

Lesson 5.5: Factoring

By the end of the lesson, we will be able to:

- ~ Factor trinomials in the form $x^2 + bx + c$
- ~ Factor trinomials in the form $ax^2 + bx + c$
- ~ Factor trinomials using substitution.

Lesson 5.5: Factoring

Factoring is essentially “undistributing”. We are trying to write a second-degree polynomial as the product of 2 first degree binomials. There is a pattern that always appears when we're factoring.

$$\text{If } x^2 + bx + c = (x + m)(x + n),$$

$$\text{then } b = m + n \text{ and } c = m \cdot n$$

$$\underline{\quad} \cdot \underline{\quad} = c$$

$$\underline{\quad} + \underline{\quad} = b$$

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Box Method of Factoring:

Step 1: In the upper left box, put your first term, In the lower right box, put your last term.

ax^2	
	c

Step 2: Multiply $A \times C$ and factor the product to find factors that add up to B . Put these factors (with an x attached) into the other two boxes. Order doesn't matter.

Step 3: Find the GCF of each row and each column. Keep the sign of the upper right and lower left boxes as part of the GCF .

Step 4: Rewrite the GCF 's of the rows in one set of parentheses, and the GCF 's of the columns in one set of parentheses. This is your final factorization.

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Ex 1: Factor

$$y^2 + 11y + 28$$

	$(y + 7)$	
$(y + 4)$	y^2	$7y$
	$4y$	28

$$(y+7)(y+4)$$

$$\underline{4} \cdot \underline{7} = 28$$

$$\underline{4} + \underline{7} = 11$$

Factor by grouping

$$y^2 + 11y + 28$$

$$y^2 + 4y + 7y + 28$$

$$y(y+4) + 7(y+4)$$

$$(y+4)(y+7)$$

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THINK!

- ~If both b and c are positive, the factors of c must both be positive.
- ~If b is negative and c is positive, both factors of c must be negative.
- ~If both b and c are negative, you must have one positive and one negative factor of c .

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Ex 2: Factor $2t^2 - 22t + 36$

(remember GCF...) $2(t^2 - 11t + 18)$

	$(t - 2)$	
t	t^2	$-2t$
-9	$-9t$	18

$$\underline{-9} \cdot \underline{-2} = 18$$

$$\underline{-9} + \underline{-2} = -11$$

$$2(t-2)(t-9)$$

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Ex 3: Factor

$$x^2 - 2xy + y^2$$

$$x^2 - 2xy + y^2$$

$$\underbrace{x^2 - 1xy} \quad \underbrace{-1xy + y^2}$$

$$x(x-y) - y(x-y)$$

$$\boxed{(x-y)(x-y)}$$

$$\begin{aligned} \underline{-1y} \cdot \underline{-1y} &= 1y^2 \\ \underline{-1y} + \underline{1y} &= -2y \end{aligned}$$

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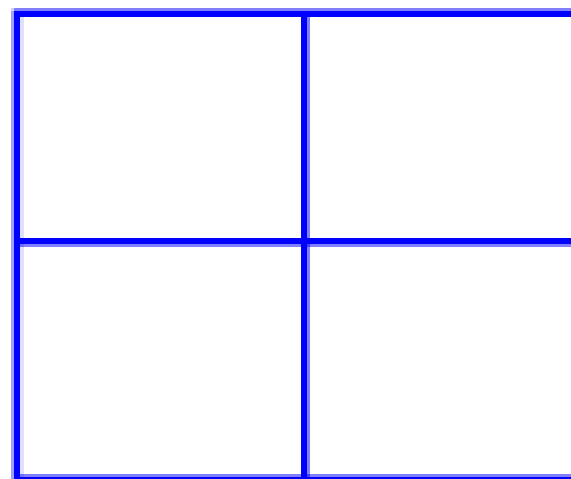
IDENTIFYING "PRIME" TRINOMIALS:

A "prime" trinomial is one that cannot be factored because there are no integer factors of c that add to b .

Ex 4: $x^2 + 5x + 10$

$$\begin{array}{r} \underline{\quad} \cdot \underline{\quad} = 10 \\ \underline{\quad} + \underline{\quad} = 5 \end{array}$$

Prime



There are no factors of 10 that sum to 5, so ... It's Prime!

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FACTORIZING WHERE THERE ARE GCF'S:

The first rule of factoring is always

LOOK FOR A GCF!!!!

Example 5: Factor $2k^3 + 6k^2 - 56k$

$$2k(k^2 + 3k - 28)$$

$$\begin{array}{r} \underline{7} \cdot \underline{-4} = -28 \\ \underline{7} + \underline{-4} = 3 \end{array}$$

$$\underbrace{k^2 + 7k}_{k(k+7)} - \underbrace{4k - 28}_{4(k-7)}$$

$$k(k+7) - 4(k-7)$$

$$2k(k+7)(k-4)$$

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FACTORIZING TRINOMIALS WITH A LEADING

COEFFICIENT: $ax^2 + bx + c$, where $a \neq 1$

There are two methods of factoring trinomials with a leading coefficient:

~ Factoring by grouping

~ Factoring by Trial and Error (also called "Guess and Check").

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FACTORING BY GROUPING:

ax^2	
	c

Step 1: Find the value of $a \cdot c$

Step 2: Find the pair of integers whose product equals ac , and whose sum equals b . Call these integers m and n , where $mn = ac$ and $m + n = b$

Step 3: Rewrite the expression as:

$$ax^2 + bx + c = ax^2 + mx + nx + c$$

Step 4: Factor the new expression by grouping.

Step 5: CHECK YOUR ANSWER!

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$$\underline{\quad} \cdot \underline{\quad} = ac$$

Example: Factor

$$6x^2 - 5x - 4$$

$$\underline{-8} \cdot \underline{3} = -24$$

$$\underline{-8} + \underline{3} = -5$$

$$(2x+1)(3x-4)$$

$$(2x + 1)$$

$3x$	$6x^2$	$3x$
4	$-8x$	-4

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Example: Factor $-15x^2 + 23x - 4$

$$\underline{20} \cdot \underline{3} = 60$$

$$\underline{20} + \underline{3} = 23$$

$$-15x^2 + 23x - 4$$

$$\underbrace{-15x^2 + 20x}_{-5x(3x-4)} + \underbrace{3x - 4}_{+1(3x-4)}$$

$$-5x(3x-4) + 1(3x-4)$$

$$(3x-4)(-5x+1)$$

$$(3x-4)(-1)(5x-1)$$

$$\boxed{-1(3x-4)(5x-1)}$$

More correct way

$$-1(15x^2 - 23x + 4)$$

$$\underline{-20} \cdot \underline{-3} = 60$$

$$\underline{-20} + \underline{-3} = -23$$

$$\underbrace{15x^2 - 20x}_{5x(3x-4)} - 3x + 4$$

$$5x(3x-4) - 1(3x-4)$$

$$\boxed{-1(3x-4)(5x-1)}$$

FACTORING BY SUBSTITUTION:

Sometimes our trinomials have variables with extra large exponents, or even use binomials in place of variables. To factor these, we can use substitution.

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Example: Factor $2n^4 - 7n^2 - 15$

Substitute: $x = n^2$
 $x^2 = (n^2)^2$

$$2x^2 - 7x - 15 \quad \begin{array}{l} \frac{-10}{-10} \cdot \frac{3}{3} = -30 \\ \frac{-10}{-10} + \frac{3}{3} = -7 \end{array}$$

$$\underbrace{2x^2 - 10x} + \underbrace{3x - 15}$$

$$2x(x-5) + 3(x-5)$$

$$(x-5)(2x+3)$$

$$(n^2-5)(2n^2+3)$$

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Example: Factor $2(x + 1)^2 + 3(x + 1) - 35$

Substitute: $z = (x + 1)$

$$2z^2 + 3z - 35 \quad \begin{array}{l} \underline{10} - \underline{7} = -70 \\ \underline{10} + \underline{-7} = 3 \end{array}$$

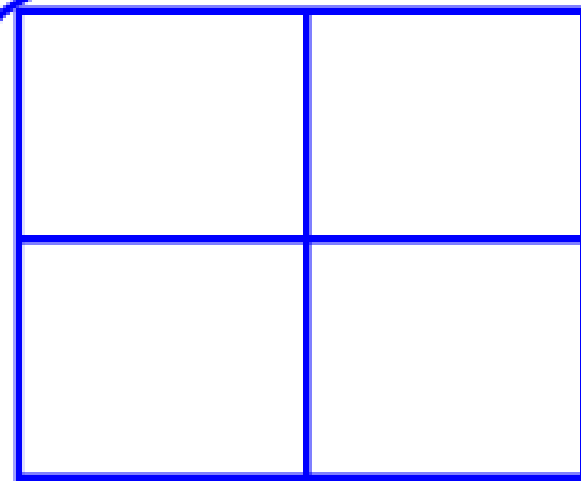
$$\underline{2z^2 + 10z} \quad \underline{-7z - 35}$$

$$2z(z + 5) - 7(z + 5)$$

$$(z + 5)(2z - 7)$$

$$((x + 1) + 5) (2(x + 1) - 7)$$

$$(x + 6) (2x - 5)$$



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Can you?

Assignment:

Page 407: #'s 9, 11, 15, 21, 25, 29, 35, 39, 43, 47, 51, 55, 57, 59, 63, 65, 67, 79, 83

AND

Page 442: #'s 65, 69, 73, 75

(23 problems)