Lesson 6.5: Rational Inequalities

By the end of the lesson, we will be able to:

~Solve Rational Inequalities

$\leq \quad <$

$\geq \quad >$

Lesson 6.5: Rational Inequalities

A **rational inequality** is simply an inequality that contains a rational expression (a fraction) on **one or both** sides of the inequality.

Examples:

$$\frac{1}{x} > 1$$

$$\frac{3}{x-5} < \frac{4x}{2x-1} + \frac{1}{x}$$

$$\frac{x-1}{x-5} \leq 0$$

Lesson 6.5: Rational Inequalities

Things to remember when solving rational inequalities:

~ A positive number divided by a positive number is always a positive value. $\frac{+}{+} = +$

~ A negative number divided by a negative number is always a positive value. $\frac{-}{-} = +$

~ A positive number divided by a negative number (and vice versa) is always a negative value.

$\frac{+}{-} = -$ $\quad \frac{-}{+} = -$

Lesson 6.5: Rational Inequalities

Things to remember when solving rational inequalities:

~ We can find important points on our graphs by finding values of the variable that makes the rational expression *equal to 0*, or that make the rational expression *undefined* (think domain).

$\frac{0}{\neq}$

$\frac{\neq}{0}$

The value of the rational expression may change signs (positive ⟷ negative) on either side of these important points.

Solving a Rational Inequality

Step 1: Simplify the inequality so that there is a single rational expression (in factored form) on one side of the inequality and 0 is on the other.

Step 2: Determine the numbers for which the rational expression is either equal to 0 or is undefined. To do this:

- To find the values where the expression will equal 0, set each factor of the *numerator* equal to 0 and simplify.
- To find the values where the expression in undefined, set each factor of the *denominator* equal to 0 and simplify.

Solving a Rational Inequality CONT…

Step 3: Graph the values found in step 2 on a number line (closed circle for values that make it equal to 0, open circles for values that make it undefined). This will separate the number line into intervals.

Step 4: Choose a test point in each interval to determine the sign (+ or -) of each factor in the numerator and denominator. Use these to determine the sign of the quotient.

- If the quotient is positive, then the rational expression will be positive for *all* numbers *x* in the interval.
- If the quotient is negative, then the rational expression will be negative for *all* numbers *x* in the interval.

≥ 0 (pos.)
≤ 0 (neg)

Solving a Rational Inequality CONT...

Step 5: Shade the intervals that make the inequality true.

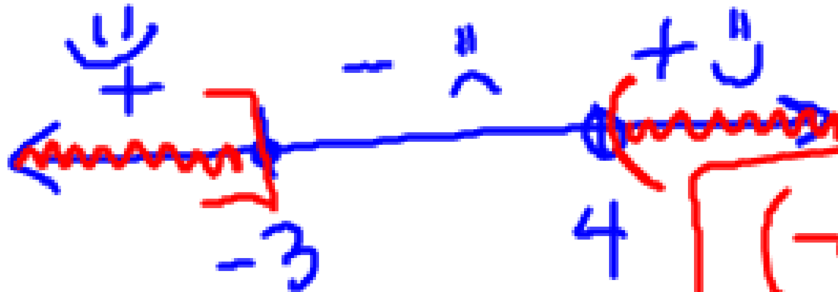
- If the rational expression is > or $\geq$ 0, shade the positive intervals.
- If the rational expression is < or $\leq$ 0, shade the negative intervals.
- Make sure you pay attention to whether you should use ( ) or [ ] to define your intervals.

Lesson 6.5: Rational Inequalities

Example 1: Solve and graph on a number line.

$$\frac{x+3}{x-4} \geq 0$$ RDS

• $x = -3$ ○ $x = 4$



$$\boxed{(-\infty, -3] \cup (4, \infty)}$$

Test $x=-4$

$\frac{-4+3}{-4-4} \geq 0$

$\frac{-}{-} \geq 0$

$+ \geq 0$ ✓

Test $x=0$

$\frac{+}{-} \geq 0$

$- \geq 0$

✗

Test $x=5$

$\frac{+}{+} \geq 0$

$+ \geq 0$

✓

Lesson 6.5: Rational Inequalities

Example 1: Solve and graph on a number line.

$$\frac{x+3}{x-4} \geq 0$$

Example 2: Solve and graph on a number line.

$$\frac{2x+3}{x-2} \le 1 \;\rightarrow\; \frac{2x+3}{x-2} - \frac{1(x-2)}{1(x-2)} \le 0 \;\rightarrow\; \frac{2x+3-1(x-2)}{(x-2)} \le 0$$

$$\rightarrow\; \frac{2x+3-x+2}{(x-2)} \le 0 \;\rightarrow\; \boxed{\frac{x+5}{x-2} \le 0}$$

• $x=-5$ o $x=2$

Test $x=-6$	Test $x=0$	Test $x=3$
$\frac{-}{-} \le 0$	$\frac{+}{-} \le 0$	$\frac{+}{+} \le 0$
$+ \le 0$	$- \le 0$	$+ \le 0$
X	✓	X

$$\boxed{[-5, 2)}$$

Lesson 6.5: Rational Inequalities

Example 2: Solve and graph on a number line.

$$\frac{2x+3}{x-2} \le 1$$

Lesson 6.5: Rational Inequalities

By the end of the lesson, we will be able to:

~Solve Rational Inequalities

Can you?

Homework:

Pg. 500: #'s

9, 15, 17, 21, 25, 27, 31, 33, 35, 37

(10 problems)

#25 $\frac{3}{x-2} \le \frac{4}{x+5}$ $\rightarrow$ $\frac{3(x+5)}{(x-2)(x+5)} - \frac{4(x-2)}{(x+5)(x-2)} \le 0$

$-\frac{4}{x+5}$ $\quad -\frac{4}{x+5}$

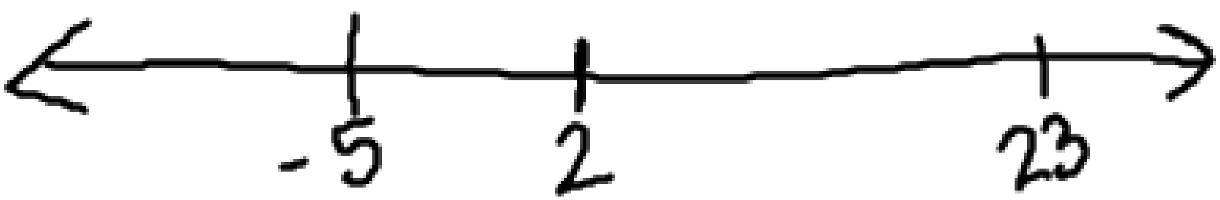
LCD: $(x-2)(x+5)$

$\rightarrow \frac{3(x+5) - 4(x-2)}{(x-2)(x+5)} \le 0 \rightarrow \frac{3x+15-4x+8}{(x-2)(x+5)} \le 0$

$\rightarrow \boxed{\frac{-x+23}{(x-2)(x+5)} \le 0}$

• $x = 23$

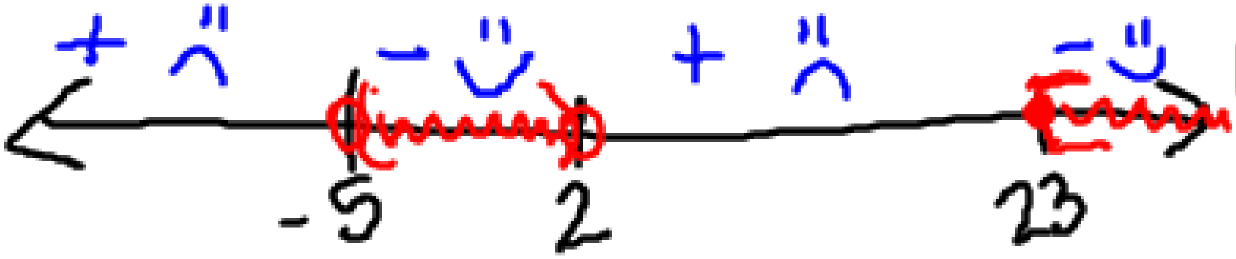
∘ $x = 2$ ∘ $x = -5$



$$\boxed{\frac{-x+23}{(x-2)(x+5)} \le 0}$$

$\bullet\ x = 23$

$\circ\ x = 2 \quad \circ\ x = -5$



$$\boxed{(-5,2) \cup [23,\infty)}$$

Test $x=-6$	Test $x=0$	Test $x=3$	Test $x=24$
$\frac{+}{(-)(-)} \le 0$	$\frac{+}{(-)(+)} \le 0$	$\frac{+}{(+)(+)} \le 0$	$\frac{-}{(+)(+)} \le 0$
$\frac{+}{+} \le 0$	$\frac{+}{-} \le 0$	$+ \le 0$	$- \le 0$
$+ \le 0$	$- \le 0$	X	✓
X	✓		