Lesson 8.2 & 8.3: Solving Quadratic Equations

Objectives:

- Solve Quadratic Equations using the Square-Root Property.
- Solve Quadratic Equations using the Quadratic Formula.
- Solve equations that are quadratic in form (using substitution).

Square Root Property

Solving Quadratic Equations Using the Square Root Property:

Step 1: Isolate the expression containing the square term.

Step 2: Use the Square Root Property: if $x^2 = p$, then $x = \pm\sqrt{p}$

**don't forget the $\pm$ symbol!

Step 3: Solve for the variable if necessary.

Step 4: Verify your solution.

check!

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Examples: Solve using the Square Root Property.

A) $z^2 - 24 = 0$

$+24 \quad +24$

$\sqrt{z^2} = \pm\sqrt{24}$

$z = \pm 2\sqrt{6}$

B) $z^2 + 16 = -4$

$-16 \quad -16$

$\sqrt{z^2} = \pm\sqrt{-20}$

$z = \pm 2i\sqrt{5}$

C) $(a-2)^2 + 12 = 0$

$-12 \quad -12$

$\sqrt{(a-2)^2} = \pm\sqrt{-12}$

$a - 2 = \pm 2i\sqrt{3}$

$+2 \quad +2$

$a = 2 \pm 2i\sqrt{3}$

The Quadratic Formula

The Quadratic Formula:

Given an equation of the form $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

To solve using the formula:

Step 1: Write the equation in standard form and identify "a", "b", and "c".

Step 2: Substitute the values of a, b, and c into the formula.

Step 3: Simplify and verify your solutions.

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Examples: Solve using the Quadratic Formula.

$a=2 \quad b=11 \quad c=15$

D) $2x^2 + 11x + 15 = 0$

$$x = \frac{-11 \pm \sqrt{(11)^2 - 4(2)(15)}}{2(2)}$$

$$x = \frac{-11 \pm \sqrt{121 - 120}}{4}$$

$$x = \frac{-11 \pm \sqrt{1}}{4}$$

$$x = \frac{-11 \pm 1}{4}$$

$$x = \frac{-11+1}{4} = \frac{-10}{4} = -\frac{5}{2}$$

$$x = \frac{-11-1}{4} = \frac{-12}{4} = -3$$

$$x = -\frac{5}{2}, -3$$

Examples: Solve using the Quadratic Formula.

$a=1 \quad b=4 \quad c=-2$

E) $y^2 - 2 = -4y$

$+4y \quad +4y$

$y^2+4y-2=0$

$$y = \frac{-4 \pm \sqrt{4^2-4(1)(-2)}}{2(1)}$$

$$y = \frac{-4 \pm \sqrt{16+8}}{2}$$

$$y = \frac{-4 \pm \sqrt{24}}{2}$$

$$y = \frac{-4 \pm 2\sqrt{6}}{2}$$

$$y = -2 \pm \sqrt{6}$$

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Examples: Solve using the Quadratic Formula.

F) $16k + \frac{9}{k} = -24$ $\cdot k$ $\cdot k$

$16k^2 + 9 = -24k$

$+24k \quad +24k$

$16k^2 + 24k + 9 = 0$

$a = 16 \quad b = 24 \quad c = 9$

$$k = \frac{-24 \pm \sqrt{24^2 - 4(16)(9)}}{2(16)}$$

$$k = \frac{-24 \pm \sqrt{576 - 576}}{32}$$

$$k = \frac{-24 \pm 0}{32}$$

$$k = -\frac{24}{32}$$

$$k = -\frac{3}{4}$$

$$\boxed{k = \frac{-3}{4}}$$

Examples: Solve using the Quadratic Formula.

G) $m^2 + m + 2 = 0$

Quadratic in Form (look quadratic)

Solving Equations That Are Quadratic in Form:

Step 1: Determine the appropriate substitution and write the equation in the form $au^2 + bu + c = 0$

Step 2: Solve the equation (using any method).

Step 3: Solve for the variable in the original equation using the value of u found in step 2. (Substitute your values back into the original substitution – you know u, now use that to find x.)

Step 4: Verify all of your solutions.

Examples: Solve using the Quadratic in form

factoring Quad. form – Method. $W = x^2 \rightarrow w^2 = x^4$

H) $x^4 - x^2 - 6 = 0$

$w^2 - w - 6 = 0$

$(w-3)(w+2) = 0$

$w-3=0$ $\quad$ $w+2=0$

$+3\ +3$ $\quad$ $-2\ -2$

$w = 3$ $\quad$ $w = -2$

$x^2 = 3$ $\quad$ $x^2 = -2$

$x = \pm\sqrt{3}$ $\quad$ $x = \pm\sqrt{-2}$

$x = \pm\sqrt{3}$, $x = \pm i\sqrt{2}$ or

$\{\sqrt{3}, -\sqrt{3}, i\sqrt{2}, -i\sqrt{2}\}$

Examples: Solve using the Quadratic in form Method.

factoring or Quad. form.

$b = (z^2+3) \rightarrow b^2 = (z^2+3)^2$

1) $(z^2+3)^2 - 2(z^2+3) - 8 = 0$

$b^2 - 2b - 8 = 0$

$(b-4)(b+2) = 0$

$b-4=0 \quad b+2=0$

$b = 4 \quad b = -2$

$z^2+3 = 4$ | $z^2+3 = -2$

$-3 \quad -3$ | $-3 \quad -3$

$z^2 = 1$ | $z^2 = -5$

$z = \pm\sqrt{1}$ | $z = \pm i\sqrt{5}$

$z = \pm 1, \; z = \pm i\sqrt{5}$

or

$\{1, -1, i\sqrt{5}, -i\sqrt{5}\}$

Examples: Solve using the Quadratic in form factoring or Quad. form. Method. $u = \sqrt{x} \rightarrow u^2 = x$

J) $2x - 5\sqrt{x} + 2 = 0$

$2u^2 - 5u + 2 = 0$

$u = \dfrac{5 \pm \sqrt{(-5)^2 - 4(2)(2)}}{2(2)}$

$u = \dfrac{5 \pm \sqrt{25 - 16}}{4}$

$u = \dfrac{5 \pm \sqrt{9}}{4}$

$u = \dfrac{5 \pm 3}{4}$

$u = \dfrac{5+3}{4} = 2$

$u = \dfrac{5-3}{4} = \dfrac{1}{2}$

$u = 2$ $\qquad$ $u = \frac{1}{2}$

$(\sqrt{x})^1 = (2)^2$ $\qquad$ $(\sqrt{x})^2 = (\frac{1}{2})^2$

$x = 4$ $\qquad$ $x = \frac{1}{4}$

Remember Factoring to solve!

Sometimes it is easiest to solve by factoring.
Here is an example:

$$a^2 - 2a - 8 = 0$$

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Homework:

Pg. 634: #'s 11-27 odd, 39-47 odd, 48, 51, 52, 65, 69 (19 problems)

on 47 & 48 Factor and on 51 & 52 use the Square Root Property

AND

Pg. 644: #'s 9-27 odd (10 problems)