Lesson 6.4: Rational Equations

Objectives:

- Solve equations containing rational expressions.
- Solve equations containing rational functions.

Lesson 6.4: Rational Equations

Solving a Rational Equation

Step 1: Find the domain of the variable in the equation. (Look at every term!)

Step 2: Find the LCD of all of the denominators.

Step 3: Multiply every term in the equation (on both sides) by the LCD. (This should get rid of all of the denominators.)

Step 4: Solve the resulting equation.

Step 5: Verify your solution.

Lesson 6.4: Rational Equations

Example 1: Solve. $\dfrac{2}{2x-5} = \dfrac{6}{2x-3}$

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Example 1: Solve. $\dfrac{2}{2x-5} = \dfrac{6}{2x-3}$

Step 1: Find the domain. $\left\{x \middle| x \neq \frac{5}{2}, \frac{3}{2}\right\}$

Step 2: Find the LCD. $LCD = (2x-5)(2x-3)$

Step 3: Multiply every term by the LCD. $\dfrac{2(2x-5)(2x-3)}{(2x-5)} = \dfrac{6(2x-5)(2x-3)}{(2x-3)}$

Step 4: Solve. $2(2x-3) = 6(2x-5)$

$4x - 6 = 12x - 30$ (combine constants & x's)

$-4x + 30 \quad -4x + 30$

$24 = 8x$ (divide by 8)

$x = 3$

Step 5: CHECK IT!

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Example 2: Solve.

$$\frac{4}{3b} + \frac{1}{6b} = \frac{7}{2b} + \frac{1}{3}$$

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Example 2: Solve.

$$\frac{4}{3b} + \frac{1}{6b} = \frac{7}{2b} + \frac{1}{3}$$

Step 1: Domain = $\{b \mid b \neq 0\}$

Step 2: LCD: $= 6b$

Step 3: Multiply through. $\frac{4(6b)}{3b} + \frac{1(6b)}{6b} = \frac{7(6b)}{2b} + \frac{1(6b)}{3}$

Step 4: Solve. $4(2) + 1 = 7(3) + 1(2b) \quad \rightarrow \quad 9 = 21 + 2b$

$\rightarrow \quad -12 = 2b \quad \rightarrow \quad \boldsymbol{b = -6}$

Step 5: CHECK IT!

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Example 3: Solve.

$$\frac{x^2+8x+6}{x^2+3x-4} = \frac{3}{x-1} - \frac{2}{x+4}$$

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Example 3: Solve.

$$\frac{x^2+8x+6}{x^2+3x-4} = \frac{3}{x-1} - \frac{2}{x+4}$$

Step 1: Domain: $x^2 + 3x - 4 = (x+4)(x-1)$ *so* $Domain = \{x | x \neq -4, 1\}$

Step 2: LCD = (x-1)(x+4)

Step 3: $(x+4)(x-1)\frac{x^2+8x+6}{(x+4)(x-1)} = \frac{3(x+4)(x-1)}{(x-1)} - \frac{2(x-1)(x+4)}{(x+4)}$

Step 4: $x^2 + 8x + 6 = (3x + 12) - (2x - 2) \quad \rightarrow \quad x^2 + 8x + 6 = x + 14$

$\rightarrow x^2 + 7x - 8 = 0 \quad \rightarrow \quad (x+8)(x-1) = 0$

$x = -8, 1$ **HOWEVER!** Our domain says $x \neq 1$, so that is what we call an ***"Extraneous Solution"***. Therefore our answer should be: $\boldsymbol{x = -8}$

Step 5: CHECK IT!

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Example 4: Solve.

$$\frac{-15}{x^2+x-6} + \frac{3}{x-2} = \frac{2}{x+3}$$

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Example 4: Solve.

$$\frac{-15}{x^2+x-6} + \frac{3}{x-2} = \frac{2}{x+3}$$

Step 1: $x^2 + x - 6 = (x+3)(x-2)$ *so* $\{x|x \neq -3, 2\}$

Step 2: $LCD = (x+3)(x-2)$

Step 3: $\frac{-15(x+3)(x-2)}{(x+3)(x-2)} + \frac{3(x+3)(x-2)}{(x-2)} = \frac{2(x+3)(x-2)}{(x+3)}$

Step 4: $-15 + 3(x+3) = 2(x-2) \quad \rightarrow \quad -15 + 3x + 9 = 2x - 4$

$3x - 6 = 2x - 4 \quad \rightarrow \quad x = 2$

Our domain says $x \neq 2$, so our solution can't work. Therefore our answer is the empty set or no solution. $\{\}$ ***or*** $\emptyset$

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Example 5: Solve.

$$\frac{3x}{x+7} - \frac{3}{x-2} = -\frac{5x+17}{x^2+5x-14}$$

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Example 5: Solve.

$$\frac{3x}{x+7} - \frac{3}{x-2} = -\frac{5x+17}{x^2+5x-14}$$

Step 1: Domain: $\{x|x \neq -7, 2\}$

Step 2: $LCD = (x+7)(x-2)$

Step 3: $\frac{3x(x+7)(x-2)}{(x+7)} - \frac{3(x+7)(x-2)}{(x-2)} = \frac{-(5x+17)(x+7)(x-2)}{(x+7)(x-2)}$

Step 4: $3x(x-2) - 3(x+7) = -5x - 17 \rightarrow 3x^2 - 6x - 3x - 21 = -5x - 17$

$\rightarrow 3x^2 - 9x - 21 = -5x - 17 \rightarrow 3x^2 - 4x - 4 = 0$

$\rightarrow (3x+2)(x-2) = 0 \rightarrow x = \frac{-2}{3}$ $x = 2$ *but x can't equal 2,so*

$x = -\frac{2}{3}$

Step 5: CHECK IT!

Lesson 6.4: Rational Equations

Example 6: For the function $f(x) = x + \frac{3}{x}$ Solve for f(x)=4.

What points are on the graph when f(x)=y=4?

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Example 6: For the function $f(x) = x + \frac{3}{x}$ Solve for f(x)=4.

For the function $f(x) = x + \frac{3}{x}$, solve f(x) = 4. What point(s) are on the graph of f at $f(x) = y = 4$?

$4 = x + \frac{3}{x}$ LCD is x, so multiply everything by x.

$4(x) = x(x) + \frac{3}{x}(x) \rightarrow 4x = x^2 + 3 \rightarrow 0 = x^2 - 4x + 3 \rightarrow 0 = (x-3)(x-1) \rightarrow \boldsymbol{x = 3, x = 1}$

Since $f(3) = 4$, and $f(1) = 4$, there are points on the graph at $\boldsymbol{(3,4)}$ $\boldsymbol{and}$ $\boldsymbol{(1,4)}$

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Can you?

Homework:

Pg. 494: # 7, 9, 11, 15, 19, 23, 27, 29, 31, 33, 37, 41, 55, 57, 63

(15 problems)