Lesson 6.1: Multiplying and Dividing Rational Expressions

By the end of the lesson, we will be able to:

~ Determine the domain of rational expressions and functions

~ Simplify, multiply, and divide rational expressions and functions

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A <u>RATIONAL EXPRESSION</u> is the quotient of two polynomials.

Examples:

$$\frac{x-5}{2x+1} \qquad \frac{x^2-7x-18}{x^2-4}$$

$$\frac{1}{x-3} \qquad \frac{2a^2+5ab+2b^2}{a^2-6ab+8b^2}$$

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A RATIONAL FUNCTION

is a function of the form $R(x) = \frac{p(x)}{q(x)}$

where $p(x)$ and $q(x)$ are polynomials

and q is not a zero polynomial.

The domain consists of all real numbers except those for which the denominator $q(x)$ is 0.

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Remember?

The Domain of an expression is all values of x that result in a defined value for y. This means that if we have a fraction, the denominator can never equal 0!

To find the domain of a rational expression, it is easier to determine what values x can't be.

Examples: Determine the domain for each of the following rational expressions or functions.

a.) $\frac{-3z}{z+5}$

$z \neq -5$

D: $\{z \mid z \neq -5\}$

b.) $\frac{n^2-2n-8}{n^2-n-12}$

$(n+3)(n-4)$

$n+3=0$, $-3\;-3$

$n-4=0$, $+4\;+4$, $n=4$

$n \neq -3,\ n \neq 4$

D: $\{n \mid n \neq -3, n \neq 4\}$

Examples: Determine the domain for each of the following rational expressions or functions.

$-4 \cdot 2 = -8$
$-4 + 2 = -2$

c.) $R(x) = \frac{x-3}{x^2-2x-8}$

$(x-4)(x+2)$

$x-4=0$ $x+2=0$

$D: \{x \mid x \neq 4, x \neq -2\}$

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SIMPLIFYING RATIONAL EXPRESSIONS/FUNCTIONS:

We simplify rational expressions and functions by dividing out any common factors.

NOTE!!! "Factors" means that we are dealing with a multiplication problem! If two terms are connected by a + or –, you CAN NOT reduce just one of the terms. You can only reduce sets of terms if the whole set is identical in both the numerator *and* the denominator.

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SIMPLIFYING RATIONAL EXPRESSIONS/FUNCTIONS:

Examples: Simplify

a.) $\dfrac{x^2+x-6}{2x^2-5x+2} = \dfrac{(x+3)(x-2)}{(x-2)(2x-1)}$

$= \dfrac{(x+3)}{(2x-1)}$

$-4 \cdot -1 = 4$

$-4 + -1 = -5$

$2x^2 - 4x - x + 2$

$2x(x-2) - 1(x-2)$

$(x-2)(2x-1)$

b.) $\dfrac{y^3+27}{2y^2+6y} = \dfrac{(y+3)(y^2-3y+9)}{2y(y+3)}$

$= \dfrac{y^2-3y+9}{2y}$

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MULTIPLYING RATIONAL EXPRESSIONS/FUNCTIONS:

Step 1: Completely factor each polynomial in the numerator and the denominator.

Step 2: Divide out common factors in the numerators and denominator.

Step 3: Multiply the remaining terms in the numerator together, and the remaining terms in the denominator together.

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Multiply Examples:

a.) $\frac{n^2-9}{n^2+5n+6} \cdot \frac{n+2}{6-2n}$ $= \frac{(n+3)(n-3)}{(n+3)(n+2)} \cdot \frac{(n+2)}{-2(n-3)}$

$6-2n = -2(-3+n)$

$= \boxed{\frac{1}{-2}}$

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Multiply Examples:

b.) $\frac{a^2-b^2}{10a^2-10ab} \cdot \frac{10a+5b}{2a^2+3ab+b^2} = \frac{(a+b)(a-b)}{10a(a-b)} \cdot \frac{5(2a+b)}{(2a+b)(a+b)}$

$2 \cdot 1 = 2$

$2 + 1 = 3$

$2a^2+2ab+1ab+b^2$

$2a(a+b)+b(a+b)$

$(a+b)(2a+b)$

$= \frac{5}{10a}$

$= \frac{1}{2a}$

DIVIDING RATIONAL EXPRESSIONS or FUNCTIONS:

To divide rational expressions, follow the rules for dividing regular fractions: Invert the second (or bottom) fraction, then multiply.

$$\frac{a}{b} \div \frac{c}{d} \rightarrow \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c}$$

DIVIDING RATIONAL EXPRESSIONS

Examples:

a.) $\dfrac{\frac{45z^4}{7y}}{\frac{5z}{21y^2}} = \dfrac{45z^4}{7y} \div \dfrac{5z}{21y^2} = \dfrac{45z^4}{7y} \cdot \dfrac{21y^2}{5z}$

$= \dfrac{9z^3 \cdot 3y}{1}$

$= \boxed{27yz^3}$

DIVIDING RATIONAL EXPRESSIONS

Examples:

b.) $\dfrac{\dfrac{p^3-8}{5p^2+15p}}{\dfrac{p^2-4}{p^2+3p}}$

$$= \frac{p^3-8}{5p^2+15p} \cdot \frac{p^2+3p}{p^2-4}$$

$$= \frac{(p-2)(p^2+2p+4)}{5p(p+3)} \cdot \frac{p(p+3)}{(p+2)(p-2)}$$

$$= \boxed{\frac{(p^2+2p+4)}{5(p+2)}}$$

WORKING WITH FUNCTIONS:

Sometimes we are given two or more functions and told to combine and simplify them.

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$$f(x) = \frac{x^2 - 9}{2x^2 - 8x} \qquad g(x) = \frac{x - 4}{x^2 + 4x + 3} \qquad h(x) = \frac{x^2 + 6x + 9}{x^2 - 5x}$$

Example: *find the given function and state the domain of each function.*

$D: \{x \mid x \neq 0, x \neq 4, x \neq -3, x \neq -1\}$

$$R(x) = f(x) \cdot g(x)$$

$$= \frac{x^2 - 9}{2x^2 - 8x} \cdot \frac{x - 4}{x^2 + 4x + 3}$$

$$= \frac{(x+3)(x-3)}{2x(x-4)} \cdot \frac{(x-4)}{(x+3)(x+1)}$$

$$R(x) = \frac{(x-3)}{2x(x+1)}$$

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$$f(x) = \frac{x^2 - 9}{2x^2 - 8x} \qquad g(x) = \frac{x - 4}{x^2 + 4x + 3} \qquad h(x) = \frac{x^2 + 6x + 9}{x^2 - 5x}$$

Example: *find the given function and state the domain of each function.*

D: {x | x ≠ 0, 4, −3}

$$A(x) = \frac{f(x)}{h(x)} = \frac{x^2-9}{2x^2-8x} \div \frac{x^2+6x+9}{x^2-5x}$$

$$= \frac{x^2-9}{2x^2-8x} \cdot \frac{x^2-5x}{x^2+6x+9} = \frac{(x+3)(x-3)}{2x(x-4)} \cdot \frac{x(x-5)}{(x+3)(x+3)}$$

$$A(x) = \frac{(x-3)(x-5)}{2(x-4)(x+3)}$$

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Can you?

Homework:

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(22 problems)