Lesson 6.2: Add & Subtract Rational Expressions

Objectives:

- Add or subtract rational expressions with a common denominator.
- Find the Least Common Denominator (LCD) of two or more rational expressions.
- Add or subtract rational expressions with unlike denominators.

ADDING OR SUBTRACTING RATIONAL EXPRESSIONS:

Step 1: Add or subtract according to the following rule:

If $\frac{a}{c}$ and $\frac{b}{c}$ are two rational expressions where $c \neq 0$, then $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$ and $\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$.

Step 2: Simplify the result. *(Factor the numerator and denominator, then reduce like terms.)*

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Example 1: Simplify.

a.) $\dfrac{x^2-42}{x+5}+\dfrac{7-2x}{x+5}$

$=\dfrac{(x^2-42)+(7-2x)}{(x+5)}$

$=\dfrac{x^2-2x-35}{(x+5)}$

$=\dfrac{(x-7)(x+5)}{(x+5)}$

$=\boxed{(x-7)}$

b.) $\dfrac{9x+1}{x+1}-\dfrac{6x-2}{x+1}$

$=\dfrac{(9x+1)-(6x-2)}{(x+1)}$

$=\dfrac{9x+1-6x+2}{(x+1)}$

$=\dfrac{3x+3}{x+1}=\dfrac{3(x+1)}{(x+1)}$

$=\boxed{3}$

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What if our denominators aren't exactly the same, but are additive inverses of each other (like x-2 and 2-x)?

We use the identity $\frac{a}{-b} = \frac{-a}{b}$ and change the signs of every term in both the numerator and the denominator (we multiply both the top and the bottom by -1). This doesn't change the value, but it gives us a common denominator.

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Example 2: Simplify.

a.) $\dfrac{3y}{y-4} + \dfrac{4}{4-y}$ (-1) (-1)

$= \dfrac{3y}{y-4} + \dfrac{-4}{y-4} = \boxed{\dfrac{3y-4}{y-4}}$

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Sometimes our rational expressions do not have common denominators, but the rules for adding and subtracting fractions require that the denominators be the same. When this is the case, we have to find the LEAST COMMON DENOMINATOR (LCD).

FINDING A LEAST COMMON DENOMINATOR:

Step 1: Factor each denominator completely. Write the factored form using powers.

Example: $x^2 + 4x + 4 = (x + 2)(x + 2) = (x + 2)^2$

Step 2: List all of the factors that are common to all of the denominators. If factors are common except for their power, list the factor with the *highest* power. Then list all factors that are not common. The resulting product is the Least Common Denominator.

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Example 3: Find the LCD.

a.) $\frac{4}{15xy^2}$ and $\frac{7}{18x^3y}$

$3 \cdot 5\, x\, y\, y$ $\quad$ $6 \cdot 3$

$2 \cdot 3$

$3 \cdot 5 \cdot x \cdot y \cdot y$ $\quad$ $2 \cdot 3 \cdot 3 \cdot x \cdot x \cdot x \cdot y$

C: $3xy$

U: $5\ y\ 2 \cdot 3\ x\ x$

LCD: $3 \cdot 5 \cdot 2 \cdot 3 x^3 y^2$

$90x^3y^2$

b.) $\frac{7}{x^2-x-2}$ and $\frac{3}{x^3-4x^2+4x}$

$(x+1)(x-2)$ $\quad$ $x(x^2-4x+4)$

$x(x-2)(x-2)$

C: $(x-2)$

U: $(x+1)(x-2)\,x$

LCD: $x(x+1)(x-2)(x-2)$

$x(x+1)(x-2)^2$

ADDING OR SUBTRACTING RATIONAL EXPRESSIONS WITH UNLIKE DENOMINATORS:

Factor

Step 1: Find the least common denominator.

Step 2: Rewrite each rational expression with the common denominator. *You will need to multiply out the numerator, but leave the denominator in factored form.*

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Step 3: Add or subtract the new rational expressions

Step 4: Simplify, if necessary.

↓ Factor

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Example 4: Simplify

a.) $\frac{x}{(x-4)} + \frac{(x-1)}{(x+2)}$

C: 1
U: $(x-4)(x+2)$
LCD: $(x-4)(x+2)$

$= \frac{x(x+2)}{(x-4)(x+2)} + \frac{(x-1)(x-4)}{(x+2)(x-4)}$

$= \frac{x^2+2x+x^2-5x+4}{(x-4)(x+2)}$

$= \boxed{\frac{2x^2-3x+4}{(x-4)(x+2)}}$

b.) $\frac{x+2}{x^2+x-6} + \frac{x-1}{x^2-4}$

C: $(x-2)$
U: $(x+3)(x+2)$
LCD: $(x-2)(x+2)(x+3)$

$\frac{x+2}{(x+3)(x-2)} \quad \frac{x-1}{(x+2)(x-2)}$

$= \frac{(x+2)(x+2)}{(x+3)(x-2)(x+2)} + \frac{(x-1)(x+3)}{(x+2)(x-2)(x+3)}$

$= \frac{x^2+4x+4+x^2+2x-3}{(x+3)(x-2)(x+2)}$

$= \boxed{\frac{2x^2+6x+1}{(x+3)(x-2)(x+2)}}$

Example 4: Simplify

c.) $\dfrac{a-2}{a^2+5a+6} - \dfrac{2a-3}{3a^2+9a}$

C: $(a+3)$

U: $3a(a+2)$

LCD: $3a(a+2)(a+3)$

$(a+3)(a+2)$ $\quad 3a(a+3)$

$$= \frac{(a-2)\,3a}{(a+3)(a+2)\,3a} - \frac{(2a-3)(a+2)}{3a(a+3)(a+2)} = \frac{3a^2-6a-(2a^2+a-6)}{3a(a+3)(a+2)}$$

$$= \frac{3a^2-6a-2a^2-a+6}{3a(a+3)(a+2)} = \frac{a^2-7a+6}{3a(a+3)(a+2)} = \frac{(a-6)(a-1)}{3a(a+3)(a+2)}$$

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Example 4: Simplify

C: $(a+2)(a-2)$
U: 1
LCD: $(a+2)(a-2)$

*common means in two or more denominators

d.) $\dfrac{5}{(a-2)}+\dfrac{5}{(a+2)}-\dfrac{6}{a^2-4}$

$(a+2)(a-2)$

$$= \frac{5(a+2)}{(a-2)(a+2)} + \frac{5(a-2)}{(a+2)(a-2)} - \frac{6}{(a-2)(a+2)} = \frac{5a+10+5a-10-6}{(a+2)(a-2)}$$

$$= \boxed{\frac{10a-16}{(a+2)(a-2)} = \frac{2(5a-3)}{(a+2)(a-2)}}$$

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Can you?

Homework:

Page 475: # 7, 9, 11, 13, 17, 19, 23, 25, 29, 33, 35, 37, 43, 45, 47, 53, 55, 67, 71, 73

(20 problems)