

Lesson 5.8: Polynomial Equations

Objectives:

- Solve polynomial equations using the Zero-Product Property.
- Solve equations involving polynomial functions.
- Model and solve problems involving polynomials.

Lesson 5.8: Polynomial Equations

SOLVING POLYNOMIAL EQUATIONS USING THE ZERO-PRODUCT PROPERTY

Zero-Product Property:

If the product of two (or more) numbers is zero, then at least one of the numbers is 0.

*If $ab = 0$, then $a = 0$
or $b = 0$, or both a and b equal 0.*

This property helps us solve polynomial equations, because when we factor, we make a *product* of factors, and if the product of factors equals 0, then one or more factors must equal 0.

Lesson 5.8: Polynomial Equations

Example 1: Solve by using the Zero Property

$$(x - 4)(3x + 2) = 0$$

$$\begin{array}{r} x - 4 = 0 \\ +4 \quad +4 \\ \hline \boxed{x = 4} \end{array}$$

$$\begin{array}{r} 3x + 2 = 0 \\ -2 \quad -2 \\ \hline 3x = -2 \\ \frac{3x}{3} = \frac{-2}{3} \\ \boxed{x = -\frac{2}{3}} \end{array}$$

Lesson 5.8: Polynomial Equations

STEPS TO SOLVING POLYNOMIAL EQUATIONS BY FACTORING:

- Step 1:** Write the equation in standard form -- all terms are on one side of the equation, and the equation is equal to zero. $ax^2 + bx + c = 0$
- Step 2:** Completely factor the polynomial expression on the left side of the equation.
- Step 3:** Set each factor found in Step 2 equal to zero (apply the zero-product property and split them up.)
- Step 4:** Solve each new equation for the variable.
- Step 5:** CHECK your answers by substituting each solution into the *original* equation.

Lesson 5.8: Polynomial Equations

Example 2: Solve by using the Zero Property

$$2x^2 + x = 6$$

$$\frac{2x^2 + x - 6 = 0}{\quad}$$

$$\underline{2x^2 + 4x} - \underline{3x - 6} = 0$$

$$2x(x+2) - 3(x+2) = 0$$

$$(x+2)(2x-3) = 0$$

$$x+2=0 \quad 2x-3=0$$

$$\boxed{x = -2 \quad x = \frac{3}{2}}$$

$$\underline{4} \cdot \underline{-3} = -12$$

$$\underline{4} + \underline{-3} = 1$$

Lesson 5.8: Polynomial Equations

Example 3: Solve by using the Zero Property

BE CAUTIOUS! Don't set each factor = 6x!

$$(2x + 5)(x - 3) = 6x$$

$-6x \quad -6x$

$$\frac{-10}{-10} = \frac{3}{3} = -30$$
$$\frac{-10}{-10} + \frac{3}{3} = -7$$

$$(2x + 5)(x - 3) - 6x = 0$$

$$2x^2 - 6x + 5x - 15 - 6x = 0$$

$$2x^2 - 7x - 15 = 0$$

$$\underbrace{2x^2 - 10x} + \underbrace{3x - 15} = 0$$

$$2x(x - 5) + 3(x - 5) = 0$$

$$(x - 5)(2x + 3) = 0$$

$$x - 5 = 0 \quad 2x + 3 = 0$$

$$x = 5 \quad x = -\frac{3}{2}$$

Lesson 5.8: Polynomial Equations

Example 4: Solve by using the Zero Property

$$n^3 + 4n^2 - 9n = 36$$

$$\frac{\quad \quad \quad -36 \quad -36}{n^3 + 4n^2 - 9n - 36 = 0}$$

$$n^2(n+4) - 9(n+4) = 0$$

$$(n+4)(n^2-9) = 0$$

$$(n+4)(n+3)(n-3) = 0$$

$$n+4=0 \quad n+3=0 \quad n-3=0$$

$$\boxed{n = -4 \quad n = -3 \quad n = 3}$$

Lesson 5.8: Polynomial Equations

SOLVING EQUATIONS CONTAINING POLYNOMIAL FUNCTIONS:

Example 5: Suppose $f(x) = x^2 - 4x + 6$

A) find the values of x such that $f(x) = 11$

$$\begin{array}{l} 11 = x^2 - 4x + 6 \\ -11 \qquad \qquad -11 \\ \hline 0 = x^2 - 4x - 5 \end{array} \quad \begin{array}{l} -5 \cdot 1 = -5 \\ -5 + 1 = -4 \end{array} \quad \left. \begin{array}{l} \text{check:} \\ f(5) = \dots \end{array} \right\}$$
$$0 = (x-5)(x+1)$$
$$x-5=0 \quad x+1=0$$

$x=5 \quad x=-1$

Lesson 5.8: Polynomial Equations

Example 5: Suppose $f(x) = x^2 - 4x + 6$

$$f(x) = 11$$

B) What points are on the graph of f ?

$$x = 5, -1$$

$$\begin{array}{l} (5, 11) \\ (-1, 11) \end{array}$$

Lesson 5.8: Polynomial Equations

Definition: A **zero** of a function $f(x)$ is any value of x such that $f(x) = 0$.

mean
same
thing

In addition, if x is a zero of a function, it is **also an x -intercept** of that function (the point on the x -axis where the graph crosses it).

Lesson 5.8: Polynomial Equations

Example 6: Find the zeros of $f(x) = 3x^2 - 8x - 35$

What are the x-intercepts of the graph? ←

$$0 = 3x^2 - 8x - 35$$

$$\begin{array}{l} \underline{-15} \cdot \underline{7} = -105 \\ \underline{-15} + \underline{7} = -8 \end{array}$$

$$0 = \underbrace{3x^2 - 15x}_{3x(x-5)} + \underbrace{7x - 35}_{7(x-5)}$$

$$0 = 3x(x-5) + 7(x-5)$$

$$0 = (x-5)(3x+7)$$

$$x-5=0$$

$$3x+7=0$$

$$x=5$$

$$x = -\frac{7}{3}$$

$$(5, 0)$$

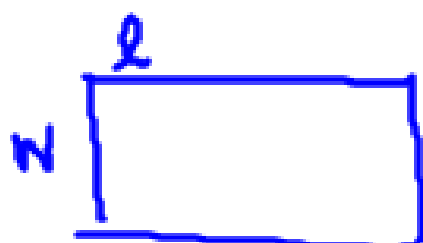
$$\left(-\frac{7}{3}, 0\right)$$

Lesson 5.8: Polynomial Equations

MODELING & SOLVING PROBLEMS INVOLVING POLYNOMIALS:

Example 7: The width of a rectangle is 7 feet less than its length. If the area of the rectangle is 78 square feet, what are the dimensions of the rectangle?

If the width (w) is 7 feet less than the length, we can say that $w = l - 7$



$$\begin{aligned} l &= 13 \text{ ft} \\ w &= 6 \text{ ft} \end{aligned}$$

$$A = l \cdot w$$

$$78 = l(l - 7)$$

$$0 = l(l - 7) - 78$$

$$0 = l^2 - 7l - 78$$

$$0 = (l + 6)(l - 13)$$

$$l + 6 = 0 \quad l - 13 = 0$$

$$\frac{6}{6} \cdot \frac{-13}{-13} = \frac{-78}{-78}$$

$$\frac{6}{6} + \frac{-13}{-13} = \frac{-7}{-7}$$

$$\cancel{l = -6} \quad l = 13$$

doesn't make sense to have length of -6

Lesson 5.8: Polynomial Equations

Objectives:

- Solve polynomial equations using the Zero-Product Property.
- Solve equations involving polynomial functions.
- Model and solve problems involving polynomials.

Lesson 5.8: Polynomial Equations

Homework:

Pg. 432: # 9, 11, 13, 17, 19, 21, 33, 37,
43, 47, 49, 51, 55, 59, 61, 73, 77, 83, 85, 91

AND

Pg. 444: # 117-129 odds