Lesson 5.5: Factoring

By the end of the lesson, we will be able to:

~ Factor trinomials in the form $x^2 + bx + c$

~ Factor trinomials in the form $ax^2 + bx + c$

~ Factor trinomials using substitution.

Lesson 5.5: Factoring

Factoring is essentially "undistributing". We are trying to write a ***second-degree*** polynomial as the product of ***2 first degree*** binomials. There is a pattern that always appears when we're factoring.

$$If\ x^2 + bx + c = (x+m)(x+n),$$

$$then\ \ b = m + n\ \ and\ \ c = m \cdot n$$

$\_\_\_ \cdot \_\_\_ = ac$

$\_\_\_ + \_\_\_ = b$

<u>**Lesson 5.5: Factoring**</u>

Box Method of Factoring:

Step 1: In the upper left box, put your first term, In the lower right box, put your last term.

ax^2	
	c

Step 2: Multiply *AxC* and factor the product to find factors that add up to B. Put these factors (with an x attached) into the other two boxes. Order doesn't matter.

Step 3: Find the *GCF* of each row and each column. Keep the sign of the upper right and lower left boxes as part of the *GCF*.

Step 4: Rewrite the *GCF*'s of the rows in one set of parentheses, and the *GCF*'s of the columns in one set of parentheses. This is your final factorization.

Lesson 5.5: Factoring

Ex 1: Factor $y^2 + 11y + 28$

$$\underline{4} \cdot \underline{7} = 28$$
$$\underline{4} + \underline{7} = 11$$

	y	$+7$
y	y^2	$7y$
$+4$	$4y$	28

$$\boxed{(y+7)(y+4)}$$

Factor by grouping

$$y^2 + 11y + 28$$
$$\underbrace{y^2 + 4y} + \underbrace{7y + 28}$$
$$y(y+4) + 7(y+4)$$
$$\boxed{(y+4)(y+7)}$$

THINK!

~If both b and c are positive, the factors of c must both be positive.

~If b is negative and c is positive, both factors of c must be negative.

~If both b and c are negative, you must have one positive and one negative factor of c.

Ex 2: Factor $2t^2 - 22t + 36$

(remember GCF...) $2(t^2 - 11t + 18)$

$(t - 2)$

	t	-2
t	t^2	$-2t$
-9	$-9t$	18

$-9 \cdot -2 = 18$

$-9 + -2 = -11$

$\boxed{2(t-2)(t-9)}$

Ex 3: Factor $x^2 - 2xy + y^2$

$-1y \cdot -1y = 1y^2$

$-1y + -1y = -2y$

$x^2 - 2xy + y^2$

$\underbrace{x^2 - 1xy}\quad\underbrace{-1xy + y^2}$

$x(x-y) - y(x-y)$

$\boxed{(x-y)(x-y)}$

IDENTIFYING "PRIME" TRINOMIALS:

A "prime" trinomial is one that cannot be factored because there are no integer factors of c that add to b.

Ex 4: $x^2 + 5x + 10$

$\_\_\_ \cdot \_\_\_ = 10$

$\_\_\_ + \_\_\_ = 5$

$\boxed{\text{Prime}}$

There are no factors of 10 that sum to 5, so ... It's Prime!

FACTORING WHERE THERE ARE GCF'S:

The first rule of factoring is always

LOOK FOR A GCF!!!!

Example 5: Factor $2k^3 + 6k^2 - 56k$

$2k(k^2 + 3k - 28)$

$\underline{7} \cdot \underline{-4} = -28$

$\underline{7} + \underline{-4} = 3$

$\underbrace{k^2 + 7k}\ \underbrace{-4k - 28}$

$k(k+7) - 4(k+7)$

$\boxed{2k(k+7)(k-4)}$

FACTORING TRINOMIALS WITH A LEADING COEFFICIENT: $ax^2 + bx + c$, where $a \neq 1$

There are two methods of factoring trinomials with a leading coefficient:

~ Factoring by grouping

~ Factoring by Trial and Error (also called "Guess and Check").

ax^2	
	c

FACTORING BY GROUPING:

Step 1: Find the value of $a \cdot c$

Step 2: Find the pair of integers whose product equals *ac*, and whose sum equals *b*. Call these integers ***m*** and ***n***, where $mn = ac$ and $m + n = b$

Step 3: Rewrite the expression as:

$$ax^2 + bx + c = ax^2 + mx + nx + c$$

Step 4: Factor the new expression by grouping.

Step 5: CHECK YOUR ANSWER!

___ • ___ = ac

Example: Factor $6x^2 - 5x - 4$

$\underline{-8} \cdot \underline{3} = -24$

$\underline{-8} + \underline{3} = -5$

$$\boxed{(2x+1)(3x-4)}$$

	$2x$	$+1$
$3x$	$6x^2$	$3x$
-4	$-8x$	-4

Lesson 5.5: Factoring

Example: Factor $-15x^2 + 23x - 4$

$\underline{20} \cdot \underline{3} = 60$

$\underline{20} + \underline{3} = 23$

$-15x^2 + 23x - 4$

$\underbrace{-15x^2 + 20x} \; \underbrace{+3x - 4}$

$-5x(3x-4) + 1(3x-4)$

$(3x-4)(-5x+1)$

$(3x-4)(-1)(5x-1)$

$\boxed{-1(3x-4)(5x-1)}$

More correct way

$-1(15x^2 - 23x + 4)$

$\underline{-20} \cdot \underline{-3} = 60$

$\underline{-20} + \underline{-3} = -23$

$\underbrace{15x^2 - 20x} \; -3x + 4$

$5x(3x-4) - 1(3x-4)$

$\boxed{-1(3x-4)(5x-1)}$

FACTORING BY SUBSTITUTION:

Sometimes our trinomials have variables with extra large exponents, or even use binomials in place of variables. To factor these, we can use substitution.

Example: Factor $2n^4 - 7n^2 - 15$

x^2 x

Substitute: $x = n^2$

$x^2 = (n^2)^2$

$2x^2 - 7x - 15$

$-10 \cdot 3 = -30$

$-10 + 3 = -7$

$\underbrace{2x^2 - 10x} + \underbrace{3x - 15}$

$2x(x-5) + 3(x-5)$

$(x-5)(2x+3)$

$\boxed{(n^2-5)(2n^2+3)}$

Lesson 5.5: Factoring

Example: Factor $2(x+1)^2 + 3(x+1) - 35$

Substitute: $z = (x+1)$

$2z^2 + 3z - 35$

$10 \cdot -7 = -70$

$10 + -7 = 3$

$2z^2 + 10z - 7z - 35$

$2z(z+5) - 7(z+5)$

$(z+5)(2z-7)$

$((x+1)+5)(2\underbrace{(x+1)}_{2x+2} - 7)$

$\boxed{(x+6)(2x-5)}$

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Can you?

Assignment:

Page 407: #'s 9, 11, 15, 21, 25, 29, 35, 39, 43, 47, 51, 55, 57, 59, 63, 65, 67, 79, 83

AND

Page 442: #'s 65, 69, 73, 75

(23 problems)