Lesson 6.6: Models using Rational Expressions

Objectives:

~ Solve for a variable in a rational expression.

~ Model and solve the following types of problems:

* Ratio & Proportion
* Work
* Uniform Motion
* Inverse Variation
* Joint or Combined Variation

SOLVING FOR A VARIABLE IN A RATIONAL EXPRESSION

Example: The formula $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$ is used in telescope and camera construction, where f is the focal length of the lens. The variable p is the distance from the object we wish to see and the lens, and q is the distance from the lens to the point of focus (the film or your eye).

a) Solve the formula for q.

LCD: fpq

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$pq = fq + fp$$

$$-fq \quad -fq$$

$$pq - fq = fp$$

$$\frac{q(p-f)}{(p-f)} = \frac{fp}{(p-f)}$$

$$q = \frac{fp}{(p-f)}$$

Lesson 6.6: Models using Rational Expressions

SOLVING FOR A VARIABLE IN A RATIONAL EXPRESSION

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

f

b) Suppose the camera has focal length 100 mm and is focusing on an object 2000 mm away. Find q.

p

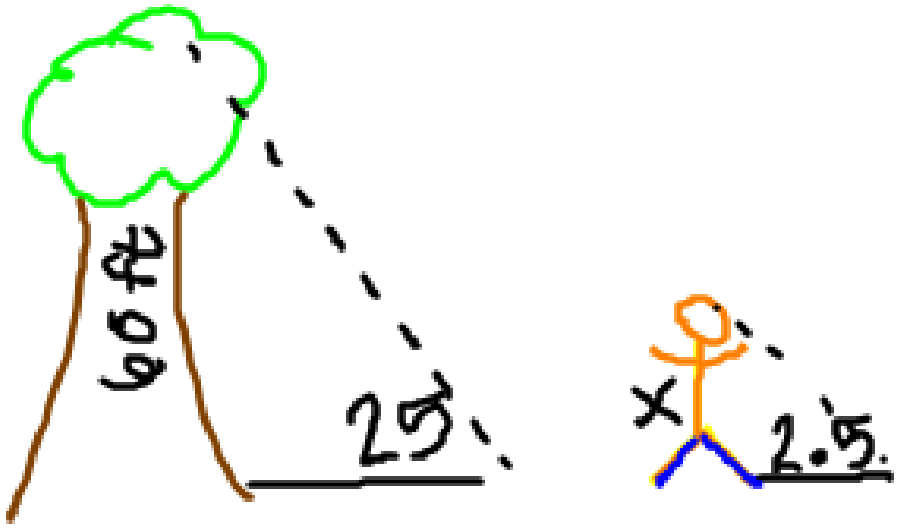
$$q = \frac{fp}{(p-f)} \rightarrow q = \frac{100(2000)}{2000-100}$$

$$= \frac{200000}{1900}$$

$$q = \frac{2000}{19} = \boxed{105.263 \text{ mm}}$$

MODEL & SOLVE RATIO AND PROPORTION PROBLEMS

Example: A 60-foot-tall tree casts a shadow of 25 feet. At the same time of day, a man casts a shadow of 2.5 feet. How tall is the man?



$$\frac{60}{x} = \frac{25}{2.5}$$

$$60(2.5) = 25x$$

$$\frac{150}{25} = \frac{25x}{25}$$

$$6 = x$$

6ft

MODEL & SOLVE RATIO AND PROPORTION PROBLEMS

Example: A horticulturist found that 175 of 1500 seedlings in an experiment did not germinate. How many seedlings, to the nearest whole, must he plant next year so that only 150 seedlings do not germinate?

$$\frac{175}{1500} = \frac{150}{x}$$

$$175x = 150(1500)$$

$$\frac{175x}{175} = \frac{225000}{175}$$

$x = 1286$ seedlings

$(x = 1285.71)$

MODEL & SOLVE WORK PROBLEMS

Example: It's Saturday, and Kevin needs to cut and edge the grass. At 9 a.m., Michael asks Kevin to go golfing at 11 a.m. (2 hrs) It usually takes Kevin 3 hours to cut and edge the grass. When Michael cuts and edges the grass, it takes 4 hours. If they work together, would they be able to finish the lawn and make the golf date?

yes, they can make it!

In one hour ...

Kevin can do 1/3 of job

Mike can do 1/4 of job

$$\frac{1}{3} + \frac{1}{4} = \frac{1}{t}$$

LCD: 12t

$$\frac{1(12t)}{3} + \frac{1(12t)}{4} = \frac{1(12t)}{t}$$

$$4t + 3t = 12$$

$$\frac{7t}{7} = \frac{12}{7}$$

$t = \frac{12}{7}$ hrs.

1.71 hrs

Lesson 6.6: Models using Rational Expressions

MODEL & SOLVE WORK PROBLEMS

x = speed of water

MODEL & SOLVE UNIFORM MOTION PROBLEMS

$d = r \cdot t$

Example: The speed of a boat in still water is 29 mph. If the boat travels 140 miles downstream in the same time it takes to travel 92 miles upstream, what is the speed of the stream? (x)

$\frac{d}{r} = \frac{r \cdot t}{r}$

$\frac{d}{r} = t$

downstream	upstream
$\frac{140}{29+x} = t$	$\frac{92}{29-x} = t$

$$\frac{140}{29+x} = \frac{92}{29-x}$$

$$140(29-x) = 92(29+x)$$

$$140(29-x) = 92(29+x)$$

$$4060 - 140x = 2668 + 92x$$

$$-2668 \quad +140x \quad -2668 \quad +140x$$

$$\frac{1392}{232} = \frac{232x}{232}$$

$$x = 6$$

6 mph (speed of stream)

MODEL & SOLVE PROBLEMS INVOLVING INVERSE VARIATION

Definition:

y varies inversely with x (or y is inversely proportional to x), if there is a nonzero number k such that $y = \frac{k}{x}$

MODEL & SOLVE PROBLEMS INVOLVING INVERSE VARIATION

Example: If the voltage in an electric circuit is held constant, the current I varies inversely with the resistance R. If the current is 60 (I) amperes when the resistance (R) is 150 ohms, find the current when the resistance is 120 ohms.

$I = \frac{k}{R}$

$60 = \frac{k}{150}$

$60(150) = k$

$9000 = k$

$I = \frac{9000}{R}$

$I = \frac{9000}{120}$

$I = 75$ amps

MODEL & SOLVE PROBLEMS INVOLVING COMBINED OR JOINT VARIATION

Definition: When a variable quantity Q is proportional to the product of 2 or more other variables, we say Q varies "jointly" with these variables.

For example, $y = kxz$ is read as "y varies jointly with x and z"

& same thing

$y = \frac{kx}{z}$ is read "y varies directly with x and inversely with z".

Note that in both cases the constant k is present.

MODEL & SOLVE PROBLEMS INVOLVING COMBINED OR JOINT VARIATION

Example: The kinetic energy ***E*** of a moving object varies jointly with its mass ***m*** and the square of its velocity ***v***. If an object weighing 25 kilograms and moving with a velocity of 10 meters per second has a kinetic energy of 1250 joules, find its kinetic energy when the velocity is 15 meters per second.

$E = k m v^2$

$1250 = k(25)(10^2)$

$1250 = k(25)(100)$

$\frac{1250}{2500} = \frac{k(2500)}{2500}$

$k = .5$

$E = .5 m v^2$

$E = .5(25)(15^2)$

$= .5(25)(225)$

$= .5(5625)$

$E = 2812.5$ joules

Lesson 6.6: Models using Rational Expressions

MODEL & SOLVE PROBLEMS INVOLVING COMBINED OR JOINT VARIATION

Lesson 6.6: Models using Rational Expressions

Objectives:

~ Solve for a variable in a rational expression.

~ Model and solve the following types of problems:

* Ratio & Proportion
* Work
* Uniform Motion
* Inverse Variation
* Joint or Combined Variation

Can you?

Homework:

Pg. 512: # 7, 9, 11, 15, 17, 19, 21, 23, 25, 27, 31, 35, 37, 41, 45, 49, 57

AND

Pg. 525: "Chapter 6 Test" # 1-13 odds.