Lesson 6.5: Rational Inequalities

By the end of the lesson, we will be able to:

~Solve Rational Inequalities

Lesson 6.5: Rational Inequalities

A <u>rational inequality</u> is simply an inequality that contains a rational expression (a fraction) on **one or both** sides of the inequality.

<u>Examples:</u>

$$\frac{1}{x} > 1$$

$$\frac{3}{x-5} < \frac{4x}{2x-1} + \frac{1}{x}$$

$$\frac{x-1}{x-5} \leq 0$$

Things to remember when solving rational inequalities:

~ A positive number divided by a positive number is always a positive value.

~ A negative number divided by a negative number is always a positive value.

~ A positive number divided by a negative number (and vice versa) is always a negative value.

Things to remember when solving rational inequalities:

~ We can find important points on our graphs by finding values of the variable that makes the rational expression *equal to 0*, or that make the rational expression *undefined* (think domain).

The value of the rational expression may change signs (positive ⟷ negative) on either side of these important points.

Solving a Rational Inequality

Step 1: Simplify the inequality so that there is a single rational expression (in factored form) on one side of the inequality and 0 is on the other.

Step 2: Determine the numbers for which the rational expression is either equal to 0 or is undefined. To do this:

- To find the values where the expression will equal 0, set each factor of the *numerator* equal to 0 and simplify.
- To find the values where the expression in undefined, set each factor of the *denominator* equal to 0 and simplify.

Solving a Rational Inequality CONT...

Step 3: Graph the values found in step 2 on a number line (closed circle for values that make it equal to 0, open circles for values that make it undefined). This will separate the number line into intervals.

Step 4: Choose a test point in each interval to determine the sign (+ or -) of each factor in the numerator and denominator. Use these to determine the sign of the quotient.

- If the quotient is positive, then the rational expression will be positive for *all* numbers *x* in the interval.
- If the quotient is negative, then the rational expression will be negative for *all* numbers *x* in the interval.

Solving a Rational Inequality CONT...

Step 5: Shade the intervals that make the inequality true.

- If the rational expression is $>$ or ≥ 0, shade the positive intervals.
- If the rational expression is $<$ or ≤ 0, shade the negative intervals.
- Make sure you pay attention to whether you should use ( ) or [ ] to define your intervals.

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Example 1: Solve and graph on a number line.

$$\frac{x+3}{x-4} \geq 0$$

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Example 2: Solve and graph on a number line.

$$\frac{2x+3}{x-2} \leq 1$$

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Can you?

Homework:

Pg. 500: #'s

9, 15, 17, 21, 25, 27, 31, 33, 35, 37

(10 problems)