Objectives:

- Evaluate the square root of negative real numbers.
- Add or subtract complex numbers.
- Multiply or divide complex numbers.
- Evaluate the powers of i.

Definitions:

- The **imaginary unit**, denoted by i, is the number whose square equals -1. That is:

$$i^2 = -1 \quad \text{or} \quad i = \sqrt{-1}$$

- **Complex numbers** are numbers in the form $a + bi$, where the real number a is called the real part, and the real number b is called the imaginary part. If $a = 0$, we call the remaining part (bi) a *pure imaginary number*.

Lesson 7.8: Complex Numbers

Evaluating Square Roots of Negative Numbers:

$$\sqrt{-N} = \sqrt{N(-1)} = \sqrt{N} \cdot \sqrt{-1} = \sqrt{N} \cdot i$$

where $i = \sqrt{-1}$

Examples: Evaluate the radicals

a.) $\sqrt{-25}$

b.) $\sqrt{-2}$

Examples: Evaluate the radicals

c.) $\sqrt{-48}$

Examples: Write in Standard Form

d.) $3 - \sqrt{-16}$

e.) $5 + \sqrt{-12}$

Examples: Write in Standard Form

f.) $\dfrac{15-\sqrt{-75}}{5}$

Adding or Subtracting Complex Numbers:

Two complex numbers are added or subtracted by combining like terms. Real parts are added, then imaginary parts are added.

Sum of complex numbers:

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

Difference of complex numbers:

$$(a + bi) - (c + di) = (a - c) + (b - d)i$$

Examples: Add or Subtract

g.) $(2 + 3i) + (-6 + 7i)$

h.) $\left(5 + \sqrt{-36}\right) - \left(2 - \sqrt{-49}\right)$

Multiplying Complex numbers:

We multiply complex numbers just like any other polynomial -- by distribution.

- $(ai)(bi) = (ab)i^2 = \ (ab)(-1) = \ -ab$
- Multiplying conjugates: $(a + bi)(a - bi) = a^2 + b^2$

Examples: Multiply

i.) $\sqrt{-49} \cdot \sqrt{-4}$

j.) $2i(5 - 3i)$

Examples: Multiply

k.) $(5 - 2i)(-1 + 3i)$

l.) $(3 + 2i)(3 - 2i)$

Dividing Complex numbers:

Step 1: Write the numerator and denominator in standard complex form $(a + bi)$.

Step 2: Multiply both the numerator and denominator by the conjugate of the denominator.

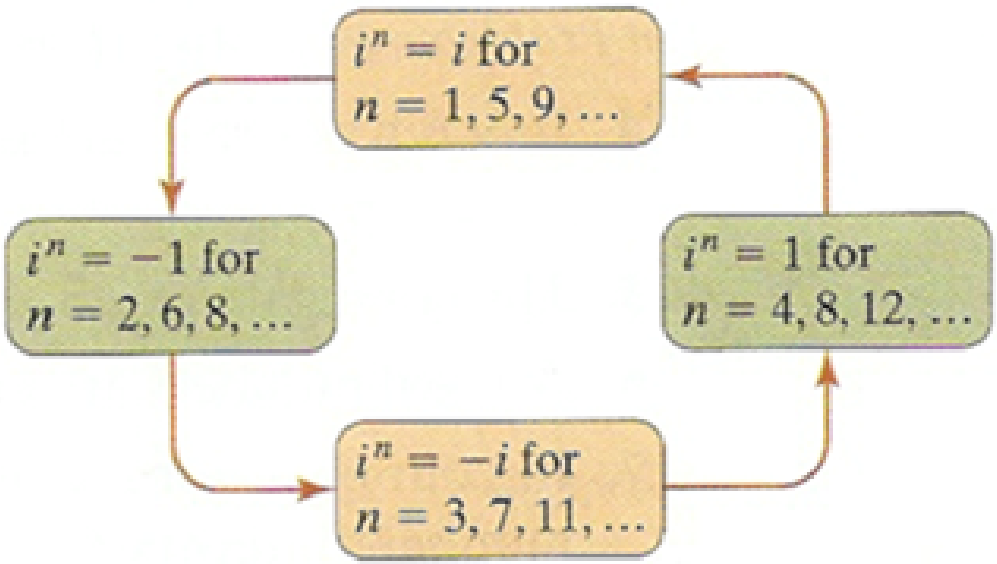
Step 3: Simplify by writing the quotient in standard form $(a + bi)$.

Examples: Divide

m.) $\frac{6+5i}{3i}$

n.) $\frac{2-i}{4+3i}$

Powers of i: The powers of i follow a pattern.



Simplifying Powers of i:

Step 1: Divide the exponent of i by 4. Rewrite i^n as $(i^4)^q \cdot i^r$ where q is the quotient and r is the remainder of the division.

Step 2: Simplify the product in Step 1 to just i^r, since $i^4 = 1$. Remember: $i^0 = 1$, $i^1 = i$, $i^2 = -1$, $i^3 = -i$ You should not have any exponents remaining in your final answer!

Examples: Simplify

o.) i^{27}

p.) i^{38}

Objectives:

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- Evaluate the powers of i.

Can You?

Homework:

Pg. 598: # 11, 15 ,21, 25, 29, 33, 37, 41,
47, 51, 55, 59, 61, 65, 67, 71, 75, 79, 81, 83
85, 87, 103, 107, 109, 113

(26 problems)