Objectives:

- Evaluate the square root of negative real numbers.
- Add or subtract complex numbers.
- Multiply or divide complex numbers.
- Evaluate the powers of i.

Definitions:

- The **imaginary unit**, denoted by i, is the number whose square equals -1. That is:

$$i^2 = -1 \quad \text{or} \quad i = \sqrt{-1}$$

- **Complex numbers** are numbers in the form $a + bi$, where the real number a is called the real part, and the real number b is called the imaginary part. If $a = 0$, we call the remaining part (bi) a *pure imaginary number*.

Lesson 7.8: Complex Numbers

Evaluating Square Roots of Negative Numbers:

$$\sqrt{-N} = \sqrt{N(-1)} = \sqrt{N} \cdot \sqrt{-1} = \sqrt{N} \cdot i$$

where $i = \sqrt{-1}$

Examples: Evaluate the radicals

a.) $\sqrt{-25}$

$i\sqrt{25}$

$5i$

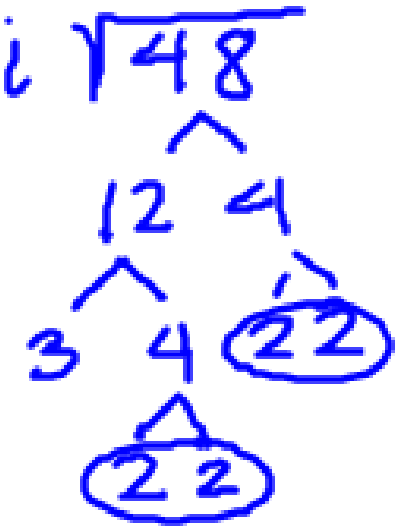
b.) $\sqrt{-2}$

$i\sqrt{2}$

Examples: Evaluate the radicals

c.) $\sqrt{-48}$

$= i\sqrt{48}$



$= 4i\sqrt{3}$

a+bi

Examples: Write in Standard Form

d.) $3 - \sqrt{-16}$

$= 3 - 4i$

e.) $5 + \sqrt{-12}$

$= 5 + i\sqrt{12}$

$12 = 4 \cdot 3$

$4 = 2 \cdot 2$

$= 5 + 2i\sqrt{3}$

Lesson 7.8: Complex Numbers

Examples: Write in Standard Form

f.) $\dfrac{15-\sqrt{-75}}{5} = \dfrac{15-i\sqrt{75}}{5} = \dfrac{\cancel{15}^{3} - \cancel{5}^{1} i\sqrt{3}}{\cancel{5}_{1}}$

$= \boxed{3 - i\sqrt{3}}$

Adding or Subtracting Complex Numbers:

Two complex numbers are added or subtracted by combining like terms. Real parts are added, then imaginary parts are added.

Sum of complex numbers:

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

Difference of complex numbers:

$$(a + bi) - (c + di) = (a - c) + (b - d)i$$

Lesson 7.8: Complex Numbers

Examples: Add or Subtract

g.) $(2 + 3i) + (-6 + 7i)$

$= 2 - 6 + 3i + 7i$

$= \boxed{-4 + 10i}$

h.) $(5 + \sqrt{-36}) - (2 - \sqrt{-49})$

$= (5 + 6i) - (2 - 7i)$

$= 5 + 6i - 2 + 7i$

$= \boxed{3 + 13i}$

$i^2 = -1$

Multiplying Complex numbers:

We multiply complex numbers just like any other polynomial -- by distribution.

- $(ai)(bi) = (ab)i^2 = (ab)(-1) = -ab$
- Multiplying conjugates: $(a + bi)(a - bi) = a^2 + b^2$

Lesson 7.8: Complex Numbers

Examples: Multiply

i.) $\sqrt{-49} \cdot \sqrt{-4}$

$= (7i)(2i)$

$= 14i^2$

$= 14(-1)$

$= \boxed{-14}$

j.) $2i(5 - 3i)$

$= 10i - 6i^2$

$= 10i - 6(-1)$

$= 10i + 6$

$\boxed{6 + 10i}$

Lesson 7.8: Complex Numbers

Examples: Multiply

k.) $(5-2i)(-1+3i)$

$= -5 + 15i + 2i - 6i^2$ ($-6(-1)$)

$= -5 + 17i + 6$

$= \boxed{1 + 17i}$

l.) $(3+2i)(3-2i)$

$= 9 - 6i + 6i - 4i^2$ ($-4(-1)$)

$= 9 + 4$

$= \boxed{13}$

Dividing Complex numbers:

Step 1: Write the numerator and denominator in standard complex form $(a + bi)$.

Step 2: Multiply both the numerator and denominator by the conjugate of the denominator.

Step 3: Simplify by writing the quotient in standard form $(a + bi)$.

Examples: Divide

m.) $\left(\frac{6+5i}{3i}\right) \div \frac{i}{i}$

$= \frac{6i+5i^2}{3i^2} = \frac{6i+5(-1)}{3(-1)}$

$= \frac{6i-5}{-3}$

$= \boxed{\frac{-5+6i}{-3} = \frac{5-6i}{3}}$

n.) $\frac{(2-i)}{(4+3i)} \cdot \frac{(4-3i)}{(4-3i)}$

$= \frac{8-6i-4i+3i^2}{16-12i+12i-9i^2}$

$= \frac{8-10i-3}{16+9}$

$= \frac{5-10i}{25} = \boxed{\frac{1-2i}{5}}$

Lesson 7.8: Complex Numbers

Powers of i:

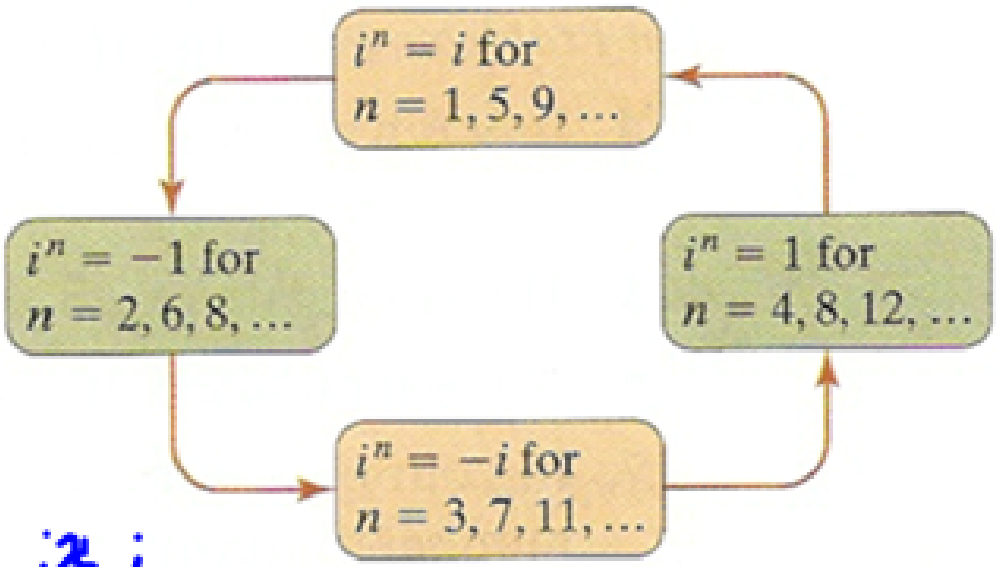
The powers of i follow a pattern.

$i^0 = 1$

$i^1 = i$

$i^2 = -1$

$i^3 = -i$



$i^2 \cdot i$

$i^2 \cdot i^2$

$i^4 = 1$

$i^5 = i$

$i^6 = -1$

Simplifying Powers of i:

Step 1: Divide the exponent of i by 4. Rewrite i^n as $(i^4)^q \cdot i^r$ where q is the quotient and r is the remainder of the division.

Step 2: Simplify the product in Step 1 to just i^r, since $i^4 = 1$. Remember: $i^0 = 1$, $i^1 = i$, $i^2 = -1$, $i^3 = -i$ You should not have any exponents remaining in your final answer!

Lesson 7.8: Complex Numbers

Examples: Simplify

o.) $i^{27} = i^3 = \boxed{-i}$

$$4\overline{)27} \quad 6 \text{ R } 3$$
$$-24$$
$$3$$

p.) $i^{38} = i^2 = \boxed{-1}$

$$4\overline{)38} \quad 9 \text{ R } 2$$
$$-36$$
$$2$$

Objectives:

- Evaluate the square root of negative real numbers.
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- Evaluate the powers of i.

Can You?

Homework:

Pg. 598: # 11, 15 ,21, 25, 29, 33, 37, 41, 47, 51, 55, 59, 61, 65, 67, 71, 75, 79, 81, 83, 85, 87, 103, 107, 109, 113

(26 problems)