Lesson 7.7: Solving Radical Equations

Objectives:

- Solve radical equations containing one radical.
- Solve radical equations containing two radicals.
- Solve for a variable in a radical equation.

Lesson 7.7: Solving Radical Equations

To solve an equation with a single radical term:

Step 1: Isolate the radical. (Get it by itself on one side of the equation – *everything* else goes to the other side.)

Step 2: Raise both sides of the equation to the <u>power of the index</u>. (If you have more than one term on the non-radical side, you may need to rewrite and distribute to simplify that side.) This should eliminate the radical.

Step 3: Solve the equation that results (find the value for the variable).

★Step 4: CHECK YOUR ANSWERS! When we solve radicals that have even indices, we often get what we refer to as *"extraneous solutions"*, or solutions that appear to be valid, but that do not work in the original equation.

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Examples: Solve the following equations.

a.) $\left(\sqrt{4x+1}\right)^2 = (5)^2$

$$4x+1 = 25$$
$$\underline{-1 \quad\quad -1}$$
$$\frac{4x}{4} = \frac{24}{4}$$

$$\boxed{x = 6}$$

Check:

$$\sqrt{4(6)+1} = 5$$
$$\sqrt{24+1} \overset{?}{=} 5$$
$$\sqrt{25} = 5 \checkmark$$

Examples: Solve the following equations.

b.) $\sqrt{5x+6} - 3 = -2$

$\phantom{\sqrt{5x+6}}\ +3\ \ \ +3$

$(\sqrt{5x+6})^2 = (1)^2$

$5x+6 = 1$

$-6\ \ \ -6$

$\dfrac{5x}{5} = \dfrac{-5}{5}$

$\boxed{x = -1}$

check:

$\sqrt{5(-1)+6}\ -3 = -2$

$\sqrt{1}\ -3 = -2$

$1-3 = -2\ \checkmark$

Lesson 7.7: Solving Radical Equations

Examples: Solve the following equations.

C.) $\sqrt{3x - 5} + 8 = 3$
      $\quad\quad\quad\underline{-8 \quad -8}$

$\left(\sqrt{3x-5}\right)^2 = (-5)^2$

$3x - 5 = 25$
$\underline{+5 \quad\quad +5}$
$\dfrac{3x}{3} = \dfrac{30}{3}$

~~$x = 10$~~ $\boxed{\text{No Solution}}$

$\rightarrow \sqrt{3x-5} = -5 \; ?$

No Solution

Check: x=10

$\sqrt{3(10)-5} + 8 = 3$

$\sqrt{25} + 8 \neq 3$

$5 + 8 \neq 3$

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Examples: Solve the following equations.

d.) $\left(\sqrt{3x-11}\right)^2 = (x-5)^2$

$3x - 11 = (x-5)(x-5)$

$3x - 11 = x^2 - 10x + 25$

$\begin{array}{cc} -3x + 11 & -3x + 11 \end{array}$

$0 = x^2 - 13x + 36$

$0 = (x-9)(x-4)$

$x - 9 = 0 \quad x - 4 = 0$

$\boxed{x = 9 \qquad x = 4}$

Check: $x = 9$

$\sqrt{3(9) - 11} = 9 - 5$

$\sqrt{16} = 4 \checkmark$

Check: $x = 4$

$\sqrt{3(4) - 11} = 4 - 5$

$\sqrt{1} \neq -1$ ✗

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Examples: Solve the following equations.

e.) $\sqrt[3]{2x+3} + 5 = 8$

$\phantom{\sqrt[3]{2x+3}}\underline{-5 \quad -5}$

$\left(\sqrt[3]{2x+3}\right)^3 = (3)^3$

$2x+3 = 27$

$\underline{-3 \quad -3}$

$\dfrac{2x}{2} = \dfrac{24}{2}$

$\boxed{x = 12}$

Check

$\sqrt[3]{2(12)+3} + 5 = 8$

$\sqrt[3]{27} + 5 = 8$

$3 + 5 = 8 \checkmark$

Examples: Solve the following equations.

f.) $(2y - 1)^{1/2} - 2 = 3$

$\sqrt{2y-1} - 2 = 3$

$\phantom{\sqrt{2y-1}} \quad +2 \quad +2$

$(\sqrt{2y-1})^2 = (5)^2$

$2y - 1 = 25$

$ +1 \quad +1$

$\dfrac{2y}{2} = \dfrac{26}{2}$

$\boxed{y = 13}$

Check:

$\sqrt{2(13)-1} - 2 = 3$

$\sqrt{25} - 2 = 3$

$5 - 2 = 3 \checkmark$

Lesson 7.7: Solving Radical Equations

Solving a radical equation containing two terms:

Step 1:   Isolate one of the radicals. (Get it by itself on one side of the
biggest ➔ equation – _everything_ else goes to the other side.)

Step 2:   Raise both sides of the equation to the power of the index.
            (If you have more than one term on the non-radical side,
            you may need to rewrite and distribute to simplify that side.)
            This should eliminate one or both radicals.

Step 3:   If a radical still remains, follow the steps we used to solve
            equations with just one radical. Solve the equation that
            results (find the value for the variable).

Step 4:   CHECK YOUR ANSWERS! Remember we may get
            extraneous solutions.

Lesson 7.7: Solving Radical Equations

Examples: Solve the following equations.

g.$\left(\sqrt[3]{a^2 - 3a + 5}\right)^3 = \left(\sqrt[3]{2a^2 - 6a - 23}\right)^3$

$a^2 - 3a + 5 = 2a^2 - 6a - 23$

$\underline{-a^2 + 3a - 5 \qquad -a^2 + 3a - 5}$

$0 = a^2 - 3a - 28$

$0 = (a - 7)(a + 4)$

$a - 7 = 0 \qquad a + 4 = 0$

$\boxed{a = 7 \qquad a = -4}$

Check on next slide

$$\sqrt[3]{a^2 - 3a + 5} = \sqrt[3]{2a^2 - 6a - 23} \qquad 7, -4$$

Check: $a = 7$

$$\sqrt[3]{(7)^2 - 3(7) + 5} = \sqrt[3]{2(7)^2 - 6(7) - 23}$$

$$\sqrt[3]{49 - 21 + 5} = \sqrt[3]{98 - 42 - 23}$$

$$\sqrt[3]{33} = \sqrt[3]{33} \checkmark$$

check $a = -4$

$$\sqrt[3]{(-4)^2 - 3(-4) + 5} = \sqrt[3]{2(-4)^2 - 6(-4) - 23}$$

$$\sqrt[3]{16 + 12 + 5} = \sqrt[3]{32 + 24 - 23}$$

$$\sqrt[3]{33} = \sqrt[3]{33} \checkmark$$

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Examples: Solve the following equations.

h.) $\sqrt{x+5} - \sqrt{x} = 1$

$\underline{ + \sqrt{x} + \sqrt{x}}$

$(\sqrt{x+5})^2 = (1+\sqrt{x})^2$

$x+5 = (1+\sqrt{x})(1+\sqrt{x})$

$x+5 = 1 + \sqrt{x} + \sqrt{x} + x$

$x+5 = 1 + x + 2\sqrt{x}$

$\underline{-x-1 -1-x}$

$\dfrac{4}{2} = \dfrac{2\sqrt{x}}{2}$

$(2)^2 = (\sqrt{x})^2$

$\boxed{4 = x}$

Check:

$\sqrt{4+5} - \sqrt{4} = 1$

$\sqrt{9} - 2 =$

$3 - 2 = 1 \checkmark$

Lesson 7.7: Solving Radical Equations

Example i.): Solve the following equations.

Given the formula $r = \sqrt{\dfrac{3V}{\pi h}}$ where r is the radius of a cone, V is the volume, and h is the height.

a) Solve the equation for h.

$$(r)^2 = \left(\sqrt{\frac{3V}{\pi h}}\right)^2$$

$$h\,r^2 = \frac{3V}{\pi h} \cdot \overset{K}{\pi K}$$

$$\frac{h\,r^2}{r^2} = \frac{\frac{3V}{\pi}}{r^2}$$

$$h = \frac{3V}{\pi} \cdot \frac{1}{r^2}$$

$$\boxed{h = \frac{3V}{\pi r^2}}$$

Example i.): Solve the following equations.

Given the formula $r = \sqrt{\dfrac{3V}{\pi h}}$ where r is the radius of a cone, V is the volume, and h is the height.

$$h = \dfrac{3V}{\pi r^2}$$

b) Find h when $r = 3cm$ and $V = 18.85$ cubic cm. (Plug values in.)

$$h = \dfrac{3(18.85)}{\pi \, (3)^2}$$

$$h = \dfrac{56.55}{(9\pi)}$$

$\rightarrow h = 2.000047118$

$$\boxed{h = 2 \text{ cm}}$$

Objectives:

- Solve radical equations containing one radical.
- Solve radical equations containing two radicals.
- Solve for a variable in a radical equation.

Can you?

Homework:

Pg. 587: # 7, 13, 17, 23, 29, 37, 41, 45, 51, 53, 55, 59, 61, 77, 81, 85, 87

(17 problems)