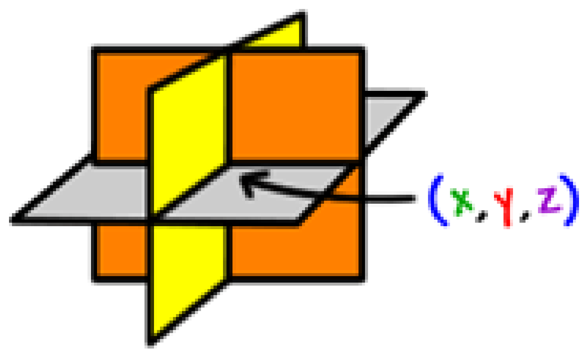


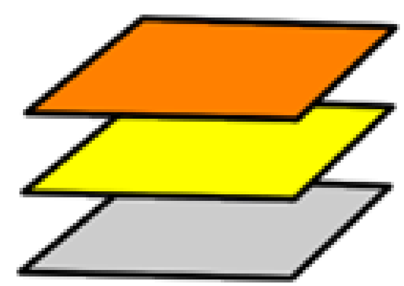




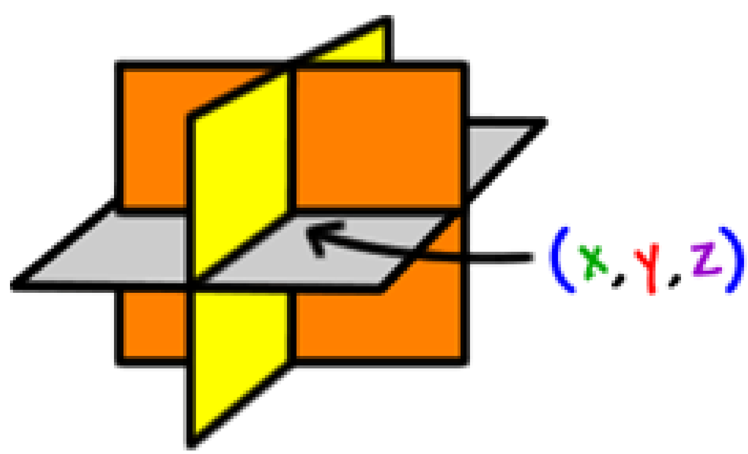








A **solution** to a system of equations consists of values for the variables that are solutions of EACH equation of the system. We write the solution to a system of three equations containing three unknowns as an **ordered triple** (x,y,z).



Determine whether the values are solutions to the system of linear equations.

$$\begin{cases} x + y + z = 0 \\ 2x - y + 3z = 17 \\ -3x + 2y - z = -21 \end{cases}$$

a.) (1, 3, -4) $\boxed{\text{NO}}$

$1 + 3 - 4 = 0$

$0 = 0$ ✓

$2(1) - 3 + 3(-4) = 17$

$-13 \neq 17$ ✗

$-3(1) + 2(3) - (-4) = -21$

b.) (3, -5, 2) $\boxed{\text{yes}}$

$3 - 5 + 2 = 0$

$0 = 0$ ✓

$2(3) - (-5) + 3(2) = 17$

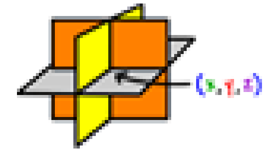
$17 = 17$ ✓

$-3(3) + 2(-5) - 2 = -21$

$-21 = -21$ ✓

Elimination with 3 Variables: (how to)

Step 1:	Select two of the equations and eliminate one of the variables from one of the equations. Select any two other equations and eliminate the *same variable* from one of the equations.
Step 2:	You will have two equations that have only two unknowns (variables). Eliminate a second variable from the two linear equations in two unknowns.
Step 3:	Solve for the remaining variable.
Step 4:	Use the value of the variable found in Step 3 to find the value of a second variable.
Step 5:	Use the two known values of the variables identified in Steps 3 & 4 to find the value of the third variable.
Step 6:	Check you answer.



Example 1:
$$\begin{cases} x + y - z = -1 \\ (2x - y + 2z = 8)2 \\ -3x + 2y + z = -9 \end{cases}$$

$$\begin{array}{r} x + y - z = -1 \\ 2x - y + 2z = 8 \\ \hline 3x \quad + z = 7 \end{array}$$

$$\begin{array}{r} 4x - 2y + 4z = 16 \\ -3x + 2y + z = -9 \\ \hline x \quad + 5z = 7 \end{array}$$

$3x + z = 7$

$(x + 5z = 7)-3 \rightarrow$

$$\begin{array}{r} 3x + z = 7 \\ -3x - 15z = -21 \\ \hline -14z = -14 \\ \overline{-14} \quad \overline{-14} \end{array}$$

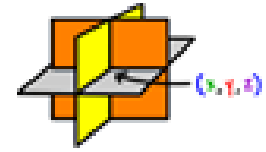
$z = 1$

$$\begin{array}{r} x + 5(1) = 7 \\ x + 5 = 7 \\ -5 \quad -5 \\ \hline x = 2 \end{array}$$

$$\begin{array}{r} 2 + y - 1 = -1 \\ y + 1 = -1 \\ -1 \quad -1 \\ \hline y = -2 \end{array}$$

$$\boxed{(2, -2, 1)}$$

Lesson 4.3: Systems of Linear Equations in Three Variables



Example 2:

$$\begin{cases} 4x + z = 4 \\ -2(2x + 3y = -4) \\ 2y - 4z = -15 \end{cases}$$

$$\begin{aligned} 4x \quad + z &= 4 \\ -4x - 6y \quad &= 8 \\ \hline -6y + z &= 12 \end{aligned}$$

$$\begin{aligned} 2y - 4z &= -15 \\ (-6y + z &= 12)4 \end{aligned}$$

$$\begin{aligned} 2y - 4z &= -15 \\ -24y + 4z &= 48 \\ \hline -22y &= 33 \\ \frac{-22y}{-22} &= \frac{33}{-22} \end{aligned}$$

$y = -\frac{3}{2}$

$$\frac{2}{1}\left(-\frac{3}{2}\right) - 4z = -15$$

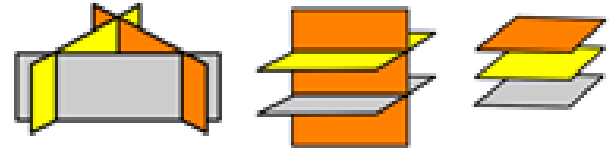
$$\begin{aligned} -3 - 4z &= -15 \\ +3 \qquad &\quad +3 \\ \hline -4z &= -12 \\ \frac{-4z}{-4} &= \frac{-12}{-4} \end{aligned}$$

$\rightarrow z = 3$

$$\begin{aligned} 4x + 3 &= 4 \\ -3 &\quad -3 \\ \hline 4x &= 1 \\ \frac{4x}{4} &= \frac{1}{4} \end{aligned}$$

$x = \frac{1}{4}$

$$\boxed{\left(\frac{1}{4}, -\frac{3}{2}, 3\right)}$$



Example 3:

$$\begin{cases} x + 2y - z = 4 \\ (-2x + 3y + z = -4)2 \\ x + 9y - 2z = 1 \end{cases}$$

$$\begin{array}{r} x + 2y - z = 4 \\ -2x + 3y + z = -4 \\ \hline -x + 5y \quad = 0 \end{array}$$

$$\begin{array}{r} -4x + 6y + 2z = -8 \\ x + 9y - 2z = 1 \\ \hline -3x + 15y \quad = -7 \end{array}$$

$$(-x + 5y = 0)-3 \rightarrow$$

$$-3x + 15y = -7$$

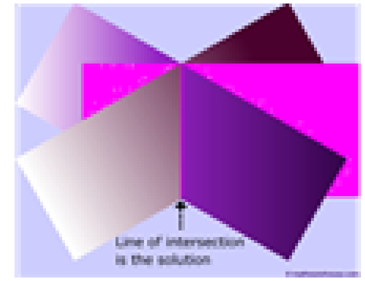
$$\begin{array}{r} 3x - 15y = 0 \\ -3x + 15y = -7 \\ \hline 0 = -7 \end{array}$$

False!

$$\boxed{\text{No Solution!}}$$

Inconsistent.

Lesson 4.3: Systems of Linear Equations in Three Variables



Example 4:
$$\begin{cases} x - 3y - z = 4 \\ x - 2y + 2z = 5 \\ 2x - 5y + z = 9 \end{cases}$$

If you get

$0=0$ ...

write

$\boxed{\text{Infinite Solutions}}$

Consistent
dependent

Example 5:

Production

A swing-set manufacturer has three different models of swing sets. The Monkey requires 2 hours to cut the wood, 2 hours to stain, and 3 hours to assemble. The Gorilla requires 3 hours to cut the wood, 4 hours to stain, and 4 hours to assemble. The King Kong requires 4 hours to cut the wood, 5 hours to stain, and 5 hours to assemble. The company has 61 hours available to cut the wood, 73 hours available to stain, and 83 hours available to assemble each day. How many of each type of swing set can be manufactured each day?

Step 1: Identify How many of each type

Step 2: Name M = monkey K = king Kong
G = gorilla

Example 5:

	Monkey	Gorilla	King Kong	Total Hrs
Cut Wood	2 m	3 g	4 k	61
Stain	2 m	4 g	5 k	73
Assemble	3 m	4 g	5 k	83

Step 3: Translate

$$2m + 3g + 4k = 61$$
$$2m + 4g + 5k = 73$$
$$3m + 4g + 5k = 83$$

Example 5:

$$\begin{cases} 2m + 3g + 4k = 61 \\ 2m + 4g + 5k = 73 \\ 3m + 4g + 5k = 83 \end{cases}$$

-1 $* -3$ $* 2$

Step 4: Solve

$2m + 3g + 4k = 61$
$-2m - 4g - 5k = -73$
$-4(-g - k = -12)$

$4g + 4k = 48$
$-4g - 5k = -53$
$-k = -5$
$k = 5$

$-6m - 12g - 15k = -219$
$6m + 8g + 10k = 166$
$-4g - 5k = -53$

$-g - 5 = -12$
$+5 \quad +5$
$-g = -7$
$g = 7$

$2m + 3(7) + 4(5) = 61$
$2m + 21 + 20 = 61$
$2m + 41 = 61$
$-41 \quad -41$
$2m = 20$
$m = 10$

Example 5:

$$\begin{cases} 2m + 3g + 4k = 61 \\ 2m + 4g + 5k = 73 \\ 3m + 4g + 5k = 83 \end{cases}$$

Step 5: Check

Step 6: Answer

We can make 10 monkey swing sets, 7 gorilla, & 5 King Kong. In the alloted time.

By the end of the lesson, we will be able to:

~Solve systems of Three Linear Equations containing three variables.

~Solve Application Problems with systems of three equations.

Homework:

Pg 293-296: # 1-5 all, 9, 11, 19,
27, 35, 41, 43;
&
Pg 330 - 332: # 3, 6, 9, 25, 27, 33