## By the end of the lesson you will:

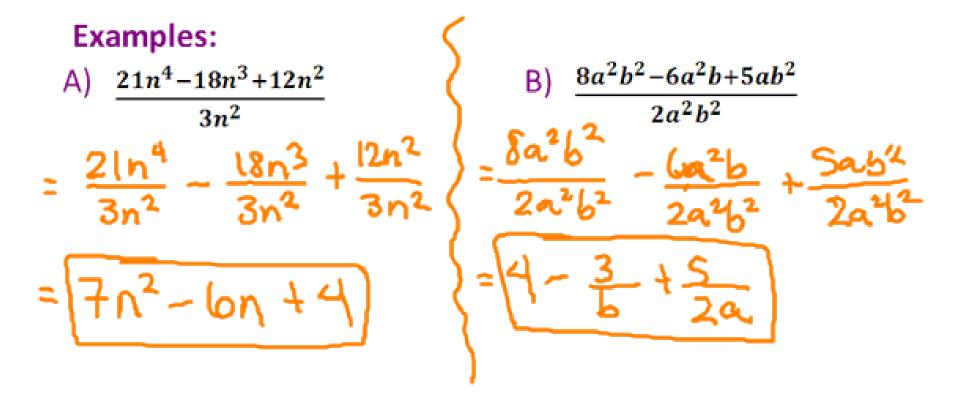
- · Divide a polynomial by a monomial.
- Divide polynomials using long division.
- Divide polynomials using synthetic division.
- Divide polynomial functions.
- Use the Remainder and Factor Theorems.

## **DIVIDING A POLYNOMIAL BY A MONOMIAL**

This method is based on the following Quotient Rule.

$$\frac{A+B}{C} = \frac{A}{C} + \frac{B}{C}$$

Since we are dealing with monomials in the numerator and denominator, we use the quotient rule of exponents to simplify.



## **DIVIDING POLYNOMIALS USING LONG DIVISION**

Review: Divide 645 by 14 using long division.

# **Dividing two polynomials follows the same steps: Example C)** Divide $4x^2 + 9x - 10$ by x + 2

**Step 1:** Divide the leading term of the dividend (4x<sup>2</sup>), by the leading term of the divisor (x). Enter the result over the term 4x<sup>2</sup>.

**Step 2:** Multiply 4x by (x+2). Vertically align like terms.

**Step 3:** Subtract (4x²+8x) from (4x²+9x-10). REMEMBER: You must switch the signs on every term when you subtract a polynomial!

**Step 4:** Repeat steps 1-3, treating each new remainder as the dividend, until no other division is possible. (The degree of the remainder is less than the degree of the divisor.)

**Step 5:** Check your answer! (Quotient)(Divisor)+Remainder = Dividend.

$$4x + 1$$
  
 $-4x^{2} + 9x - 10$   
 $-4x^{2} + 8x$   
 $-4x^{2} + 8x$   
 $-4x^{2} + 8x$   
 $-x + 2$   
 $-12$  remainder  
 $4x + 1 - \frac{12}{x + 2}$ 

 $4x^2 + 9x - 10$  by x + 2

## **Example D)** Simplify by performing long division:

(Note: Start by rewriting the numerator with its powers in descending order!)  $\frac{2}{2}$ 

#### DIVIDING POLYNOMIALS USING SYNTHETIC DIVISION

Synthetic Division is a shortened version of long division. THIS IS ONLY USED WHEN THE DIVISOR IS IN THE FORM x-c or x+c. (There are no exponents.) Synthetic division is essentially writing the long division in a compact form by not writing terms that are exactly the same as the term above.

#### LONG DIVISION:

$$\begin{array}{c} 2x^2 + x - 4 \\ x - 3)2x^3 - 5x^2 - 7x + 20 \\ -\underline{(2x^3 - 6x^2)} \\ x^2 - 7x \\ -\underline{(x^2 - 3x)} \\ -4x + 20 \\ -\underline{(-4x + 12)} \\ 8 \end{array} \qquad \text{Remainder}$$

#### NOW REMOVE UNNECESSARY TERMS:

$$\begin{array}{r}
2x^{2} + x - 4 \\
x - 3)2x^{3} - 5x^{2} - 7x + 20 \\
\underline{-6x^{2}} \\
x^{2}$$

$$\underline{-3x} \\
-4x$$

$$\underline{12} \\
8$$

#### NOW REMOVE X'S (keeping like terms aligned):

$$\begin{array}{r}
2x^2 + x - 4 \\
x - 3)2 - 5 - 7 20 \\
\underline{-6} \\
1 \\
\underline{-3} \\
-4 \\
\underline{-12} \\
\hline$$

#### NOW REMOVE LEVELS (move all the lines up):

#### STEPS FOR SYNTHETIC DIVISION:

- Step 1: Rewrite the dividend in descending order of power (if necessary). Copy the coefficients of the dividend, putting in a zero for any missing powers of x.
- Step 2: Insert the division symbol. Rewrite the divisor in the form x-c (for instance, if it's x+c, rewrite as x-(-c)). Insert the value of c to the left of the division symbol.
- Step 3: Bring the first coefficient of the dividend down two rows and enter it in row 3.
- Step 4: Multiply the last entry in row 3 by the value of c and place the result in row 2, but one column to the right.
- Step 5: Add the entry in Row 2 to the entry above it in Row 1 and enter the sum in row 3.
- Step 6: Repeat steps 4 and 5 until there aren't any more entries left in Row 1.
- Step 7: The last entry in row 3 is the remainder. The other entries in row 3 are the coefficients of the answer in descending order.

**Example E)** Use synthetic division to find the quotient and remainder.  $2x^3 - 3x^2 - 4x + 11$  divided by x + 2

## DIVIDING POLYNOMIAL FUNCTIONS

If f and g are functions, then the quotient f/g is the function

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \qquad g(x) \neq 0$$

$$g(x) \neq 0$$

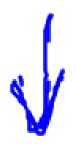
Example F) 
$$f(x) = x^3 - 2x^2 - 4x - 5$$
 and  $g(x) = x + 2$ .

Find 
$$\left(\frac{f}{g}\right)(x)$$
 and  $\left(\frac{f}{g}\right)(2)$ 

Find 
$$\left(\frac{f}{g}\right)(x)$$
 and  $\left(\frac{f}{g}\right)(2)$ 

$$\left(\frac{f}{g}\right)(x) = \frac{x^3 - 2x^2 - 4x - 5}{x + 2} \qquad \left(\frac{f}{g}\right)(x) = x^2 - 4x + 4 - \frac{13}{x + 2}$$

$$\left(\frac{f}{g}\right)(x) = x^2 - 4x + 4 - \frac{13}{x+2}$$



$$\left(\frac{f}{g}\right)(x) = \frac{\chi^2 - 4\chi + 4 - \frac{13}{\chi + 2}}{g}$$

$$\left(\frac{f}{g}\right)(2) = (2)^{2} - 4(2) + 4 - \frac{13}{2+2}$$

$$= 4 - 8 + 4 - \frac{13}{4}$$

$$= -4 + 4 - \frac{13}{4}$$

$$\left(\frac{f}{g}\right)(2) = -\frac{13}{4}$$

## THE REMAINDER THEOREM

Let f be a polynomial function. If f(x) is divided by x-c, then the remainder is f(c).

**Example G)** Use the Remainder Theorem to find the remainder if  $f(x) = 3x^3 - 2x + 6$  is divided by x + 2.

$$f(-2) = 3(-2)^{3} - 2(-2) + 6$$

$$= 3(-8) + 4 + 6$$

$$f(-2) = -24 + 4 + 6$$

$$f(-2) = -14$$

## THE FACTOR THEOREM

Let f be a polynomial function. Then x-c is a factor of f(x) if and only if f(c) = 0.

(remainder is zero.)

**Example H)** Use the Factor Theorem to determine whether the function  $f(x) = 2x^3 - x^2 - 16x + 15$  Has the factor

a) 
$$x-2$$
  $C=2$ 
b)  $x+3$   $C=-3$ 

$$f(2) = 2(2)^3 - (2)^2 - 14(2) + 15 f(-3) = 2(-3)^3 - (-3)^2 - 14(-3) + 15$$

$$= 2(8) - 4 - 32 + 15$$

$$= 16 - 4 - 32 + 15$$

$$= -54 - 9 + 48 + 15$$

$$f(3) = 0$$
Not a factor
$$f(3) = 0$$
Ues Factor

## **Homework:**

**Pg. 386:** #13, 17, 19, 25,29, 31, 37, 39, 43, 49, 51, 55, 61,63, 67, 71, 73, 77, 81, 85, 95

And

**Pg. 390:** #2-10 all