By the end of the lesson, you will be able to:

- Define the terms Polynomial and Monomial, and determine the degree of a monomial & polynomial.
- Simplify Polynomials by combining like terms.
- Evaluate Polynomial functions.
- Add, Subtract and Multiply Monomials, Polynomials and special functions.

TERMS	*DEFINITION*	*EXAMPLES*
Monomial	The product of a constant and a variable raised to a nonnegative integer power in the form ax^k, where a is a constant, x is a variable, and $k \geq 0$ is an integer. The constant a is called the ***coefficient*** of the monomial.	$3x^2y$ $4z$ -7
Degree of a Monomial	Is equivalent to the exponent k in the monomial. If there is more than one variable in the monomial, the degree is equal to the sum of the exponents. (Example: in $x^m y^n$, the degree is $m+n$.) The degree of a constant value is 0, and the number 0 has no degree.	$4x^2y^3z$ The degree is equal to $2+3+1=6$
Polynomial	A polynomial is a monomial or the sum of monomials.	$6x-y+3z+5$
Degree of a Polynomial	The highest degree of all the terms of the polynomial.	x^2y^2-xy+6 Degree is 4.
Binomial	A polynomial with two monomials that are not like terms.	$x-2$
Trinomial	A polynomial with three monomials that are not like terms.	$3x^2y-2xy+xy^2$

EXAMPLES
$3x^2y$ $4z$ -7
$4x^2y^3z$ The degree is equal to $2+3+1=6$
$6x-y+3z+5$
x^2y^2-xy+6 Degree is 4.
$x-2$
$3x^2y-2xy+xy^2$

EXAMPLES: Identify whether or not the expression is a polynomial. Identify the type of polynomial, the degree of the polynomial, and if it's a monomial, identify what its coefficient is.

A) $2x^4$ Monomial
D: 4
Coeff: 2

B) $7a^3 - 4a^2 + 6a - 2$
Polynomial
D: 3

C) $2m^3n^2 - mn^3 + 4mn^2$
trinomial
D: 5

D) 8 Monomial
D: 0
Coeff: 8

E) $\frac{3}{x^2} = 3x^{-2}$
Not a Polynomial

F) $5a^{\frac{2}{3}} - 6a^{\frac{1}{3}} + 4$
Not a Polynomial

SIMPLIFYING POLYNOMIALS ~ ADDITION & SUBTRACTION

To add (or subtract) polynomials, we combine the coefficients of like terms. The variables and exponents themselves *do not change*!

Horizontal Addition: Rearrange like terms so they are side by side, then combine.

G) Simplify: $(-5x^3 + 6x^2 + 2x - 7) + (3x^3 + 4x + 1)$

$= -5x^3 + 3x^3 + 6x^2 + 2x + 4x - 7 + 1$

$= \boxed{-2x^3 + 6x^2 + 6x - 6}$

Vertical Addition: Line like terms up into columns, and add the coefficients in the columns of like terms.

H) Simplify: $(5x^2y - 3xy + 12xy^2) + (x^2y + 12xy - 5xy^2)$

$+ \; x^2y + 12xy - 5xy^2$

$\boxed{6x^2y + 9xy + 7xy^2}$

Subtraction: Either the horizontal or vertical method works here as well. The difference here is that to add, you must change the sign of every term of the second polynomial (essentially you are multiplying everything by -1).

I) Simplify: $(6z^3 + 2z^2 - 5) - (-3z^3 + 9z^2 - z + 1)$

$= 6z^3 + 2z^2 - 5 + 3z^3 - 9z^2 + z - 1$

$$
\begin{array}{r}
= 6z^3 + 2z^2 - 5 \\
+3z^3 - 9z^2 + z - 1 \\
\hline
\boxed{9z^3 - 7z^2 + z - 6}
\end{array}
$$

MULTIPLYING POLYNOMIALS: Several methods are used depending on the type of polynomials we have.

Multiplying monomials and polynomials: Use the distributive property.

J) Simplify: $2x^2(x^2 + 3x + 5)$

$= 2x^2 \cdot x^2 + 2x^2 \cdot 3x + 2x^2 \cdot 5$

$= \boxed{2x^4 + 6x^3 + 10x^2}$

Multiplying 2 binomials: Use distribution (sometimes called FOIL).

K) Simplify: $(2y-3)(5y+6)$

$= 10y^2 + 12y - 15y - 18$

$= \boxed{10y^2 - 3y - 18}$

Multiplying 2 polynomials: Use repeated use of the distributive property.

L) Simplify: $(2x+3)(x^2+5x-1)$

$= 2x^3 + 10x^2 - 2x + 3x^2 + 15x - 3$

$= \boxed{2x^3 + 13x^2 + 13x - 3}$

Multiplying Special Products: *Certain products end up following specific patterns every time. The following are special product formulas that make it easy to find a product quickly.*

DIFFERENCE OF TWO SQUARES:

$$(A - B)(A + B) = A^2 - B^2$$

M) $(2x + 5)(2x - 5)$ $= 2x^2 \cancel{-10x} \cancel{+10x} - 25$

$= \boxed{4x^2 - 25}$

SQUARES OF BINOMIALS , or PERFECT SQUARE BINOMIALS:

$(A+B)^2 = A^2 + 2AB + B^2$ and $(A-B)^2 = A^2 - 2AB + B^2$

N) $(n+8)^2 = (n+8)(n+8)$

$= n^2 + 8n + 8n + 64$

$= \boxed{n^2 + 16n + 64}$

or use formula

O) $(7z-2)^2 = (7z-2)(7z-2)$

$= 49z^2 - 28z + 4$

(we used the formula)

POLYNOMIAL FUNCTIONS: A function whose rule is a polynomial. (Example: $f(x) = 3x^2 - 2x^2 + 6x + 1$)

ADDING, SUBTRACTING, &MULTIPLYING POLYNOMIAL FUNCTIONS:

If f and g are functions, then

$$(f+g)(x) = f(x) + g(x)$$

$$(f-g)(x) = f(x) - g(x)$$

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

$(fg)(x)$

EXAMPLES: Let $f(x) = 2x^2 + x - 3$ and $g(x) = -x^2 - 2x + 1$

P) $(f+g)(x) = (2x^2 + x - 3) + (-x^2 - 2x + 1)$

$f(x) + g(x) = x^2 - x - 2$

$\boxed{(f+g)(x) = x^2 - x - 2}$

Q) $(f-g)(x) = (2x^2 + x - 3) - (-x^2 - 2x + 1)$

$= 2x^2 + x - 3 + x^2 + 2x - 1$

$\boxed{(f-g)(x) = 3x^2 + 3x - 4}$

* R) $(f+g)(2) =$

$f(2) = 2(2)^2 + 2 - 3$
$= 8 + 2 - 3$
$f(2) = 7$

$g(2) = -(2)^2 - 2(2) + 1$
$= -4 - 4 + 1$
$g(2) = -7$

$(f+g)(2) = 7 + -7$

$\boxed{(f+g)(2) = 0}$

*Note: When evaluateing functions at a specific value of x, evaluate ***each*** function at that point FIRST, then combine.

S) Let $f(x) = 3x^2$ and $g(x) = x^2 - 2x + 1$

Find: $(f \cdot g)(x) = 3x^2(x^2 - 2x + 1)$

$$\boxed{(f \cdot g)(x) = 3x^4 - 6x^3 + 3x^2}$$

APPLICATIONS: The Profit Function.

Profit is defined as total revenue minus total cost. The profit function of a company is shown as $P(x) = R(x) - C(x)$

Example: If a company sells sunglasses for $20, the revenue function is $R(x) = 20x$. If the company's variable cost is $8 per pair of sunglasses and fixed costs are $1000 per week, the cost function is $C(x) = 8x + 1000$

a) Find the profit function $P(x)$

$$= R(x) - C(x)$$
$$= 20x - (8x + 1000)$$
$$= 20x - 8x - 1000$$
$$\boxed{P(x) = 12x - 1000}$$

b) Determine and interpret $P(750)$

$$P(750) = 12(750) - 1000$$
$$= 9000 - 1000 = 8000$$

$P(750) = 8000$

$P(750) = 8000$

If I sell 750 sunglasses, my profit will be \$8000.

Homework:

Pg 364: #13-33 odds, 37, 39, 45, 47, 53, 55, 57, 61, 65, 69, 75, 81, 85

Pg 374 (assigned on Dec $\overset{A}{13}$/$\overset{B}{14}$)