Lesson 7.5: Rationalizing Radical Expressions

Objectives:

- Rationalize a denominator containing one term.
- Rationalize a denominator containing two terms.

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The term "**Rationalizing the Denominator**" means that we are making the denominator of a fraction be a rational number (no radicals).

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To Rationalize a Denominator with a single term:

1. Multiply both the numerator and denominator by the radical in the denominator. Follow this rule:

   If the radical in the denominator is in the form $\sqrt[n]{a}$, multiply both the numerator and denominator by $\sqrt[n]{a^{n-1}}$.

   Example: to rationalize $\frac{1}{\sqrt[3]{a}}$ multiply by $\frac{\sqrt[3]{a^2}}{\sqrt[3]{a^2}}$

2. Simplify the numerator and denominator if necessary.

   Why? Multiplying a square root by the identical square root will "cancel" out the radical, but multiplying a cube root ($\sqrt[3]{\square}$) by the same cube root just squares what's inside, it doesn't cube it, so it doesn't cancel the radical. Whatever value is inside the radical must eventually be raised to the power of the index in order to remove the radical.

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Examples: Rationalize the denominator of each expression.

a.) $\frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{\sqrt{5}}{\sqrt{25}} = \boxed{\frac{\sqrt{5}}{5}}$

b.) $\frac{\sqrt{3}}{\sqrt{32}} = \frac{\sqrt{3}}{4\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$16 \cdot 2$

$= \frac{\sqrt{6}}{4\sqrt{4}} = \frac{\sqrt{6}}{4(2)} = \boxed{\frac{\sqrt{6}}{8}}$

Examples: Rationalize the denominator of each expression.

c.) $\dfrac{3}{2\sqrt{5x}} \cdot \dfrac{\sqrt{5x}}{\sqrt{5x}} = \dfrac{3\sqrt{5x}}{2\sqrt{25x^2}}$

$= \dfrac{3\sqrt{5x}}{2(5x)} = \boxed{\dfrac{3\sqrt{5x}}{10x}}$

d.) $\dfrac{5}{\sqrt[3]{2}} \cdot \dfrac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}}$

$= \dfrac{5\sqrt[3]{4}}{\sqrt[3]{2^3}} = \boxed{\dfrac{5\sqrt[3]{4}}{2}}$

Examples: Rationalize the denominator of each expression.

e.) $\sqrt[3]{\frac{5}{12}} = \frac{\sqrt[3]{5}}{\sqrt[3]{12}} \cdot \frac{\sqrt[3]{12^2}}{\sqrt[3]{12^2}}$

$$= \frac{\sqrt[3]{5 \cdot 144}}{\sqrt[3]{12^3}} = \frac{\sqrt[3]{5 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 2 \cdot 2}}{12}$$

$$= \frac{\overset{1}{\cancel{2}}\sqrt[3]{90}}{\underset{6}{\cancel{12}}} = \boxed{\frac{\sqrt[3]{90}}{6}}$$

f.) $\frac{10}{\sqrt[4]{2z^2}} \cdot \frac{\sqrt[4]{2^3 z^2}}{\sqrt[4]{2^3 z^2}}$

$$= \frac{10\sqrt[4]{8z^2}}{\sqrt[4]{2^4 z^4}}$$

$$= \frac{\overset{5}{\cancel{10}}\sqrt[4]{8z^2}}{\underset{1}{\cancel{2}}z}$$

$$= \boxed{\frac{5\sqrt[4]{8z^2}}{z}}$$

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$3+\sqrt{2}$ or $\sqrt{x}-\sqrt{5}$

conj. $3-\sqrt{2}$ $\sqrt{x}+\sqrt{5}$

Rationalizing a Denominator containing two terms:

1. Multiply both the numerator and denominator by the "conjugate" of the denominator. We're using the property $(a+b)(a-b)=a^2-b^2$ here. The conjugate of $(a+b)$ is $(a-b)$ and vice versa.
2. Multiply by distributing the conjugate. (Use FOIL).
3. Simplify. Remember that you can only reduce by factors, so you would need a GCF that is the same in both the top and the bottom.

Examples: Rationalize the denominator of each expression.

g.) $\dfrac{11}{(6-\sqrt{3})} \cdot \dfrac{(6+\sqrt{3})}{(6+\sqrt{3})} = \dfrac{66+11\sqrt{3}}{36+\cancel{6\sqrt{3}}-\cancel{6\sqrt{3}}-\sqrt{9}}$

$= \dfrac{66+11\sqrt{3}}{36-3} = \dfrac{66+11\sqrt{3}}{33} = \dfrac{\cancel{11}(6+\sqrt{3})}{\cancel{11}\cdot 3}$

$= \boxed{\dfrac{6+\sqrt{3}}{3}}$

$\dfrac{\overset{6}{\cancel{66}}+\overset{1}{\cancel{11}}\sqrt{3}}{\underset{3}{\cancel{33}}}$

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Examples: Rationalize the denominator of each expression.

h.) $\frac{(\sqrt{5}-2)}{(\sqrt{5}+\sqrt{3})} \cdot \frac{(\sqrt{5}-\sqrt{3})}{(\sqrt{5}-\sqrt{3})}$

$$= \frac{\sqrt{25}-\sqrt{15}-2\sqrt{5}+2\sqrt{3}}{\sqrt{25}-\cancel{\sqrt{15}}+\cancel{\sqrt{15}}-\sqrt{9}} = \frac{5-\sqrt{15}-2\sqrt{5}+2\sqrt{3}}{5-3}$$

$$= \boxed{\frac{5-\sqrt{15}-2\sqrt{5}+2\sqrt{3}}{2}}$$

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Can you?

Late hmk 7.1 - 7.5
Due March 21st!

Homework:

Pg. 571: # 5, 9, 13, 17, 23, 29, 33, 37, 43, 47, 51, 55, 57, 65, 69, 73, 77

(17 problems)