By the end of the lesson you will:

- Divide a polynomial by a monomial.
- Divide polynomials using long division.
- Divide polynomials using synthetic division.
- Divide polynomial functions.
- Use the Remainder and Factor Theorems.

DIVIDING A POLYNOMIAL BY A MONOMIAL

This method is based on the following Quotient Rule.

$$\frac{A+B}{C} = \frac{A}{C} + \frac{B}{C}$$

Since we are dealing with monomials in the numerator and denominator, we use the quotient rule of exponents to simplify.

Examples:

A) $\frac{21n^4-18n^3+12n^2}{3n^2}$

$= \frac{21n^4}{3n^2} - \frac{18n^3}{3n^2} + \frac{12n^2}{3n^2}$

$= \boxed{7n^2 - 6n + 4}$

B) $\frac{8a^2b^2-6a^2b+5ab^2}{2a^2b^2}$

$= \frac{8a^2b^2}{2a^2b^2} - \frac{6a^2b}{2a^2b^2} + \frac{5ab^2}{2a^2b^2}$

$= \boxed{4 - \frac{3}{b} + \frac{5}{2a}}$

DIVIDING POLYNOMIALS USING LONG DIVISION

Review: Divide 645 by 14 using long division.

$$\begin{array}{r} 46 \\ 14\overline{)645} \\ -56\downarrow \\ \hline 85 \\ -84 \\ \hline \boxed{1} \end{array} \leftarrow \text{remainder}$$

$$46\frac{1}{14}$$

Dividing two polynomials follows the same steps:

Example C) Divide $4x^2 + 9x - 10 \;\; by \;\; x + 2$

Step 1: Divide the leading term of the dividend ($4x^2$), by the leading term of the divisor (x). Enter the result over the term $4x^2$.
Step 2: Multiply 4x by (x+2). Vertically align like terms.
Step 3: Subtract ($4x^2$+8x) from ($4x^2$+9x-10). REMEMBER: You must switch the signs on every term when you subtract a polynomial!
Step 4: Repeat steps 1-3, treating each new remainder as the dividend, until no other division is possible. (The degree of the remainder is less than the degree of the divisor.)
Step 5: Check your answer! (Quotient)(Divisor)+Remainder = Dividend.

$4x + 1$

$x+2 \,\overline{)\, 4x^2 + 9x - 10}$

$-4x^2 \mp 8x \quad \downarrow$

$x - 10$

$-x \mp 2$

$\boxed{-12}$ ← remainder

$$\boxed{4x + 1 - \frac{12}{x+2}}$$

$$4x^2 + 9x - 10 \quad by \quad x + 2$$

Example D) Simplify by performing long division:

$$\frac{8-9x+2x^2+12x^3+5x^5}{x^2+3}$$

$$
\begin{array}{r|l}
 & 5x^3 \qquad -3x \quad +2 \\
\hline
x^2+3 & 5x^5+0x^4+12x^3+2x^2-9x+8 \\
 & -5x^5 \qquad \mp 15x^3 \downarrow \quad \downarrow \quad \downarrow \\
\cline{2-2}
 & \qquad -3x^3+2x^2-9x+8 \\
 & \qquad \pm 3x^3 \qquad \pm 9x \\
\cline{2-2}
 & \qquad\qquad 2x^2 \qquad +8 \\
 & \qquad\qquad -2x^2 \qquad \mp 6 \\
\cline{2-2}
 & \qquad\qquad\qquad \boxed{2}
\end{array}
$$

$$\boxed{5x^3-3x+2+\frac{2}{x^2+3}}$$

(Note: Start by rewriting the numerator with its powers in descending order!)

DIVIDING POLYNOMIALS USING SYNTHETIC DIVISION

Synthetic Division is a shortened version of long division. **THIS IS ONLY USED WHEN THE DIVISOR IS IN THE FORM *x-c* or *x+c*.** (There are no exponents.) Synthetic division is essentially writing the long division in a compact form by not writing terms that are exactly the same as the term above.

LONG DIVISION:

$$\begin{array}{r} 2x^2 + x - 4 \quad \longleftarrow \text{Quotient} \\ x - 3\overline{)2x^3 - 5x^2 - 7x + 20} \\ -(2x^3 - 6x^2) \\ \hline x^2 - 7x \\ -(x^2 - 3x) \\ \hline -4x + 20 \\ -(-4x + 12) \\ \hline 8 \quad \longleftarrow \text{Remainder} \end{array}$$

NOW REMOVE UNNECESSARY TERMS:

$$\begin{array}{r} 2x^2 + x - 4 \\ x - 3\overline{)2x^3 - 5x^2 - 7x + 20} \\ -6x^2 \\ \hline x^2 \\ -3x \\ \hline -4x \\ 12 \\ \hline 8 \end{array}$$

NOW REMOVE X'S (keeping like terms aligned):

$$\begin{array}{r} 2x^2 + x - 4 \\ x - 3\overline{)2 \quad -5 \quad -7 \quad 20} \\ -6 \\ \hline \boxed{1} \\ -3 \\ \hline \boxed{-4} \\ 12 \\ \hline \boxed{8} \end{array}$$

NOW REMOVE LEVELS (move all the lines up):

$$\begin{array}{rrrrr} & \multicolumn{4}{l}{2x^2 + x - 4} \\ x - 3) & 2 & -5 & -7 & 20 \\ & & -6 & -3 & 12 \\ \hline & \square & 1 & -4 & 8 \end{array}$$

STEPS FOR SYNTHETIC DIVISION:

Step 1: Rewrite the dividend in descending order of power (if necessary). Copy the coefficients of the dividend, putting in a zero for any missing powers of x.

Step 2: Insert the division symbol. Rewrite the divisor in the form x-c (for instance, if it's x+c, rewrite as x-(-c)). Insert the value of c to the left of the division symbol.

Step 3: Bring the first coefficient of the dividend down two rows and enter it in row 3.

Step 4: Multiply the last entry in row 3 by the value of c and place the result in row 2, but one column to the right.

Step 5: Add the entry in Row 2 to the entry above it in Row 1 and enter the sum in row 3.

Step 6: Repeat steps 4 and 5 until there aren't any more entries left in Row 1.

Step 7: The last entry in row 3 is the remainder. The other entries in row 3 are the coefficients of the answer in descending order.

$x - 4$

$c = 4$

$x + 6 = x - -6$

$c = -6$

$\boxed{x - c}$ *

$x + 2$

$x - -2$

$c = -2$

Example E) Use synthetic division to find the quotient and remainder. $2x^3 - 3x^2 - 4x + 11$ *divided by* $x + 2$

$c = -2$

$$\begin{array}{r|rrrr} -2 & 2 & -3 & -4 & +11 \\ & \downarrow & -4 & 14 & -20 \\ \hline & 2 & -7 & 10 & \boxed{-9} \end{array}$$

← remainder

$$\boxed{2x^2 - 7x + 10 - \frac{9}{x+2}}$$

DIVIDING POLYNOMIAL FUNCTIONS

If f and g are functions, then the quotient f/g is the function defined by $$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \qquad g(x) \neq 0$$

Example F) $f(x) = x^3 - 2x^2 - 4x - 5$ and $g(x) = x + 2$.

$c = -2$

Find $\left(\frac{f}{g}\right)(x)$ and $\left(\frac{f}{g}\right)(2)$

$$\left(\frac{f}{g}\right)(x) = \frac{x^3 - 2x^2 - 4x - 5}{x+2}$$

$$\boxed{\left(\frac{f}{g}\right)(x) = x^2 - 4x + 4 - \frac{13}{x+2}}$$

$$\begin{array}{r|rrrr} -2 & 1 & -2 & -4 & -5 \\ & \downarrow & -2 & 8 & -8 \\ \hline & 1 & -4 & 4 & \boxed{-13} \end{array}$$

$$\left(\frac{f}{g}\right)(x) = x^2 - 4x + 4 - \frac{13}{x+2}$$

$$\left(\frac{f}{g}\right)(2) = (2)^2 - 4(2) + 4 - \frac{13}{2+2}$$

$$= 4 - 8 + 4 - \frac{13}{4}$$

$$= -4 + 4 - \frac{13}{4}$$

$$\boxed{\left(\frac{f}{g}\right)(2) = -\frac{13}{4}}$$

THE REMAINDER THEOREM

Let *f* be a polynomial function. If *f(x)* is divided by *x-c*, then the remainder is *f(c)*.

Example G) Use the Remainder Theorem to find the remainder if $f(x) = 3x^3 - 2x + 6$ is divided by $x + 2$.

$c = -2$

$f(-2) = 3(-2)^3 - 2(-2) + 6$

$= 3(-8) + 4 + 6$

$= -24 + 4 + 6$

$f(-2) = -14$

$\boxed{\text{The remainder is } -14. \quad R = -14}$

THE FACTOR THEOREM

Let *f* be a polynomial function. Then *x-c* is a factor of *f(x)* if and only if *f(c) = 0*. (remainder is zero)

Example H) Use the Factor Theorem to determine whether the function $f(x) = 2x^3 - x^2 - 16x + 15$ Has the factor

a) $x-2$ $c=2$

$f(2) = 2(2)^3 - (2)^2 - 16(2) + 15$

$= 2(8) - 4 - 32 + 15$

$= 16 - 4 - 32 + 15$

$f(2) = -5$

Not a factor

b) $x+3$ $c=-3$

$f(-3) = 2(-3)^3 - (-3)^2 - 16(-3) + 15$

$= 2(-27) - 9 + 48 + 15$

$= -54 - 9 + 48 + 15$

$f(-3) = 0$

yes Factor

Homework:

Pg. 386: #13, 17, 19, 25,29, 31, 37, 39, 43, 49, 51, 55, 61,63, 67, 71, 73, 77, 81, 85, 95

And

Pg. 390: #2-10 all