Lesson 8.4: Graphing Quadratic Equations

Objectives:

- Graph quadratic functions in any form, using properties of $y = x^2$.
- Find the vertex of a parabola, given the quadratic equation, and determine if it's a maximum or minimum.

Lesson 8.4: Graphing Quadratic Equations

Definition:

A quadratic function is a function in the form $f(x) = ax^2 + bx + c$ where a, b, c are real numbers, and $a \neq 0$.

The domain of a quadratic consists of all real numbers.

D: $\mathbb{R}$

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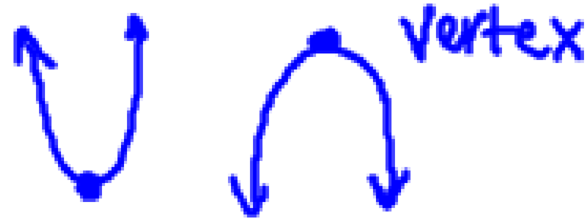
When we graph quadratic functions, we can rewrite the function in the form

$f(x) = a(x-h)^2 + k$, (Called the "vertex form")

and use the $a, h,$ and k values to graph the function without plotting points.

This is called "graphing using transformations" because the $a, h,$ and k values move (or transform) the placement of the graph on the coordinate plane.

Lesson 8.4: Graphing Quadratic Equations



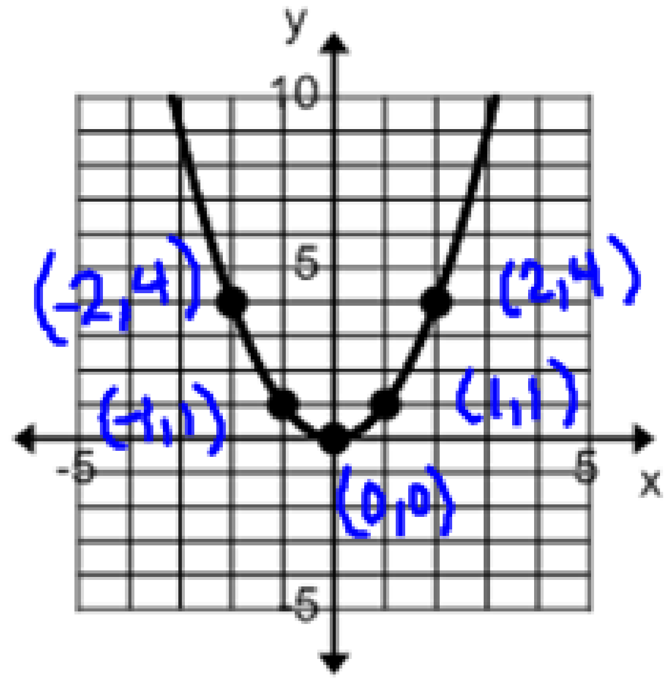
Properties of the Vertex Form $(f(x) = a(x-h)^2 + k)$

- The vertex of the parabola is at the point (h, k).
- The h value tells you how far left or right the vertex of the graph will shift. (If you see $(x-h)^2$, the graph will shift to the RIGHT, if you see $(x+h)^2$ the graph will shift to the LEFT.)
- The a value describes the vertical stretch or compression (how tall and skinny, or wide and fat it gets).
- The k value tells you how far up or down the vertex of the parabola will shift.

Lesson 8.4: Graphing Quadratic Equations

Look at $y = x^2$. We call this our "parent graph". This is the graph that we always start with.

X	Y	(x, y)
0	$0^2 = 0$	(0, 0)
1	$1^2 = 1$	(1, 1)
-1	$(-1)^2 = 1$	(-1, 1)
2	$2^2 = 4$	(2, 4)
-2	$(-2)^2 = 4$	(-2, 4)



Lesson 8.4: Graphing Quadratic Equations

When we use transformations to shift the graph we start with this basic form.

Graphing Quadratic Functions in Vertex Form:

Step 1: Find the vertex (h, k) and mark it on the coordinate plane.

Step 2: Identify the "stretch" (the a value). If your stretch is negative, your graph will open downwards. If your stretch is any value other than 1, you will have to adjust the points of your new graph. (For example, if $a = 2$, you would go over one unit from the vertex and up 2(1)=2 units. Then go over 2 units from the vertex and go up 2(4)=8 units.

Step 3: Graph the function $y = x^2$ at that new vertex, adjusting your points according to the stretch.

EXAMPLES: Graph the following functions using transformations. Identify the vertex, and the vertical stretch if any. Find the domain and the range of the function.

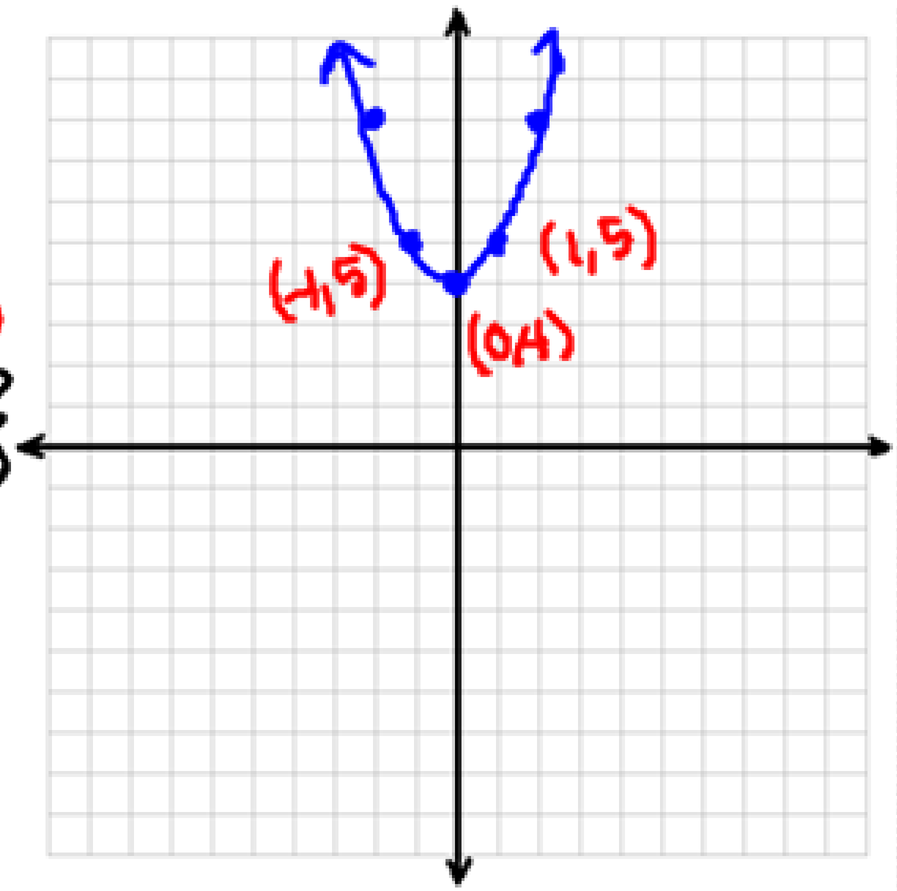
A) $f(x) = x^2 + 4$ $a=1$ $h=0$ $k=4$ label 3 points!

vertex: $(0,4)$

x	y ($y=x^2$)	stretch $*1$
-2	4	4
-1	1	1
0	0	0
1	1	1
2	4	4

D: $\mathbb{R}$ or $(-\infty, \infty)$

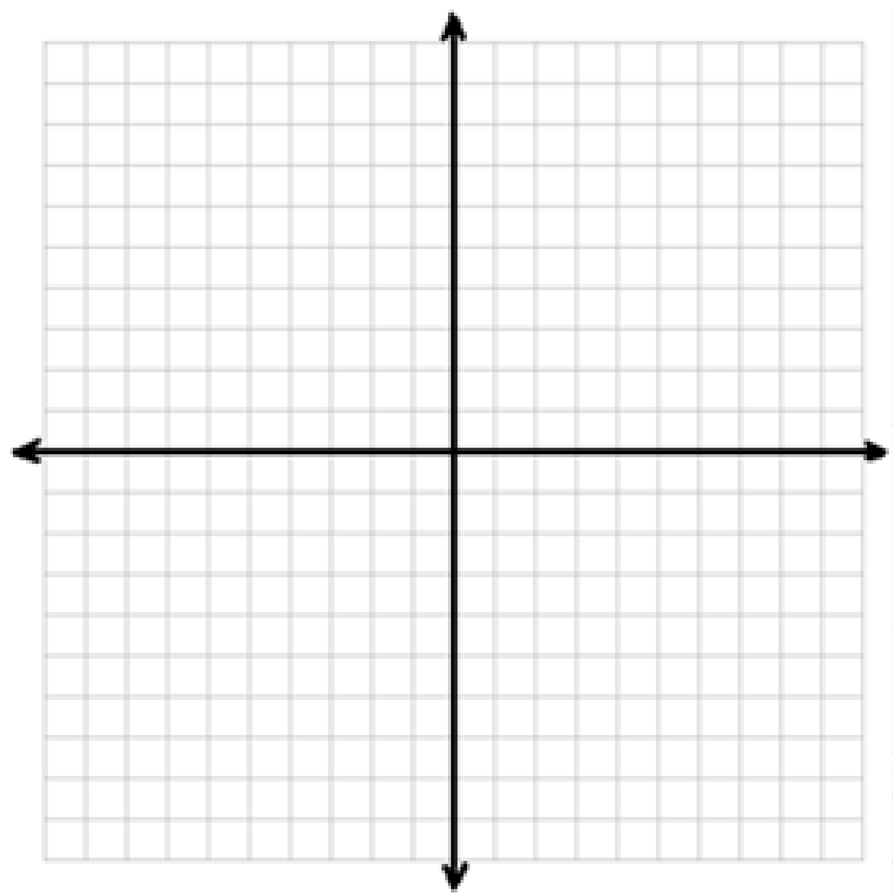
R: $\{y \geq 4\}$ or $[4, \infty)$



<u>Lesson 8.4: Graphing Quadratic Equations</u>

EXAMPLES: Graph the following functions using transformations. Identify the vertex, and the vertical stretch if any. Find the domain and the range of the function.

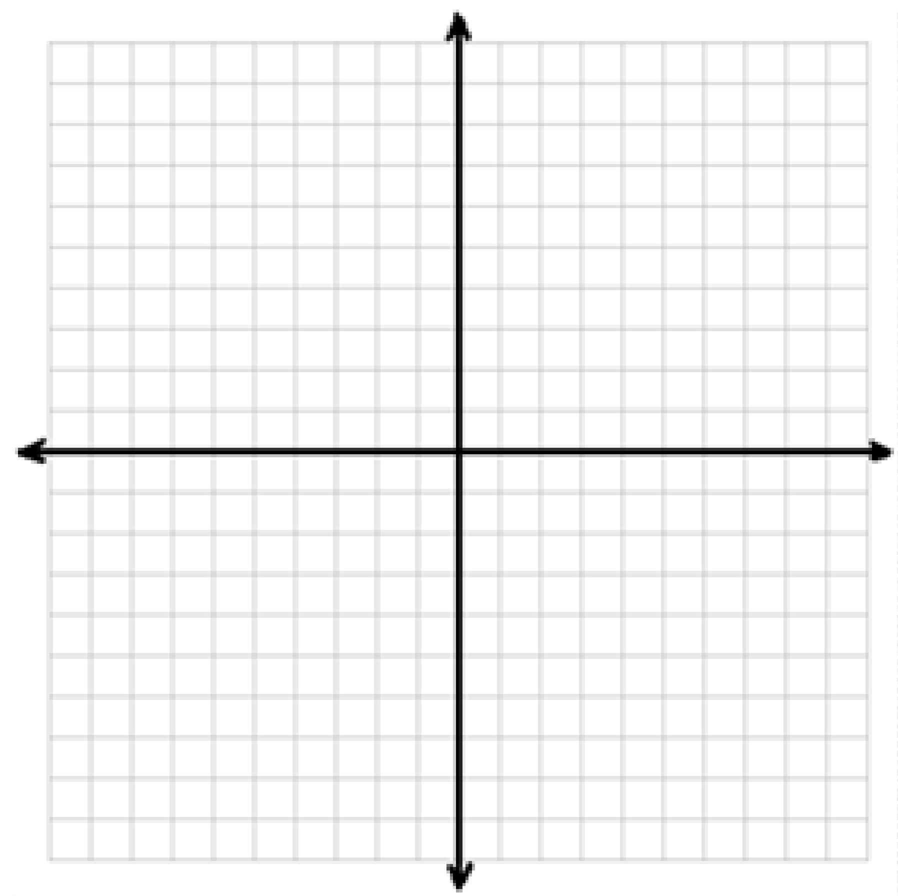
B) $f(x) = (x - 3)^2$



Lesson 8.4: Graphing Quadratic Equations

EXAMPLES: Graph the following functions using transformations. Identify the vertex, and the vertical stretch if any. Find the domain and the range of the function.

C) $f(x) = (x+4)^2 - 1$



EXAMPLES: Graph the following functions using transformations. Identify the vertex, and the vertical stretch if any. Find the domain and the range of the function.

D) $f(x) = 3x^2$

$a=3$ $h=0$ $k=0$

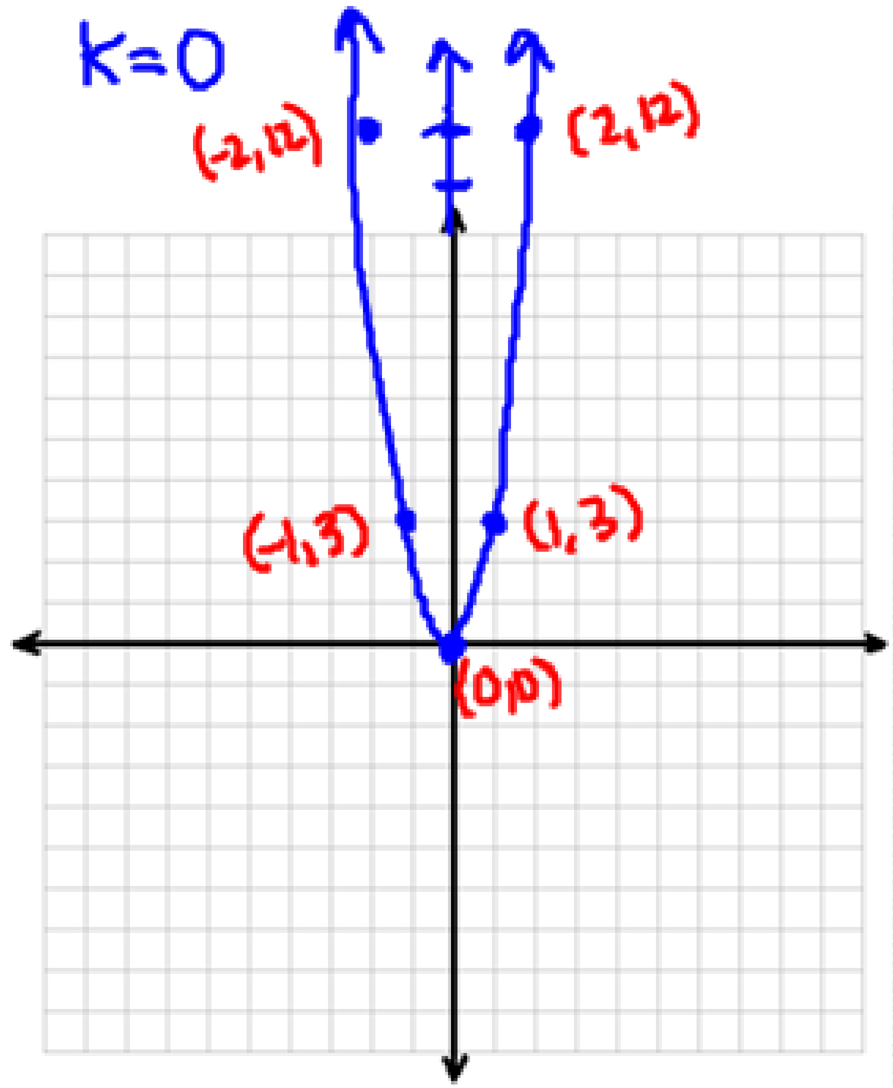
vertex: $(0,0)$

stretch $*3$

x	y	$*3$
-2	4	12
-1	1	3
0	0	0
1	1	3
2	4	12

D: $\mathbb{R}$ $(-\infty, \infty)$

R: $\{y \geq 0\}$ $[0, \infty)$



Lesson 8.4: Graphing Quadratic Equations

EXAMPLES: Graph the following functions using transformations. Identify the vertex, and the vertical stretch if any. Find the domain and the range of the function.

E) $f(x) = -2(x-1)^2 + 3$ $\quad a=-2 \quad h=1 \quad k=3$

Vertex: $(1,3)$

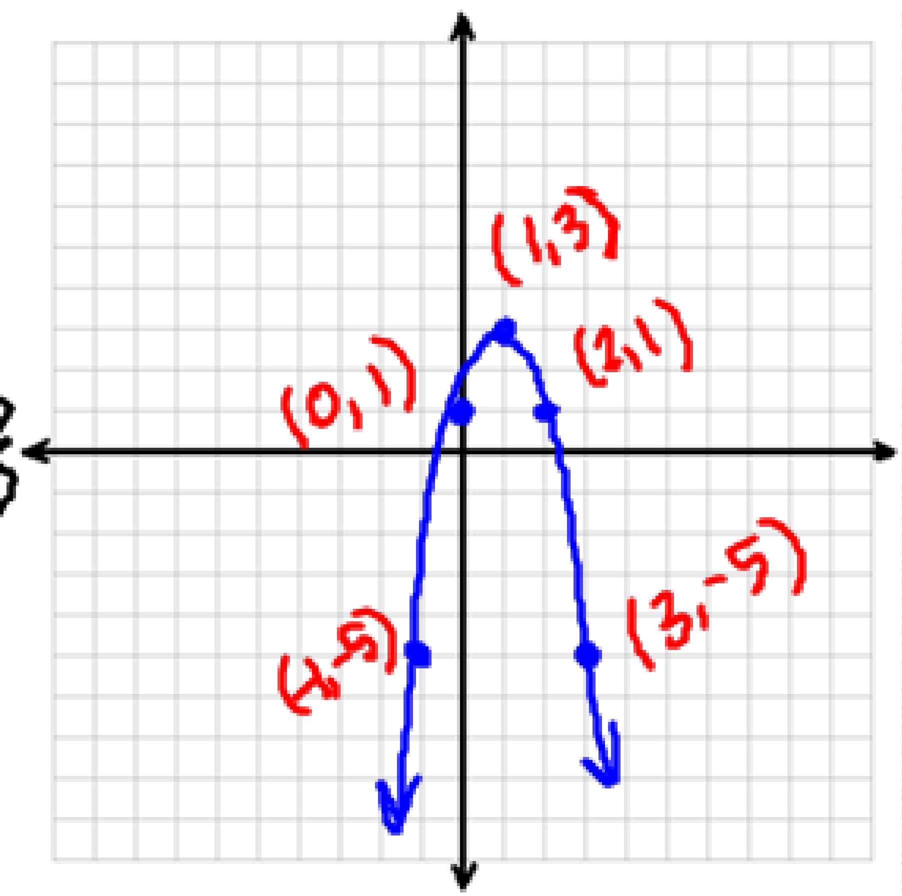
x	y ($y=x^2$)	Stretch $*-2$
-2	4	-8
-1	1	-2
0	0	0
1	1	-2
2	4	-8

D: $\mathbb{R}$

$(-\infty, \infty)$

R: $\{y \leq 3\}$

$(-\infty, 3]$



Lesson 8.4: Graphing Quadratic Equations

What happens if our function equation isn't in vertex form, but it's in standard form instead?

We can find the vertex by another method.

The Vertex of a Parabola:

Any quadratic function $f(x) = ax^2 + bx + c$, $a \neq 0$, will have the vertex

$h = \frac{-b}{2a}$

$$(h, k) = \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$$

In other words, you can find the x-value of the vertex (h) by finding $\frac{-b}{2a}$, then plug that value into the equation and it will give you the y-value (k) of the vertex.

Lesson 8.4: Graphing Quadratic Equations

Example:

& put in vertex form

F) Find the vertex of the function: $f(x) = 3x^2 - 5x + 2$

$h = \frac{-b}{2a} = \frac{5}{2(3)} = \frac{5}{6}$

$h = \frac{5}{6}$

$k = -\frac{1}{12}$

$k = f\left(\frac{5}{6}\right) = \frac{3}{1}\left(\frac{5}{6}\right)^2 - \frac{5}{1}\left(\frac{5}{6}\right) + 2$

$= \frac{\cancel{3}^{1}}{1}\left(\frac{25}{\cancel{36}_{12}}\right) - \frac{25}{6} + \frac{2}{1}$

$= \frac{25}{12} - \frac{25 \cdot 2}{6 \cdot 2} + \frac{2 \cdot 12}{1 \cdot 12}$

$= \frac{25}{12} - \frac{50}{12} + \frac{24}{12}$

$k = -\frac{1}{12}$

$\boxed{\text{vertex: } \left(\frac{5}{6}, -\frac{1}{12}\right)}$

$\boxed{f(x) = 3\left(x - \frac{5}{6}\right)^2 - \frac{1}{12}}$

Lesson 8.4: Graphing Quadratic Equations

Graphing a Quadratic Function in Standard Form:

Step 1: Decide if the parabola opens up or down (look at the a value – it defines your stretch regardless of the form the function is in).

Step 2: Find your vertex using the formula. Using this new vertex, rewrite the equation in vertex form

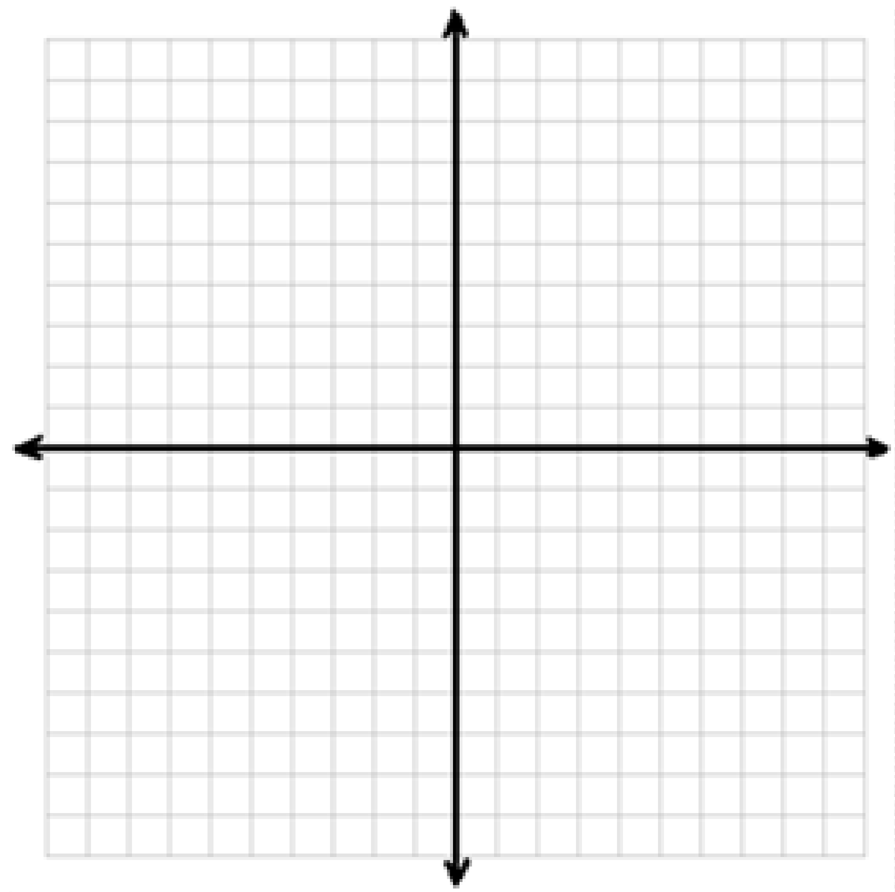
$$f(x) = a(x-h)^2 + k.$$

Step 3: Graph as before.

Lesson 8.4: Graphing Quadratic Equations

Examples: Graph. Find the Domain and Range of the function.

G) $f(x) = x^2 - 2x - 3$



Lesson 8.4: Graphing Quadratic Equations

Examples: Graph. Find the Domain and Range of the function.

open down

vertex $(-2,9)$

H) $f(x) = -2x^2 - 8x + 1 \rightarrow f(x) = -2(x+2)^2 + 9$

$h = \frac{-b}{2a} = \frac{8}{2(-2)} = -2$

$K = f(-2) = -2(-2)^2 - 8(-2) + 1$

$= -2(4) + 16 + 1$

$= -8 + 16 + 1$

$K = 9$

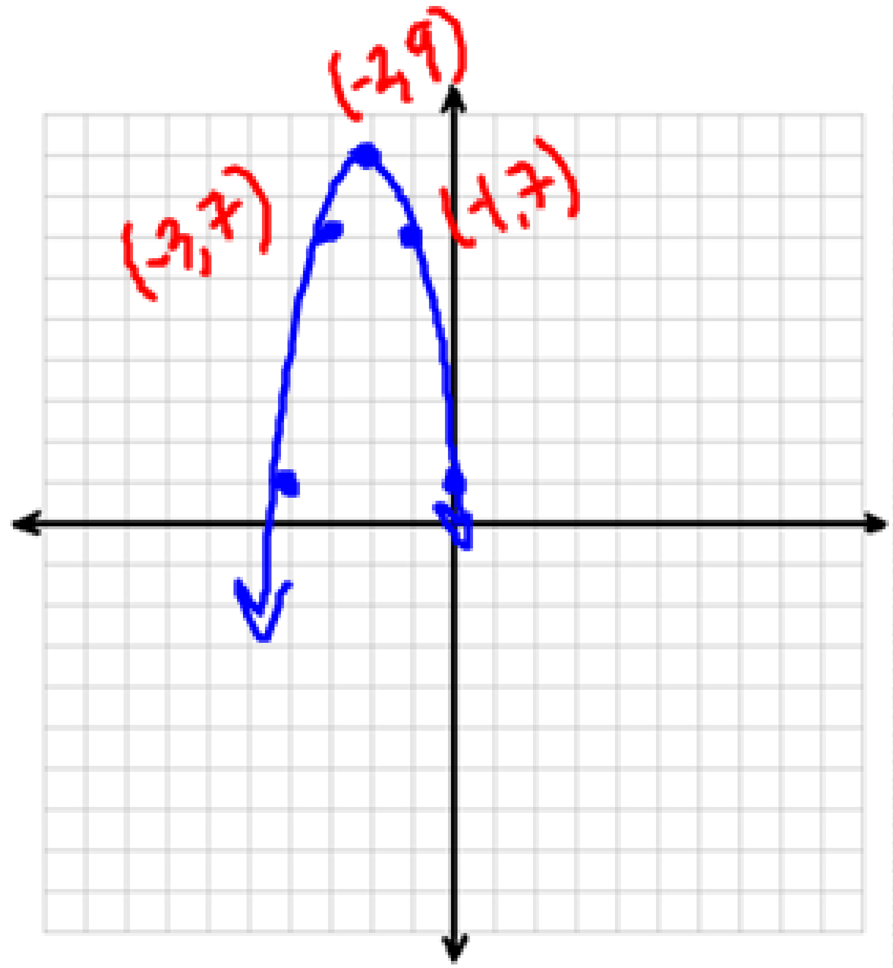
D: $\mathbb{R}$

$(-\infty, \infty)$

R: $\{y \le 9\}$

$(-\infty, 9]$

x	y	×-2
-2	4	-8
-1	1	-2
0	0	0
1	1	-2
2	4	-8



Lesson 8.4: Graphing Quadratic Equations

Verbally explain how to obtain the graph of the given quadratic functions from the graph of $y = x^2$.

I.) $g(x) = (x-9)^2$ $a=1$ $h=9$ $k=0$

- right 9

J.) $g(x) = x^2 - 8$ $a=1$ $h=0$ $k=-8$

- down 8

Lesson 8.4: Graphing Quadratic Equations

Verbally explain how to obtain the graph of the given quadratic functions from the graph of $y = x^2$.

K.) $h(x) = 4(x+7)^2$ $a=4$ $h=-7$ $k=0$

- 4 times as tall, opens up
- left 7

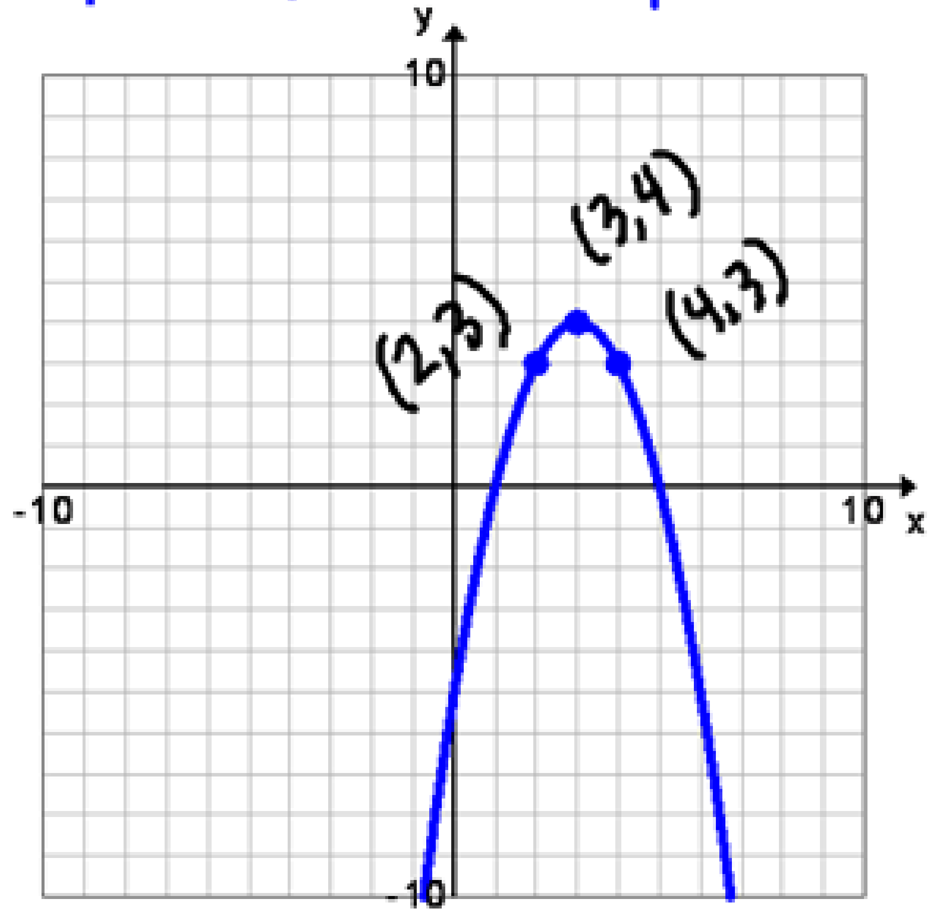
Lesson 8.4: Graphing Quadratic Equations

Write the equation of the quadratic from its graph in vertex form.

$a=-1 \quad h=3 \quad k=4$

L.)

$$\boxed{f(x) = -(x-3)^2+4}$$



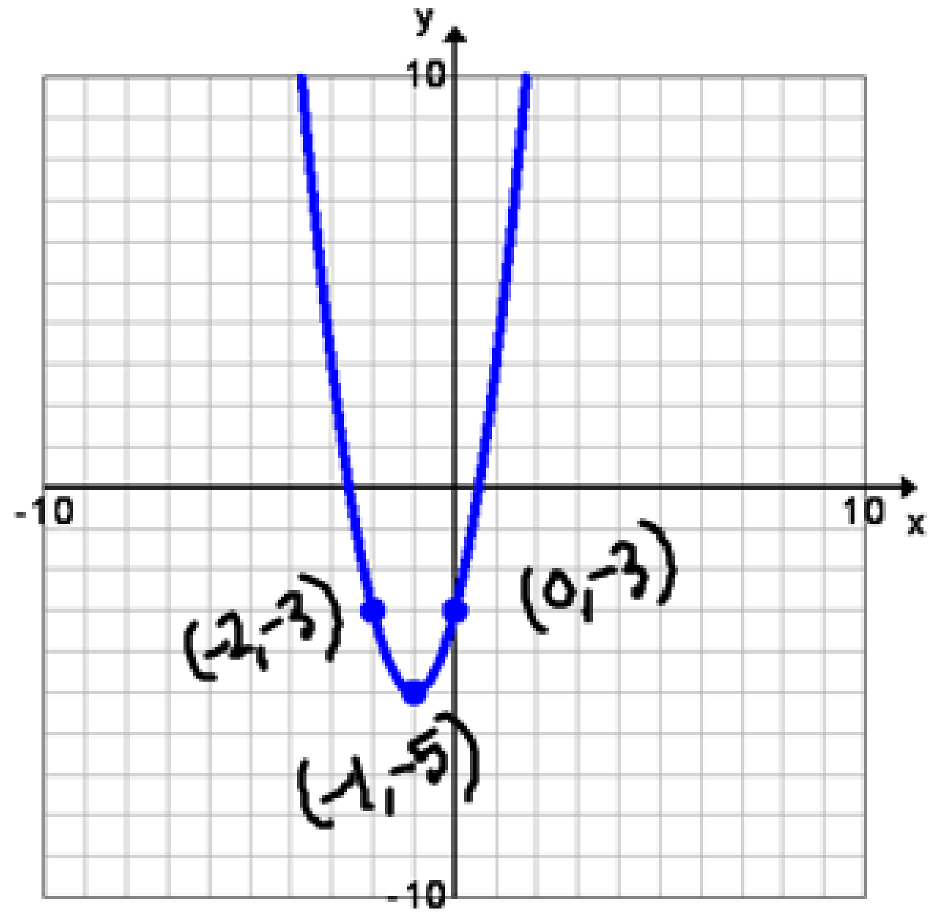
Lesson 8.4: Graphing Quadratic Equations

Write the equation of the quadratic from its graph in vertex form.

M.) up 2

$a=2$ $h=-1$ $k=-5$

$$\boxed{f(x)=2(x+1)^2-5}$$



Lesson 8.4: Graphing Quadratic Equations

Objectives:

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Can you?

Lesson 8.4: Graphing Quadratic Equations

Homework:

Page 658: # 9, 11, 17, 19-27 odds, 33-39 odds, 45-53 odd - find vertex form only - don't graph, 61 - find vertex form only - don't graph, 65, 71, 75, 77, 79

(18 problems)