Lesson 6.3: Complex Rational Expressions

Objectives:

- Simplify a complex rational expression by simplifying the numerator and denominator separately.
- Simplify a complex rational expression by using the Least Common Denominator (LCD).

METHOD 1:

Simplify the Numerator and Denominator Separately

Step 1: Write the numerator (top) of the expression as a single rational expression (add or subtract).

Step 2: Write the denominator of the expression as a single rational expression (add or subtract).

Step 3: Rewrite the complex rational expression using your new numerator and denominator you found in steps 1 and 2.

Step 4: Simplify. (Invert the denominator and multiply, then factor and simplify.) KFC

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Example 1: Simplify.

LCD: $5x$

a.) $\dfrac{\frac{1}{5}+\frac{1}{x}}{\frac{x+5}{2}}$

Top: $\frac{1}{5}\cdot\frac{x}{x} + \frac{1}{x}\cdot\frac{5}{5} = \frac{x}{5x} + \frac{5}{5x} = \frac{x+5}{5x}$ Top

$\dfrac{\frac{x+5}{5x}}{\frac{x+5}{2}} = \dfrac{x+5}{5x} \cdot \dfrac{2}{x+5} = \dfrac{2}{5x}$

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LCD: $(x-2)(x+2)$

Example 1: Simplify.

b.) $\dfrac{\dfrac{x}{x-2}+\dfrac{1}{x^2-4}}{x+\dfrac{1}{x+2}}$ $\rightarrow$ $\dfrac{\dfrac{x(x+2)}{(x-2)(x+2)}+\dfrac{1}{(x-2)(x+2)}}{} = \dfrac{x^2+2x+1}{(x-2)(x+2)}$

METHOD 2:

Simplify using the Least Common Denominator

Step 1: Find the LCD among all the denominators in the complex rational expression.

Step 2: Multiply both the numerator and denominator of the complex rational expression by the LCD found in Step 1.

Step 3: Simplify the expression.

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Example 2: Simplify. LCD: $10x$

a.) $\dfrac{\frac{1}{5}+\frac{1}{x}}{\frac{x+5}{2}}$

$10x\left(\frac{1}{5}+\frac{1}{x}\right)$ and $10x\left(\frac{x+5}{2}\right)$

$$= \frac{\frac{10x\cdot 1}{5}+\frac{10x}{x}}{5\cdot 10x\left(\frac{x+5}{x}\right)} = \frac{2x+10}{5x(x+5)}$$

$$= \frac{2(x+5)}{5x(x+5)}$$

$$= \boxed{\frac{2}{5x}}$$

Example 2: Simplify.

C: $(x-3)$

U: $(x+3)(x)$

LCD: $x(x-3)(x+3)$

b.) $\dfrac{\frac{1}{x}+\frac{4}{(x-3)}}{\frac{x}{x^2-9}+\frac{1}{(x-3)}}$ $\quad (x+3)(x-3)$

$$= \frac{x(x-3)(x+3)\left(\frac{1}{x}+\frac{4}{(x-3)}\right)}{x(x-3)(x+3)\left(\frac{x}{(x+3)(x-3)}+\frac{1}{(x-3)}\right)} = \frac{x(x-3)(x+3)\left(\frac{1}{x}\right)+x(x-3)(x+3)\left(\frac{4}{(x-3)}\right)}{x(x-3)(x+3)\left(\frac{x}{(x-3)(x+3)}\right)+x(x-3)(x+3)\left(\frac{1}{(x-3)}\right)}$$

$$= \frac{(x-3)(x+3)+4x(x+3)}{x^2+x(x+3)} = \frac{x^2-9+4x^2+12x}{x^2+x^2+3x} = \frac{5x^2+12x-9}{2x^2+3x}$$

$$= \frac{5x^2+12x-9}{2x^2+3x}$$

$$= \boxed{\frac{(x+3)(5x-3)}{x(2x+3)}}$$

$15 \cdot -3 = -45$

$15 + -3 = 12$

$5x^2+15x-3x-9$

$5x(x+3)-3(x+3)$

$(x+3)(5x-3)$

Example 3: Simplify using BOTH methods.

$$\frac{x^{-2}-y^{-2}}{x^{-1}+y^{-1}} = \frac{\frac{1}{x^2}-\frac{1}{y^2}}{\frac{1}{x}+\frac{1}{y}}$$

Top: $\frac{1y^2}{x^2y^2} - \frac{1x^2}{y^2x^2}$

$= \frac{y^2-x^2}{x^2y^2} = \frac{(y+x)(y-x)}{x^2y^2}$

bottom: $\frac{1y}{xy} + \frac{1x}{yx} = \frac{y+x}{xy}$

$= \frac{\frac{(y+x)(y-x)}{x^2y^2}}{\frac{(y+x)}{xy}} =$

$$\frac{x^{-2}-y^{-2}}{x^{-1}+y^{-1}} = \frac{\left(\frac{1}{x^2}-\frac{1}{y^2}\right)x^2y^2}{\left(\frac{1}{x}+\frac{1}{y}\right)x^2y^2}$$

LCD: x^2y^2

$$= \frac{\frac{x^2y^2}{x^2} - \frac{x^2y^2}{y^2}}{\frac{x^2y^2}{x} + \frac{x^2y^2}{y}} =$$

$$= \frac{y^2-x^2}{xy^2+x^2y} = \frac{(y-x)(y+x)}{xy(y+x)}$$

$$= \boxed{\frac{y-x}{xy}}$$

Solution: $\frac{y-x}{xy}$

$$= \frac{(y-x)(y+x)}{x^2y^2} \cdot \frac{xy}{(y+x)}$$

$x \quad y$

$$= \boxed{\frac{y-x}{xy}}$$

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Can you?

Homework:

Pg. 484: # 5, 7, 9, 13, 17, 19, 21, 23, 25, 27, 31, 33, 35, 37, 39

AND

Pg. 487: "Putting the Concepts Together" #1 - 10 all (25 problems)