By the end of the lesson you will:

- Divide a polynomial by a monomial.
- Divide polynomials using long division.
- Divide polynomials using synthetic division.
- Divide polynomial functions.
- Use the Remainder and Factor Theorems.

DIVIDING A POLYNOMIAL BY A MONOMIAL

This method is based on the following Quotient Rule.

$$\frac{A+B}{C} = \frac{A}{C} + \frac{B}{C}$$

Since we are dealing with monomials in the numerator and denominator, we use the quotient rule of exponents to simplify.

Examples:

A) $\frac{21n^4-18n^3+12n^2}{3n^2}$

B) $\frac{8a^2b^2-6a^2b+5ab^2}{2a^2b^2}$

DIVIDING POLYNOMIALS USING LONG DIVISION

Review: Divide 645 by 14 using long division.

Dividing two polynomials follows the same steps:

Example C) Divide $4x^2 + 9x - 10 \quad by \quad x + 2$

Step 1: Divide the leading term of the dividend ($4x^2$), by the leading term of the divisor (x). Enter the result over the term $4x^2$.
Step 2: Multiply 4x by (x+2). Vertically align like terms.
Step 3: Subtract ($4x^2$+8x) from ($4x^2$+9x-10). REMEMBER: You must switch the signs on every term when you subtract a polynomial!
Step 4: Repeat steps 1-3, treating each new remainder as the dividend, until no other division is possible. (The degree of the remainder is less than the degree of the divisor.)
Step 5: Check your answer! (Quotient)(Divisor)+Remainder = Dividend.

Example D) Simplify by performing long division:

$$\frac{8-9x+2x^2+12x^3+5x^5}{x^2+3}$$

(Note: Start by rewriting the numerator with its powers in descending order!)

DIVIDING POLYNOMIALS USING SYNTHETIC DIVISION

Synthetic Division is a shortened version of long division. **THIS IS ONLY USED WHEN THE DIVISOR IS IN THE FORM *x-c* or *x+c*.** (There are no exponents.) Synthetic division is essentially writing the long division in a compact form by not writing terms that are exactly the same as the term above.

LONG DIVISION:

$$\begin{array}{r} 2x^2 + x - 4 \\ x - 3\overline{)2x^3 - 5x^2 - 7x + 20} \\ -(2x^3 - 6x^2) \qquad\qquad \\ x^2 - 7x \qquad \\ -(x^2 - 3x) \qquad \\ -4x + 20 \\ -(-4x + 12) \\ 8 \end{array}$$

Quotient: $2x^2 + x - 4$ ⟵ Quotient

Remainder: 8 ⟵ Remainder

NOW REMOVE UNNECESSARY TERMS:

$$\begin{array}{r} 2x^2 + x - 4 \\ x - 3\overline{)2x^3 - 5x^2 - 7x + 20} \\ -6x^2 \qquad\qquad \\ x^2 \qquad\qquad \\ -3x \qquad \\ -4x \qquad \\ 12 \\ 8 \end{array}$$

NOW REMOVE X'S (keeping like terms aligned):

$$\begin{array}{r} 2x^2 + x - 4 \\ x - 3\overline{)2 \quad -5 \quad -7 \quad 20} \\ -6 \qquad\qquad \\ \boxed{1} \qquad\qquad \\ -3 \qquad \\ \boxed{-4} \qquad \\ 12 \\ \boxed{8} \end{array}$$

NOW REMOVE LEVELS (move all the lines up):

$$\begin{array}{r} 2x^2 + x - 4 \\ x - 3\overline{)2 \quad -5 \quad -7 \quad 20} \\ -6 \quad -3 \quad 12 \\ \hline \square \quad 1 \quad -4 \quad 8 \end{array}$$

STEPS FOR SYNTHETIC DIVISION:

Step 1: Rewrite the dividend in descending order of power (if necessary). Copy the coefficients of the dividend, putting in a zero for any missing powers of x.

Step 2: Insert the division symbol. Rewrite the divisor in the form x-c (for instance, if it's x+c, rewrite as x-(-c)). Insert the value of c to the left of the division symbol.

Step 3: Bring the first coefficient of the dividend down two rows and enter it in row 3.

Step 4: Multiply the last entry in row 3 by the value of c and place the result in row 2, but one column to the right.

Step 5: Add the entry in Row 2 to the entry above it in Row 1 and enter the sum in row 3.

Step 6: Repeat steps 4 and 5 until there aren't any more entries left in Row 1.

Step 7: The last entry in row 3 is the remainder. The other entries in row 3 are the coefficients of the answer in descending order.

Example E) Use synthetic division to find the quotient and remainder. $2x^3 - 3x^2 - 4x + 11 \quad divided\ by\ x + 2$

DIVIDING POLYNOMIAL FUNCTIONS

If *f* and *g* are functions, then the quotient *f/g* is the function defined by

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \qquad g(x) \neq 0$$

Example F) $f(x) = x^3 - 2x^2 - 4x - 5$ and $g(x) = x + 2$.

Find $\left(\frac{f}{g}\right)(x) \quad and \quad \left(\frac{f}{g}\right)(2)$

THE REMAINDER THEOREM

Let *f* be a polynomial function. If *f(x)* is divided by *x-c*, then the remainder is *f(c).*

Example G) Use the Remainder Theorem to find the remainder if $f(x) = 3x^3 - 2x + 6$ is divided by $x + 2$.

THE FACTOR THEOREM

Let *f* be a polynomial function. Then *x-c* is a factor of *f(x)* if and only if *f(c) = 0*.

Example H) Use the Factor Theorem to determine whether the function $f(x) = 2x^3 - x^2 - 16x + 15$ Has the factor

a) *x-2*

b) *x+3*

Homework:

Pg. 386: #13, 17, 19, 25,29, 31, 37, 39, 43, 49, 51, 55, 61,63, 67, 71, 73, 77, 81, 85, 95

And

Pg. 390: #2-10 all