Lesson 7.1: Radicals and Rational Exponents

Objectives:

- ~ Evaluate nth roots
- ~ Simplify Expressions of the form $\sqrt[n]{a^n}$
- ~ Evaluate Expressions of the form $a^{\frac{1}{n}}$
- ~ Evaluate Expressions of the form $a^{\frac{m}{n}}$

Lesson 7.1: Radicals and Rational Exponents

The principal nth root of a number, a, is symbolized by $\sqrt[n]{a}$ where $n \geq 2$ and an integer.

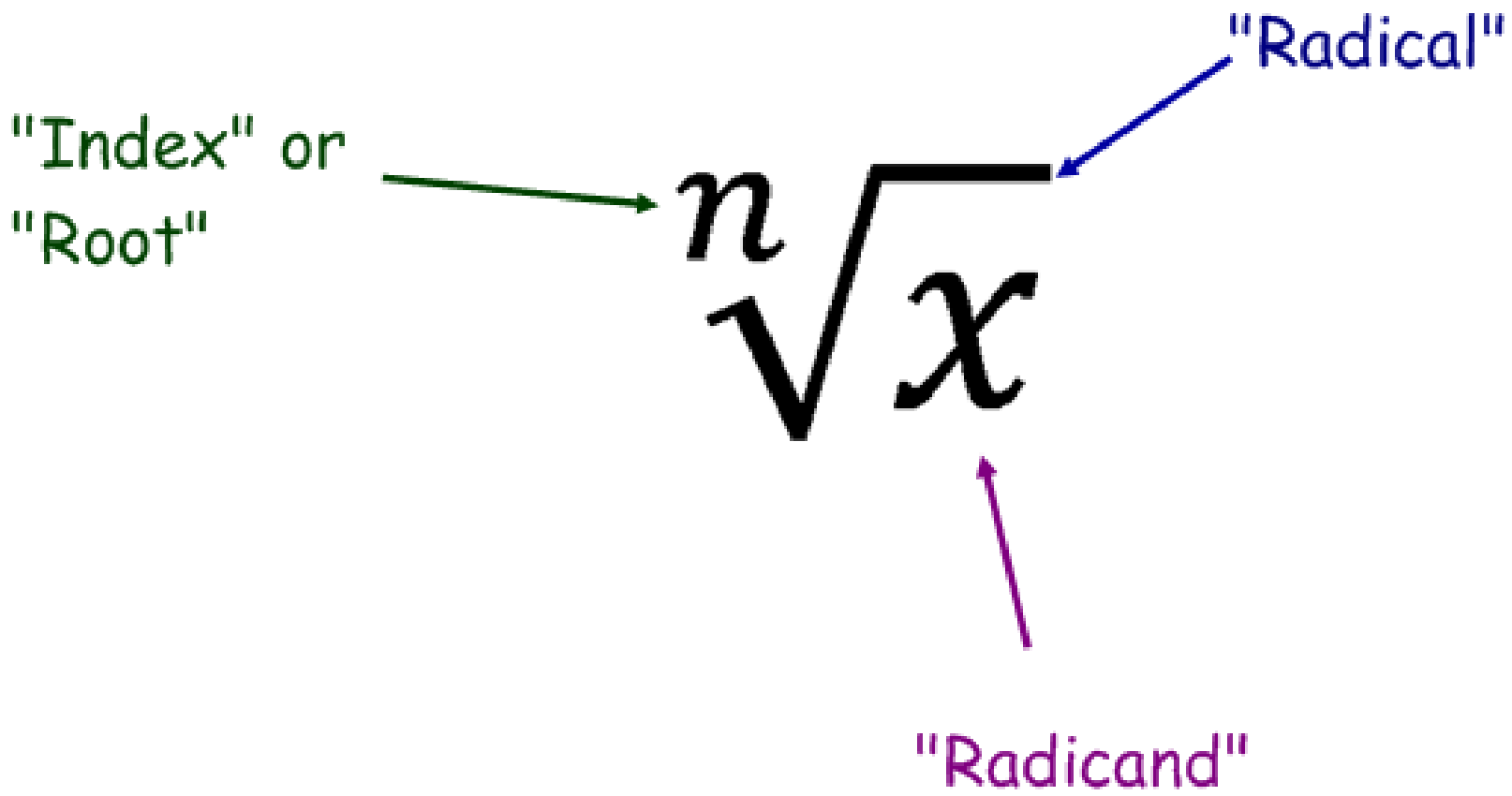
$$\sqrt[n]{a} = b \text{ means } a = b^n$$

~ If n is even, then a and b must be greater than or equal to 0 (pos.).

~ If n is odd, then a and b can be any real number (pos. or neg.).

Just a quick reminder...

Parts of a Radical:



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When we evaluate radicals, we really like "perfect" numbers. These are numbers than have exact answers. Here are some examples of perfect Squares, Cubes, Fourths, Fifths…

x	x^2	x^3	x^4	x^5	x^6	x^7
2	4	8	16	32	64	128
3	9	27	81	243	729	2187
4	16	64	256	1024	4096	
5	25	125	625	3125		
6	36	216	1296			
7	49	343	2401			
8	64	512	4096			
9	81	729				

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Use the what you know about "perfect" numbers to evaluate.

Example 1: Evaluate the following:

a.) $\sqrt[3]{27} = 3$

b.) $\sqrt[4]{256} = 4$

c.) $\sqrt[3]{-125} = -5$

d.) $\sqrt[4]{16} = 2$

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Simplify Expressions of the form $\sqrt[n]{a^n}$.

$$\sqrt[n]{a^n} = a \quad if\ n\ odd$$

$$\sqrt[n]{a^n} = |a| \quad if\ n\ is\ even$$

$\sqrt[3]{a^3} = a$

$\sqrt[2]{a^2} = |a|$

* This is not that important for us, because we are going to assume ALL variables are positive. But in the back of the book, you will see answers with absolute values.

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Example 2: Simplify the following:

a.) $\sqrt[5]{z^5} = z$

b.) $\sqrt[4]{(n-3)^4} = n-3$

book: $|n-3|$

c.) $-\sqrt[6]{(2)^6} = -2$

d.) $\sqrt[3]{\frac{27}{8}} = \frac{\sqrt[3]{27}}{\sqrt[3]{8}} = \frac{3}{2}$

Writing Radicals with Rational Exponents

Definition:

If a is a real number and n is an integer with $n \geq 2$, then

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

Example 3: Rewrite the following radicals with a rational exponent.

a.) $\sqrt[3]{9z}$

$= (9z)^{\frac{1}{3}}$

$= 9^{\frac{1}{3}} z^{\frac{1}{3}}$

b.) $\sqrt[4]{\frac{a^3 b}{7}}$

$= \left(\frac{a^3 b}{7}\right)^{\frac{1}{4}}$

$= \frac{a^{\frac{3}{4}} b^{\frac{1}{4}}}{7^{\frac{1}{4}}}$

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Evaluate Expressions of the Form $a^{\frac{m}{n}}$

Definition:

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$$

* Generally, taking the root FIRST is easier.

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*root first (bottom)

Example 4: Evaluate the following expressions.

a.) $9^{\frac{3}{2}} = 3^3 = 27$

$\sqrt[2]{9^3} = (\sqrt{9})^3$

b.) $125^{\frac{2}{3}} = 5^2 = 25$

$(\sqrt[3]{125})^2$

c.) $-36^{\frac{3}{2}} = -1(6^3)$

$-1 \cdot 36^{\frac{3}{2}} = -216$

d.) $(-27)^{\frac{4}{3}} = (-3)^4 = 81$

$(\sqrt[3]{-27})^4$

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Example 5: Rewrite the following with Rational Exponents

a.) $\sqrt[5]{x^3} = x^{\frac{3}{5}}$

b.) $2\sqrt[3]{a^2} = 2\left(a^{\frac{2}{3}}\right)$

$= 2a^{\frac{2}{3}}$

c.) $\left(\sqrt[3]{24xy^2}\right)^4 = (24xy^2)^{\frac{4}{3}}$

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What if the Exponent is negative? What happens?

$$a^{-\frac{m}{n}} = \frac{1}{a^{\frac{m}{n}}} \text{ or } \frac{1}{a^{-\frac{m}{n}}} = a^{\frac{m}{n}}$$

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Example 6: Rewrite each of the following with POSITIVE exponents and then completely simplify if possible.

a.) $49^{-\frac{1}{2}} = \frac{1}{49^{\frac{1}{2}}} = \boxed{\frac{1}{7}}$

b.) $\frac{1}{64^{-\frac{2}{3}}} = 64^{\frac{2}{3}} = 4^2 = \boxed{16}$

c.) $(3x)^{-\frac{3}{4}} = \boxed{\frac{1}{(3x)^{\frac{3}{4}}}}$

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Homework:

Pg. 540: #'s 9-67 odds

* If you do the Exponent Monster at home, you can get 15 points extra credit.

* A-day - we will be taking the Exponent Monster in class next class (timed - 15 minutes) for 15 more points Extra Credit.