Lesson 7.2: Simplifying Expressions using Laws of Exponents

Objectives:

- Use the Laws of Exponents to simplify expressions that contain rational exponents.
- Use the Laws of Exponents to simplify radical expressions. $\sqrt{}$
- Factor expressions containing rational exponents.

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The Laws of Exponents that we learned before when we worked with exponents that were integers, will also work for rational exponents. Here is a quick overview:

LAWS OF EXPONENTS:

Assuming that a and b are real numbers, and assuming the expression is defined (there aren't any denominators equal to zero)...

Zero Exponent Rule:	$a^0 = 1$	if $a \neq 0$
Negative-Exponent Rule:	$a^{-\frac{m}{n}} = \frac{1}{a^{\frac{m}{n}}}$	if $a \neq 0$

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Product Rule: $a^{\frac{m}{n}} \cdot a^{\frac{r}{s}} = a^{\left(\frac{m}{n}+\frac{r}{s}\right)}$

Quotient Rule: $\dfrac{a^{m/n}}{a^{r/s}} = a^{\left(\frac{m}{n}-\frac{r}{s}\right)} = \dfrac{1}{a^{\left(\frac{r}{s}-\frac{m}{n}\right)}}$ if $a \neq 0$

Power Rule: $\left(a^{\frac{m}{n}}\right)^{\frac{r}{s}} = a^{\frac{m}{n}\cdot\frac{r}{s}}$

Product to Power Rule: $(a \cdot b)^{\frac{m}{n}} = a^{\frac{m}{n}} \cdot b^{\frac{m}{n}}$

Quotient to Power Rule: $\left(\frac{a}{b}\right)^{\frac{m}{n}} = \dfrac{a^{\frac{m}{n}}}{b^{\frac{m}{n}}}$ if $b \neq 0$

Quotient to a Negative Power Rule: $\left(\frac{a}{b}\right)^{-\frac{m}{n}} = \left(\frac{b}{a}\right)^{\frac{m}{n}}$

if $a \neq 0,\ b \neq 0$

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"To Simplify" means the following:

- All exponents are positive.
- Each base occurs only once (we combine all x's, y's, numerical coefficients, etc.). $x^2 \cdot x^3 = \boxed{x^5}$
- There are no parentheses left in the expression.
- There are no powers written to powers left in the expression.

$$(y^2)^3 = \boxed{y^6}$$

$$y^2 y^2 y^2$$

EXAMPLES: Simplify each of the following.

A) $16^{2/3} \cdot 16^{5/6}$

$= 16^{\frac{2 \cdot 2}{3 \cdot 2} + \frac{5}{6}}$

$= 16^{\frac{4}{6} + \frac{5}{6}}$

$= 16^{\frac{9}{6}} = 16^{\frac{3}{2}} = 4^3$

$= \boxed{64}$

B) $\dfrac{4^{2/3}}{4^{-5/6}}$

$= 4^{\frac{2 \cdot 2}{3 \cdot 2} + \frac{5}{6}} = 4^{\frac{4}{6} + \frac{5}{6}}$

$= 4^{\frac{9}{6}} = 4^{\frac{3}{2}} = 2^3$

$= \boxed{8}$

EXAMPLES: Simplify each of the following.

C) $\left(4^{3/2}\right)^{5/3}$

$= 4^{\frac{\cancel{3}^{1}}{2} \cdot \frac{5}{\cancel{3}_{1}}}$

$= 4^{\frac{5}{2}} = 2^{5}$

$= \boxed{32}$

D) $\left(a^{-3/2}b^{1/4}\right)^{\frac{8}{1}}$

$= a^{\frac{-3}{\cancel{2}_1} \cdot \frac{\cancel{8}^4}{1}} \cdot b^{\frac{1}{\cancel{4}_1} \cdot \frac{\cancel{8}^2}{1}}$

$= a^{-12} \cdot b^{2}$

$= \boxed{\dfrac{b^2}{a^{12}}}$

EXAMPLES: Simplify each of the following.

E) $(x^{-4/3}y^{-2})(x^{2}y^{1/2})^{4/3}$

$$= \left(x^{-\frac{4}{3}}y^{-2}\right)\left(x^{\frac{2}{1}\cdot\frac{4}{3}}\, y^{\frac{1}{\cancel{2}_{1}}\cdot\frac{\cancel{4}^{2}}{3}}\right)$$

$$= \left(x^{-\frac{4}{3}}y^{-2}\right)\left(x^{\frac{8}{3}}y^{\frac{2}{3}}\right)$$

$$= x^{-\frac{4}{3}}\cdot x^{\frac{8}{3}}\cdot y^{-2}\cdot y^{2/3}$$

$$= x^{-\frac{4}{3}+\frac{8}{3}}\, y^{-\frac{2\cdot 3}{1\cdot 3}+\frac{2}{3}}$$

$$= x^{\frac{4}{3}}y^{-\frac{4}{3}}$$

$$\boxed{\frac{x^{\frac{4}{3}}}{y^{\frac{4}{3}}}}$$

EXAMPLES: Simplify each of the following.

F) $$\frac{(2x^{-1}y^{2/5})^5}{x^2y^2} = \frac{2^5 x^{-5} \cancel{y^2}}{x^2 \cancel{y^2}} = \frac{32}{x^2 x^5} = \boxed{\frac{32}{x^7}}$$

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EXAMPLE: Use Rational Exponents to simplify the radicals.

G) $\sqrt[6]{9^3} = 9^{\frac{3}{6}}$

$= 9^{\frac{1}{2}}$

$= \boxed{3}$

H) $\sqrt[3]{27a^3b^9}$

$= 27^{\frac{1}{3}} a^{\frac{3}{3}} \cdot b^{\frac{9}{3}}$

$= \boxed{3ab^3}$

EXAMPLE: Use Rational Exponents to simplify the radicals.

I) $\dfrac{\sqrt[4]{x^3}}{\sqrt{x}} = \dfrac{x^{\frac{3}{4}}}{x^{\frac{1}{2}}} = x^{\frac{3}{4}-\frac{1}{2}}$

$= x^{\frac{3}{4}-\frac{2}{4}} = x^{\frac{1}{4}}$

$= \boxed{\sqrt[4]{x}}$

J) $\sqrt{\sqrt[3]{n}} = \left(\sqrt[3]{n}\right)^{\frac{1}{2}}$

$= \left(n^{\frac{1}{3}}\right)^{1/2} = n^{\frac{1}{6}}$

$= \boxed{\sqrt[6]{n}}$

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Homework:

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(15 problems)