Objectives:

- Use the Product Property to Multiply and/or Simplify Radical Expressions.
- Use the Quotient Property to Simplify Radical Expressions.
- Multiply Radicals with Unlike Indices (like a Square Root times a Cube Root).

Product Property of Radicals:

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, then

$$\sqrt[n]{\boldsymbol{a}} \cdot \sqrt[n]{\boldsymbol{b}} = \sqrt[n]{\boldsymbol{a} \cdot \boldsymbol{b}}$$

Lesson 7.3: Simplifying Radical Expressions

Examples: Multiply

a.) $\sqrt{7} \cdot \sqrt{2}$

$= \boxed{\sqrt{14}}$

b.) $\sqrt[3]{3} \cdot \sqrt[3]{11}$

$= \boxed{\sqrt[3]{33}}$

Examples: Multiply

c.) $\sqrt{(n+4)} \cdot \sqrt{(n-4)}$

$= \sqrt{(n+4)(n-4)}$

$= \boxed{\sqrt{n^2 - 16}}$

Can't $\sqrt{\ }$ because connected w/a minus...

d.) $\sqrt[5]{7a^3} \cdot \sqrt[5]{3a}$

$= \boxed{\sqrt[5]{21a^4}}$

Simplifying a Radical Expression:

Step 1: Write each factor of the radicand as the product of 2 factors – one of which is a perfect power of the index.

Step 2: Write the radicand as the product of two radicals, one of which contains all the perfect powers of the index, the other that contains everything else (all multiplied back together).

Step 3: Take the nth root of each perfect power. Multiply this by the radical with leftover factors.

Lesson 7.3: Simplifying Radical Expressions

Examples: Multiply.

a.) $2\sqrt[3]{128}$

$2 \cdot 64$

$4 \cdot 4 \cdot 4$

$= 2\sqrt[3]{2} \cdot \sqrt[3]{64}$

$= 2 \cdot \sqrt[3]{2} \cdot 4$

$= \boxed{8\sqrt[3]{2}}$

b.) $\sqrt{75z^2}$

$25 \quad 3$

$= \sqrt{25} \cdot \sqrt{3} \cdot \sqrt{z^2}$

$= \boxed{5z\sqrt{3}}$

Lesson 7.3: Simplifying Radical Expressions

Examples: Multiply

c.) $\sqrt[4]{50}$

5 10

2 5

$\boxed{\text{Simplified } \sqrt[4]{50}}$

d.) $\dfrac{6-\sqrt{63}}{3}$

$\sqrt{63}$

21 3

7 3

$= \dfrac{\cancel{6}^{2}-\cancel{3}^{1}\sqrt{7}}{\cancel{3}_{1}}$

$= \boxed{2-\sqrt{7}}$

Lesson 7.3: Simplifying Radical Expressions

Simplifying Radicals with Variable Radicands:

When the radicand contains a variable that is not a perfect power of the index, it is simplest to rewrite that variable as a product of the perfect power of the index and the leftover value.

Shortcut: If you divide the exponent of the variable by the value of the index, the quotient (the whole number answer) will be the value of the exponent on the variable taken outside of the radical, and the remainder will be the exponent on the variable left inside the radical.

Shortcut: If you divide the exponent of the variable by the value of the index, the quotient (the whole number answer) will be the value of the exponent on the variable taken outside of the radical, and the remainder will be the exponent on the variable left inside the radical.

For example: $\sqrt[3]{x^7} = x^{\frac{7}{3}}$

$$7 \div 3 = 2\ remainder\ 1$$

$$x^{\frac{7}{3}} = x^2 \cdot x^{\frac{1}{3}} = x^2 \cdot \sqrt[3]{x^1} = x^2 \cdot \sqrt[3]{x}$$

Lesson 7.3: Simplifying Radical Expressions

Examples: Simplify.

a.) $\sqrt[3]{\overset{2^3}{8}a^5b^{10}}$

$= 2\sqrt[3]{a^3a^2\underbrace{b^9}_{b^3b^3b^3}b^1}$

$= \boxed{2ab^3\sqrt[3]{a^2b}}$

b.) $2\sqrt[3]{9x^2}\cdot\sqrt[3]{3x^2}$

$= 2\sqrt[3]{\underbrace{27}_{3^3}\underbrace{x^4}_{x^3x}}$

$= 2\cdot 3x\sqrt[3]{x}$

$= \boxed{6x\sqrt[3]{x}}$

Quotient Property of Radicals:

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, then

same index

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

Lesson 7.3: Simplifying Radical Expressions

Examples: Simplify.

a.) $\sqrt{\frac{24}{49}} = \frac{\sqrt{24}}{\sqrt{49}}$

$= \frac{\sqrt{24}}{7} = \frac{\sqrt{6(4)}}{7}$

$= \boxed{\frac{2\sqrt{6}}{7}}$

b.) $\frac{-2\sqrt[3]{250n}}{\sqrt[3]{2n^4}}$

$= -2\sqrt[3]{\frac{250n}{2n^4}}$

$= -2\sqrt[3]{\frac{125}{n^3}}$

$= \frac{-2\sqrt[3]{125}}{\sqrt[3]{n^3}} = \frac{-2 \cdot 5}{n} = \boxed{\frac{-10}{n}}$

Multiplying Radicals with Unlike Indices:

$x^{\frac{\text{power}}{\text{root}}}$

Step 1: Rewrite as Rational Exponents.

Step 2: Find the LCD of the terms exponents, and convert all exponents to the same denominator.

Step 3: Rewrite each term as a radical (they should all have the same index now).

Step 4: Combine all the terms together in one radical and simplify.

Example: $\sqrt[3]{4} \cdot \sqrt[4]{2} = 4^{\frac{1}{3}} \cdot 2^{\frac{1}{4}}$

$= 4^{\frac{4}{12}} \cdot 2^{\frac{3}{12}} = \sqrt[12]{4^4 \cdot 2^3} = \sqrt[12]{256 \cdot 8} = \boxed{\sqrt[12]{2048}}$

Lesson 7.3: Simplifying Radical Expressions

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Can you?

Lesson 7.3: Simplifying Radical Expressions

Homework:

Pg. 556: # 9, 13, 17, 19, 23, 27, 33, 37, 41, 47, 51, 53, 57, 61, 67, 73, 75, 79, 83, 89, 91, 95