Lesson 9.1 - Composite & Inverse Functions

Objectives:

- Form Composite Functions.
- Determine whether a function is one-to-one.
- Determine the inverse of a function given a set of ordered pairs or an equation.
- Graph the inverse of a function.

Composite Functions:

Given two functions f and g, the COMPOSITE FUNCTION, denoted by $f \circ g$ (read "f composed with g"), is defined by

$$(f \circ g)(x) \quad f \circ g = f(g(x))$$

BE SMART! The $\circ$ means "composed with", not multiply by. We are inserting a function in place of the x's in another function – we are *not* multiplying functions together.

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To evaluate a composite function:

Step 1: Evaluate the second or inside function at the given x-value. $f(g(3))$

(So if you have $(f \circ g)(3)$, find the value for $g(3)$.

Note that the "inside" or second function is always evaluated *first*.

Step 2: Evaluate the first function using the value you found in step 1.

(So if $g(3) = 4$ in our previous step, then we would now find $f(4)$.)

Example 1: Let $f(x) = x^2 - 2x$ and $g(x) = x + 1$.
Find each of the following.

A) $(f \circ g)(x) = f(x+1) = (x+1)^2 - 2(x+1) = (x+1)(x+1) - 2x - 2$

$f(g(x))$

$= x^2 + \cancel{2x} + 1 - \cancel{2x} - 2 = x^2 - 1$

$\boxed{(f \circ g)(x) = x^2 - 1}$

B) $(g \circ f)(x) = g(f(x)) = g(x^2 - 2x) = (x^2 - 2x) + 1$

$\boxed{(g \circ f)(x) = x^2 - 2x + 1}$

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Example 1: Let $f(x) = x^2 - 2x$ and $g(x) = x + 1$.

Find each of the following.

C) $(f \circ g)(-3)$

$(f \circ g)(x) = x^2 - 1$

$(f \circ g)(-3) = (-3)^2 - 1$

$= 9 - 1$

$\boxed{(f \circ g)(-3) = 8}$

$(f \circ g)(-3) = f(g(-3))$

$g(-3) = -3 + 1$

$g(-3) = -2$

$f(g(-3)) = f(-2) = (-2)^2 - 2(-2)$

$= 4 + 4$

$\boxed{(f \circ g)(-3) = 8}$

D) $(g \circ f)(-2)$

$(g \circ f)(x) = x^2 - 2x + 1$

$(g \circ f)(-2) = (-2)^2 - 2(-2) + 1$

$= 4 + 4 + 1$

$\boxed{(g \circ f)(-2) = 9}$

Determining if Functions are One-To-One:

A function is one-to-one if each input in the domain (x) corresponds to exactly one output in the range (y).

No two inputs will ever have the same output - they are each unique.

Knowing if a function is one-to-one allows us to decide if a function has an inverse. *

Pass vertical line test + Horizontal line test

We have two ways of deciding:

- Given a map or a set of points, we look at the y-values. If any y-value appears more than once, it is not one-to-one.

- Given a graph, perform the HORIZONTAL LINE TEST. If every possible horizontal line that can be drawn on the graph intersects the graph in at most one point, then the function is one-to-one.

Example 2: Determine if the functions shown are one-to-one.

E) $\{(3,-5),(2,-1),(1,0),(0,7),(-1,8)\}$

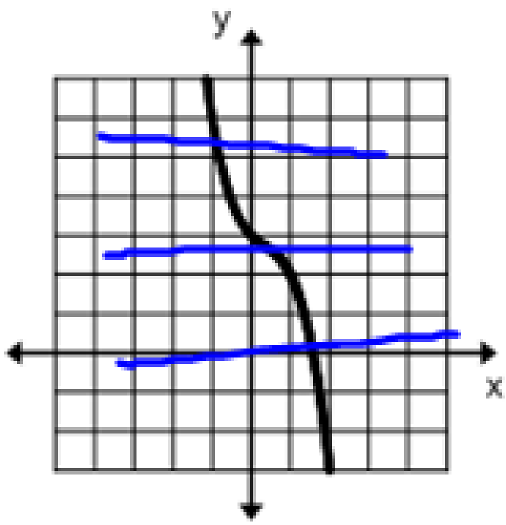
yes

F) $\{(3,4),(2,5),(1,7),(0,5),(-1,4)\}$

No

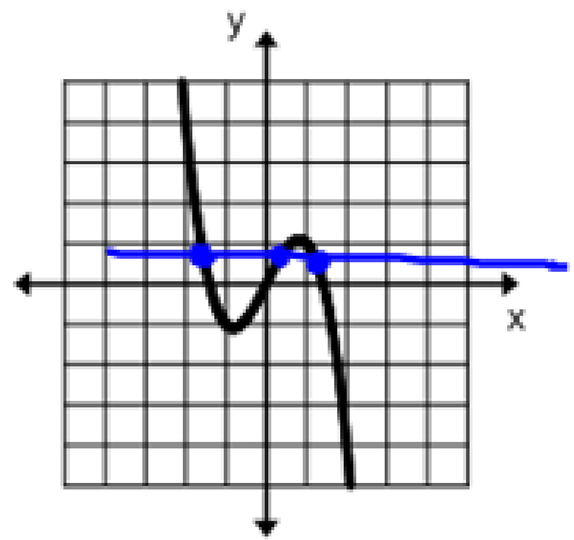
Example 2: Determine if the functions shown are one-to-one.

G.)



yes

H.)



No

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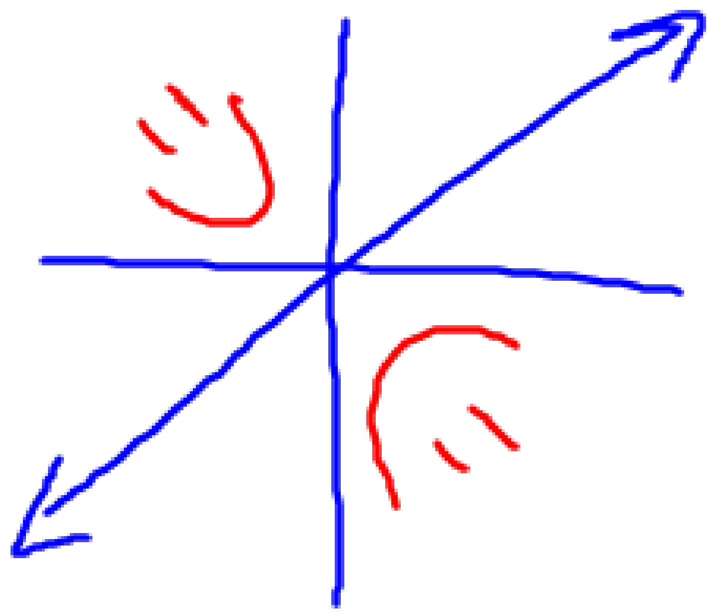
Relationship between the Domain and Range of a Function $f(x)$ and Its Inverse $f^{-1}(x)$:

- **DEFINITION:** If $f(x)$ is a one-to-one function with ordered pairs of the form (a, b), then the **INVERSE** function (denoted by f^{-1}), is the set of ordered pairs of the form (b, a).
- All elements in the domain of f are in the range of f^{-1}. Likewise, all elements in the range of f are in the domain of f^{-1}. This means that we simply switch the x's and y's of each coordinate point in f to find its inverse.

The graph of a function and the graph of its inverse are symmetric around the line $y = x$. If we reverse the x and y in each coordinate point of a graph and plot them, we will have the graph of the inverse.



Examples 3: Find the inverse of the function.

I) $\{(3,-1),(2,7),(1,-4),(0,8),(-1,5)\}$.

$\{(-1,3),(7,2),(-4,1),(8,0),(5,-1)\}$

State the domain and range of the inverse function.

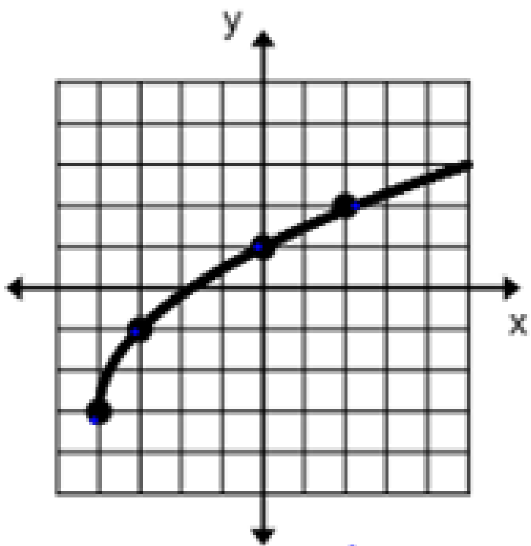
Inverse:

Domain: $\{-1,7,-4,8,5\}$

Range: $\{3,2,1,0,-1\}$

Examples 3: Find the inverse of the function.

J) Graph the inverse function on the coordinate plane.

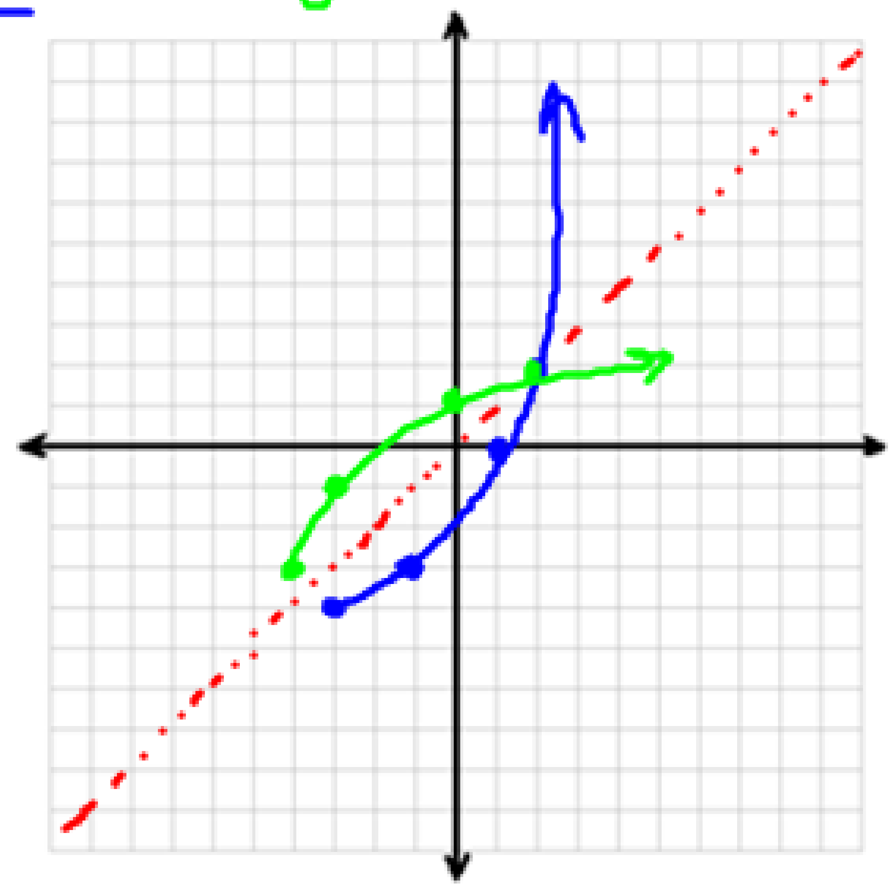


(-4,-3) (2,2)
(-3,-1)
(0,1)

Inverse

(-3,-4)
(-1,-3)
(1,0)
(2,2)

green-original



Finding the Inverse of a Function Algebraically:

Because an inverse "undoes" what the original function does, we have the following relationship:

$$f\left(f^{-1}(x)\right) = x \quad \text{and} \quad f^{-1}(f(x)) = x$$

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How to find an inverse:

Step 1: Replace $f(x)$ with y in the equation.

(Example: $f(x) = x + 3$ goes to $y = x + 3$).

Step 2: Switch the variables x and y in the equation.

(so $y = x + 3$ becomes $x = y + 3$)

Step 3: Solve your new equation for y in terms of x. (subtract 3, $y = x - 3$)

Step 4: Replace y with $f^{-1}(x)$. (So $y = x - 3$ becomes $f^{-1}(x) = x - 3$)

Step 5: VERIFY your answer. Show that both $f(f^{-1}(x)) = x$ AND $f^{-1}(f(x)) = x$. ✓ ✓

Example 4:

$y = 3x + 4$

K) Find the inverse of $f(x) = 3x + 4$

Inverse

$x = 3y + 4$

$-4 \qquad -4$

$\frac{x-4}{3} = \frac{3y}{3}$

$y = \frac{x-4}{3}$

$\boxed{f^{-1}(x) = \frac{x-4}{3}}$

verify:

$f(f^{-1}(x)) = \cancel{3}\left(\frac{x-4}{\cancel{3}}\right) + 4$

$= x - \cancel{4} + \cancel{4}$

$f(f^{-1}(x)) = x$ ✓

$f^{-1}(f(x)) = \frac{3x\cancel{+4}\cancel{-4}}{3} = \frac{\cancel{3}x}{\cancel{3}}$

$f^{-1}(f(x)) = x$ ✓

Example 4:

$y = x^3 - 2$

L) Find the inverse of $f(x) = x^3 - 2$

Inverse:

$x = y^3 - 2$

$+2 \qquad +2$

$\sqrt[3]{x+2} = \sqrt[3]{y^3}$

$y = \sqrt[3]{x+2}$

$$\boxed{f^{-1}(x) = \sqrt[3]{x+2}}$$

verify:

$f(f^{-1}(x)) = \left(\cancel{\sqrt[3]{}}\,x+2\right)^{\cancel{3}} - 2 = x + \cancel{2} - \cancel{2}$

$f(f^{-1}(x)) = x$ ✓

$f^{-1}(f(x)) = \sqrt[3]{x^3 \cancel{-2} + \cancel{2}} = \cancel{\sqrt[3]{}}\,x^{\cancel{3}}$

$f^{-1}(f(x)) = x$ ✓

Example 4:

M) Verify that $f(x) = 10x - 1$ and $g(x) = \frac{x+1}{10}$ are inverses of each other.

$f(g(x)) = \cancel{10}\left(\frac{x+1}{\cancel{10}}\right) - 1$

$= x + \cancel{1} \cancel{-1}$

$f(g(x)) = x$ ✓

$g(f(x)) = \frac{10x \cancel{-1} \cancel{+1}}{10}$

$= \frac{\cancel{10}x}{\cancel{10}}$

$g(f(x)) = x$ ✓

$\boxed{\text{yes, inverses}}$

Objectives:

- Form Composite Functions.
- Determine whether a function is one-to-one.
- Determine the inverse of a function given a set of ordered pairs or an equation.
- Graph the inverse of a function.

Can you?

Homework:

Pg. 707: # 11, 15, 17, 19, 23, 29, 31, 33, 35, 37, 39, 41, 43, 45, 49, 51, 53, 55, 57, 59, 64, 67, 69, 77, 81, 87, 91, 93, 99

(29 problems)