Lesson 8.5: Graphing Quadratic Equations

Objectives:

- Graph quadratic functions in any form, using properties of $y = x^2$.
- Find the vertex of a parabola, given the quadratic equation, and determine if it's a maximum or minimum.

Quadratic Function:

The maximum or minimum value of a quadratic function always occurs at the vertex, since that will be the highest point on the graph (the maximum) if the graph opens downward, or the lowest point of the graph (the minimum) if the graph opens upward. Since we are looking at height, this value relates to the y-axis, so the y-value of the vertex (k) will be the maximum or minimum value.

Examples: Determine whether the quadratic function has a maximum or minimum value, then find that value.

opens up – min

A.) $f(x) = 2x^2 + 12x - 3$

$h = \frac{-b}{2a} = \frac{-12}{2(2)} = \frac{-12}{4} = -3$

$k = f(-3) = 2(-3)^2 + 12(-3) - 3$

$= 2(9) - 36 - 3$

$= 18 - 36 - 3$

$= -18 - 3$

$k = -21$

$\boxed{\text{Min of } -21}$

Examples: Determine whether the quadratic function has a maximum or minimum value, then find that value.

opens down - max

B.) $g(x) = -3x^2 + 6x + 4$

$h = \frac{-6}{2(-3)} = \frac{-6}{-6} = 1$

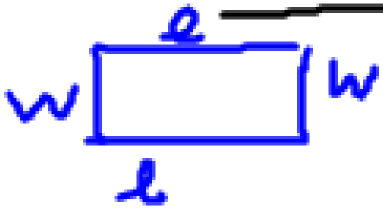
$k = g(1) = -3(1)^2 + 6(1) + 4$

$= -3 + 6 + 4$

$k = 7$

$\boxed{\text{Max is } 7}$

C.) A farmer has 2000 feet of fencing to enclose a rectangular field. What is the maximum area that can be enclosed by the fence? What are the dimensions of the rectangle that encloses the most area?



$P = 2l + 2w$

$2000 = 2l + 2w$

$-2l \quad -2l$

$\frac{2000 - 2l}{2} = \frac{2w}{2}$

$w = 1000 - l$

$A = l \cdot w$

$A(l) = l(1000 - l)$

$A(l) = 1000l - l^2$

$A(l) = -l^2 + 1000l$

$h = \frac{-b}{2a} = \frac{-1000}{2(-1)} = 500 = l$

$K = A(500) = -(500)^2 + 1000(500)$

$= -250000 + 500000$

$K = 250{,}000$ Max

Max area is $250{,}000 \text{ ft}^2$

$l = 500$ ft
$w = 500$ ft

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D.) Suppose that the marketing department of Dell Computers has found that, when a certain model of computer is sold at a price of p dollars, the daily revenue R (in dollars) as a function of the price p is $R(p) = -\frac{1}{4}p^2 + 400p$. *

a.) For what price will the revenue be maximized?

$$h = \frac{-b}{2a} = \frac{-400}{2(-\frac{1}{4})} = \frac{-400}{-\frac{1}{2}} = \frac{-400}{1} \cdot \frac{-2}{1} = 800$$

$\boxed{p = \$800 \text{ will maximize Revenue}}$

$k = R(800)$

Gives max revenue...

D.) Suppose that the marketing department of Dell Computers has found that, when a certain model of computer is sold at a price of p dollars, the daily revenue R (in dollars) as a function of the price p is $R(p) = -\frac{1}{4}p^2 + 400p$.

b.) What is the maximum daily revenue?

$$k = R(800) = -\frac{1}{4}(800)^2 + 400(800)$$

$$= -\frac{1}{\underset{1}{\cancel{4}}}\left(\overset{160000}{\cancel{640000}}\right) + 320000$$

$$= -160000 + 320000$$

$$k = 160000$$

$$\boxed{\text{Max revenue is \$160,000.}}$$

x y

E.) The difference of two numbers is 18. Find the numbers such that their product is a minimum.

$x - y = 18$ $\quad$ $xy = \text{min.}$

$x = 18 + y$ $\rightarrow$ $(18+y)y = f(y)$

$\boxed{f(y) = y^2 + 18y}$

$y = -9$

$x = 18 + -9$

$x = 9$

$h = \frac{-18}{2(1)} = -9$ *

$k = f(-9) = (-9)^2 + 18(-9)$

$= 81 - 162$

$k = -81$

$9 - (-9) = 18$

$9 + 9 = 18$ ✓

$9(-9) = -81$ ✓

Min product is −81

The two numbers are 9 and −9.

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Can you?

Homework:

Page 672: #17, 19, 27, 31, 57-69 odds, 73, 77, 79, 83, 85
(16 problems)