

Lesson 6.4: Rational Equations

Objectives:

- ~ Solve equations containing rational expressions.
- ~ Solve equations containing rational functions.

Lesson 6.4: Rational Equations

Solving a Rational Equation

Step 1: Find the domain of the variable in the equation.
(Look at every term!)

Step 2: Find the LCD of all of the denominators.

Step 3: Multiply every term in the equation (on both sides) by the LCD. (This should get rid of all of the denominators.) (mult on top only)

Step 4: Solve the resulting equation.

Step 5: Verify your solution.

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Example 1: Solve.

$$\frac{2}{(2x-5)} = \frac{6}{(2x-3)}$$

$$\frac{2 \cancel{(2x-5)} \cancel{(2x-3)}}{\cancel{(2x-5)}} = \frac{6 \cancel{(2x-5)} \cancel{(2x-3)}}{\cancel{(2x-3)}}$$

$$2(2x-3) = 6(2x-5)$$

$$\begin{array}{r} 4x - 6 = 12x - 30 \\ -4x + 30 \quad -4x + 30 \\ \hline \end{array}$$

$$\frac{24}{8} = \frac{8x}{8}$$

$$\boxed{x=3}$$

$$D: x \neq \frac{5}{2}, \frac{3}{2}$$

$$\begin{array}{r} 2x-5 < 0 \\ +5 \quad +5 \\ \hline 2x = 5 \\ x = \frac{5}{2} \end{array}$$

C: 1

$$U: (2x-5)(2x-3)$$

$$LCD: (2x-5)(2x-3)$$

Check

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Example 1: Solve. $\frac{2}{2x-5} = \frac{6}{2x-3}$

Step 1: Find the domain. $\left\{x \mid x \neq \frac{5}{2}, \frac{3}{2}\right\}$

Step 2: Find the LCD. $LCD = (2x - 5)(2x - 3)$

Step 3: Multiply every term by the LCD. $\frac{2(2x-5)(2x-3)}{(2x-5)} = \frac{6(2x-5)(2x-3)}{(2x-3)}$

Step 4: Solve. $2(2x - 3) = 6(2x - 5)$

$$4x - 6 = 12x - 30$$

(combine constants & x's)

$$-4x + 30 = -4x + 30$$

$$24 = 8x$$

(divide by 8)

$$x = 3$$

Step 5: CHECK IT!

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$\text{D: } b \neq 0$

Example 2: Solve.

$$\frac{4}{3b} + \frac{1}{6b} = \frac{7}{2b} + \frac{1}{3}$$

\uparrow
2 · 3

C: 2 · 3 · b
U:
LCD: 6b

$$\frac{4 \cdot \overset{2}{\cancel{6b}}}{3\cancel{b}} + \frac{1 \cdot \overset{1}{\cancel{6b}}}{\cancel{6b}} = \frac{7 \cdot \overset{3}{\cancel{6b}}}{2\cancel{b}} + \frac{1 \cdot \overset{2}{\cancel{6b}}}{\cancel{b}}$$

$$4 \cdot 2 + 1 = 7 \cdot 3 + 1 \cdot 2b$$

$$8 + 1 = 21 + 2b$$

$$\begin{array}{r} 9 = 21 + 2b \\ -21 \quad -21 \\ \hline \end{array}$$

$$\frac{-12}{2} = \frac{2b}{2}$$

$b = -6$

check

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Example 2: Solve.

$$\frac{4}{3b} + \frac{1}{6b} = \frac{7}{2b} + \frac{1}{3}$$

Step 1: Domain = $\{b | b \neq 0\}$

Step 2: LCD: = $6b$

Step 3: Multiply through. $\frac{4(6b)}{3b} + \frac{1(6b)}{6b} = \frac{7(6b)}{2b} + \frac{1(6b)}{3}$

Step 4: Solve. $4(2) + 1 = 7(3) + 1(2b) \rightarrow 9 = 21 + 2b$
 $\rightarrow -12 = 2b \rightarrow b = -6$

Step 5: CHECK IT!

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$D: x \neq 1, x \neq -4$

Example 3: Solve.

$$\frac{x^2 + 8x + 6}{x^2 + 3x - 4} = \frac{3}{x-1} - \frac{2}{x+4}$$

$(x-1)(x+4)$

$C: (x-1)(x+4)$

U:

LCD: $(x-1)(x+4)$

$$\frac{(x^2 + 8x + 6) \cancel{(x-1)(x+4)}}{\cancel{(x-1)(x+4)}} = \frac{3 \cancel{(x-1)(x+4)}}{(x-1)} - \frac{2 \cancel{(x-1)(x+4)}}{\cancel{(x+4)}}$$

$(x-1)(x+8) = 0$

$x-1=0$ $x+8=0$

$x=1$ $x=-8$

$$x^2 + 8x + 6 = 3(x+4) - 2(x-1)$$

$$x^2 + 8x + 6 = 3x + 12 - 2x + 2$$

$$x^2 + 8x + 6 = x + 14$$

$$x^2 + 7x - 8 = 0$$

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Example 3: Solve.

$$\frac{x^2 + 8x + 6}{x^2 + 3x - 4} = \frac{3}{x - 1} - \frac{2}{x + 4}$$

Step 1: Domain: $x^2 + 3x - 4 = (x + 4)(x - 1)$ so Domain = $\{x | x \neq -4, 1\}$

Step 2: LCD = $(x - 1)(x + 4)$

$$\text{Step 3: } (x + 4)(x - 1) \frac{x^2 + 8x + 6}{(x + 4)(x - 1)} = \frac{3(x + 4)(x - 1)}{(x - 1)} - \frac{2(x - 1)(x + 4)}{(x + 4)}$$

$$\text{Step 4: } x^2 + 8x + 6 = (3x + 12) - (2x - 2) \rightarrow x^2 + 8x + 6 = x + 14 \\ \rightarrow x^2 + 7x - 8 = 0 \rightarrow (x + 8)(x - 1) = 0$$

$x = -8, 1$ **HOWEVER!** Our domain says $x \neq 1$, so that is what we call an "Extraneous Solution". Therefore our answer should be: $x = -8$

Step 5: CHECK IT!

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$D: x \neq -3, x \neq 2$

Example 4: Solve.

$$\frac{-15}{x^2+x-6} + \frac{3}{x-2} = \frac{2}{x+3}$$

(x+3)(x-2)
(x+3)(x-2)
(x+3)(x-2)
(x+3)(x-2)

C: $(x+3)(x-2)$
U:
LCD: $(x+3)(x-2)$

$$-15 + 3(x+3) = 2(x-2)$$
$$-15 + 3x + 9 = 2x - 4$$

$$\begin{array}{r} 3x - 6 = 2x - 4 \\ -2x \quad +6 \quad -2x \quad +6 \\ \hline \end{array}$$

~~$x = 2$~~

no solution

\emptyset

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Example 4: Solve.

$$\frac{-15}{x^2+x-6} + \frac{3}{x-2} = \frac{2}{x+3}$$

Step 1: $x^2 + x - 6 = (x + 3)(x - 2)$ so $\{x|x \neq -3, 2\}$

Step 2: $LCD = (x + 3)(x - 2)$

$$\text{Step 3: } \frac{-15(x+3)(x-2)}{(x+3)(x-2)} + \frac{3(x+3)(x-2)}{(x-2)} = \frac{2(x+3)(x-2)}{(x+3)}$$

$$\begin{aligned} \text{Step 4: } -15 + 3(x + 3) &= 2(x - 2) \quad \rightarrow \quad -15 + 3x + 9 = 2x - 4 \\ 3x - 6 &= 2x - 4 \quad \rightarrow \quad x = 2 \end{aligned}$$

Our domain says $x \neq 2$, so our solution can't work. Therefore our answer is the empty set or no solution. $\{ \}$ or \emptyset

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Example 5: Solve.

$$\frac{3x}{x+7} - \frac{3}{x-2} = -\frac{5x+17}{x^2+5x-14}$$

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Example 5: Solve.

$$\frac{3x}{x+7} - \frac{3}{x-2} = -\frac{5x+17}{x^2+5x-14}$$

Step 1: Domain: $\{x|x \neq -7, 2\}$

Step 2: LCD = $(x+7)(x-2)$

Step 3: $\frac{3x(x+7)(x-2)}{(x+7)} - \frac{3(x+7)(x-2)}{(x-2)} = \frac{-(5x+17)(x+7)(x-2)}{(x+7)(x-2)}$

Step 4: $3x(x-2) - 3(x+7) = -5x-17 \rightarrow 3x^2 - 6x - 3x - 21 = -5x - 17$
 $\rightarrow 3x^2 - 9x - 21 = -5x - 17 \rightarrow 3x^2 - 4x - 4 = 0$
 $\rightarrow (3x+2)(x-2) = 0 \rightarrow x = -\frac{2}{3} \quad x = 2 \text{ but } x \text{ can't equal } 2, \text{ so}$
 $x = -\frac{2}{3}$

Step 5: CHECK IT!

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Example 6: For the function $f(x) = \frac{x}{1} + \frac{3}{x}$ Solve for $f(x)=4$.

What points are on the graph when $f(x)=y=4$?

$$\frac{4}{1} = \frac{x}{1} + \frac{3}{x}$$

$$\begin{array}{r} 4x = x^2 + 3 \\ -4x \quad -4x \\ \hline \end{array}$$

$$0 = x^2 - 4x + 3$$

$$0 = (x-3)(x-1)$$

$$x-3=0 \quad x-1=0$$

$$\boxed{x=3 \quad x=1}$$

$$\text{D: } x \neq 0$$

$$\text{LCD: } x$$

$$\boxed{\begin{array}{c} (3, 4) \\ (1, 4) \end{array}}$$

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Example 6: For the function $f(x) = x + \frac{3}{x}$ Solve for $f(x)=4$.

For the function $f(x) = x + \frac{3}{x}$, solve $f(x) = 4$.

What point(s) are on the graph of f at $f(x) = y = 4$?

$$4 = x + \frac{3}{x}$$

LCD is x , so multiply everything by x .

$$4(x) = x(x) + \frac{3}{x}(x) \rightarrow 4x = x^2 + 3 \rightarrow 0 = x^2 - 4x + 3 \rightarrow 0 = (x - 3)(x - 1) \rightarrow x = 3, x = 1$$

Since $f(3) = 4$, and $f(1) = 4$, there are points on the graph at $(3, 4)$ and $(1, 4)$

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Objectives:

- ~ Solve equations containing rational expressions.
- ~ Solve equations containing rational functions.

Can you?

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Homework:

Pg. 494: # 7, 9, 11, 15, 19, 23, 27,
29, 31, 33, 37, 41, 55, 57, 63
(15 problems)