Lesson 9.2: Exponential Functions

Objectives:

- Evaluate, Graph, Solve, and Model Exponential Expressions.
- Define the Number e.

Definition:

An **exponential function** is a function of the form $f(x) = a^x$, where $a > 0$ and $a \neq 1$. $a \geq 2$

The DOMAIN of an exponential function is the set of all real numbers.

$D: \mathbb{R}$

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Examples: Using a calculator, evaluate each of the following expressions. Write as many decimal places as possible.

A) $2^{1.4}$ = 2^(1.4)

2.639015822

B) $2^{1.41}$ = 2^(1.41)

2.657371628

C) $2^{1.414}$

= 2.66474965

D) $2^{\sqrt{2}}$ = 2^√(2)

= 2.665144143

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Properties of the Graph of an Exponential Function

$f(x) = a^x,\ \ x > 0$:

$\mathbb{R}$

1. The domain is the set of all real numbers. The range is the set of all positive real numbers.
2. There are no x-intercepts (the x-axis is an asymptote), and the y-intercept is always at 1.
3. The graph of f will always contain the following three special points: $\left(-1, \frac{1}{a}\right)$, $(0,1)$, $(1,a)$

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Note: We have similar transformations on the exponential graphs as we saw with the quadratic graphs. If $a < 0$, then the graph is reflected over the x-axis. If there is a constant added to a^x, then the graph moves up or down on the y-axis. If there is a constant added to the x in the exponent itself, the graph will shift left or right.

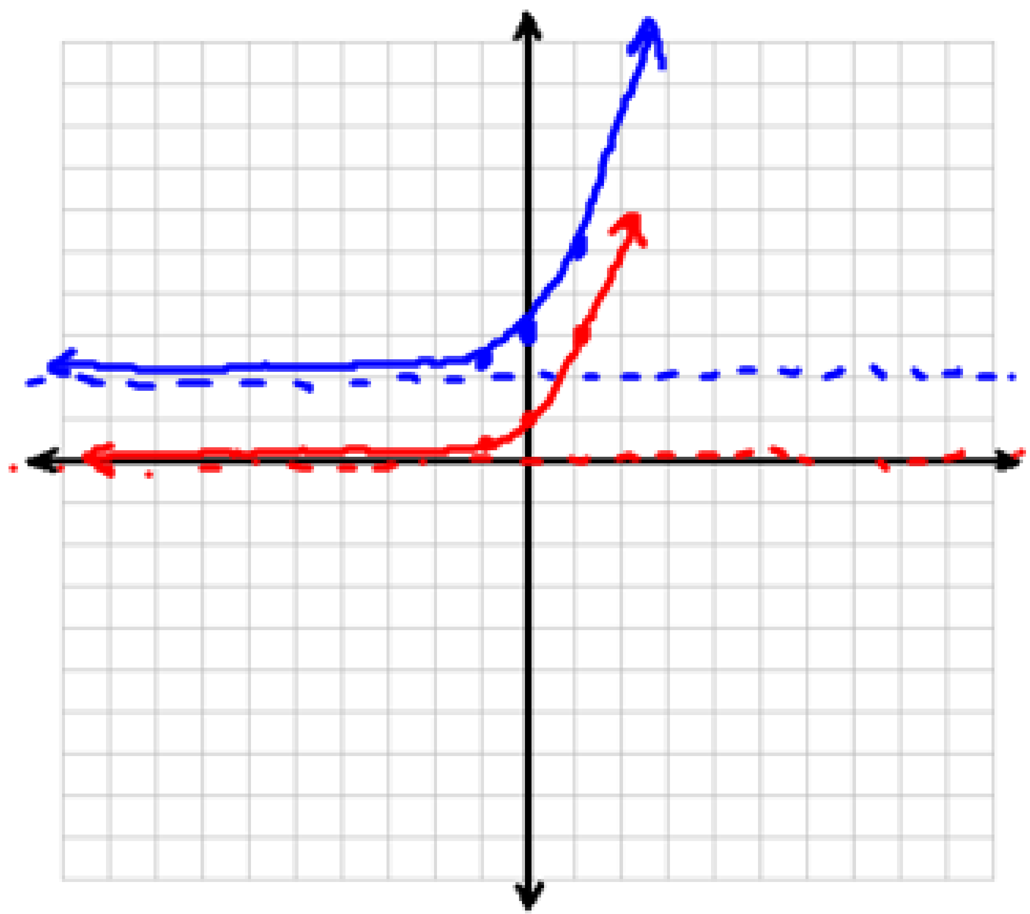
$$f(x) = a^{(x-h)} + k$$

Examples: Graph the following functions using the special points. Give the domain and range of each function.

k up 2

Red graph = $f(x) = 3^x$

E.) $f(x) = 3^x + 2$



Points: $(-1, 2\frac{1}{3}), (0,3), (1,5)$

Domain: $\mathbb{R}$ or $(-\infty, \infty)$

Range: $\{y \mid y > 2\}$ or $(2, \infty)$

$(-1, \frac{1}{3}) \xrightarrow{k=2 \text{ to y's}} (-1, 2\frac{1}{3})$

$(0,1) \rightarrow (0, 3)$

$(1,3) \rightarrow (1,5)$

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Examples: Graph the following functions using the special points. *Give the domain and range of each function.*

F) $f(x) = \left(\frac{1}{3}\right)^x$

Points: $(-1,3),(0,1),(1,\frac{1}{3})$

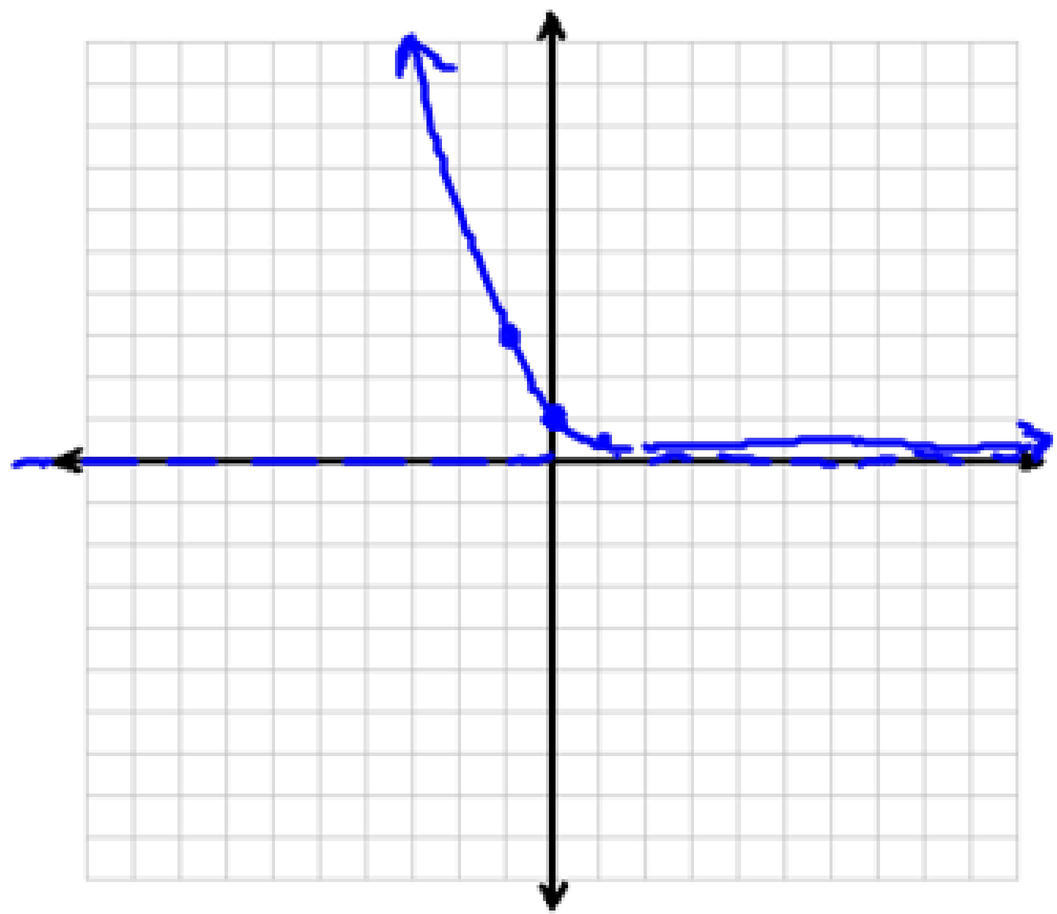
Domain: $\mathbb{R}$ or $(-\infty,\infty)$

Range: $\{y \mid y>0\}$ or $(0,\infty)$

$(-1,\frac{1}{1/3}) = (-1,3)$

$(0,1)$

$(1,\frac{1}{3})$



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Examples: Graph the following functions using the special points. Give the domain and range of each function.

Flip upside down

$h = 1$ Right 1 (+1 to x's)

G.) $g(x) = -4^{(x-1)}$

mult y's by -1

Points:_____________

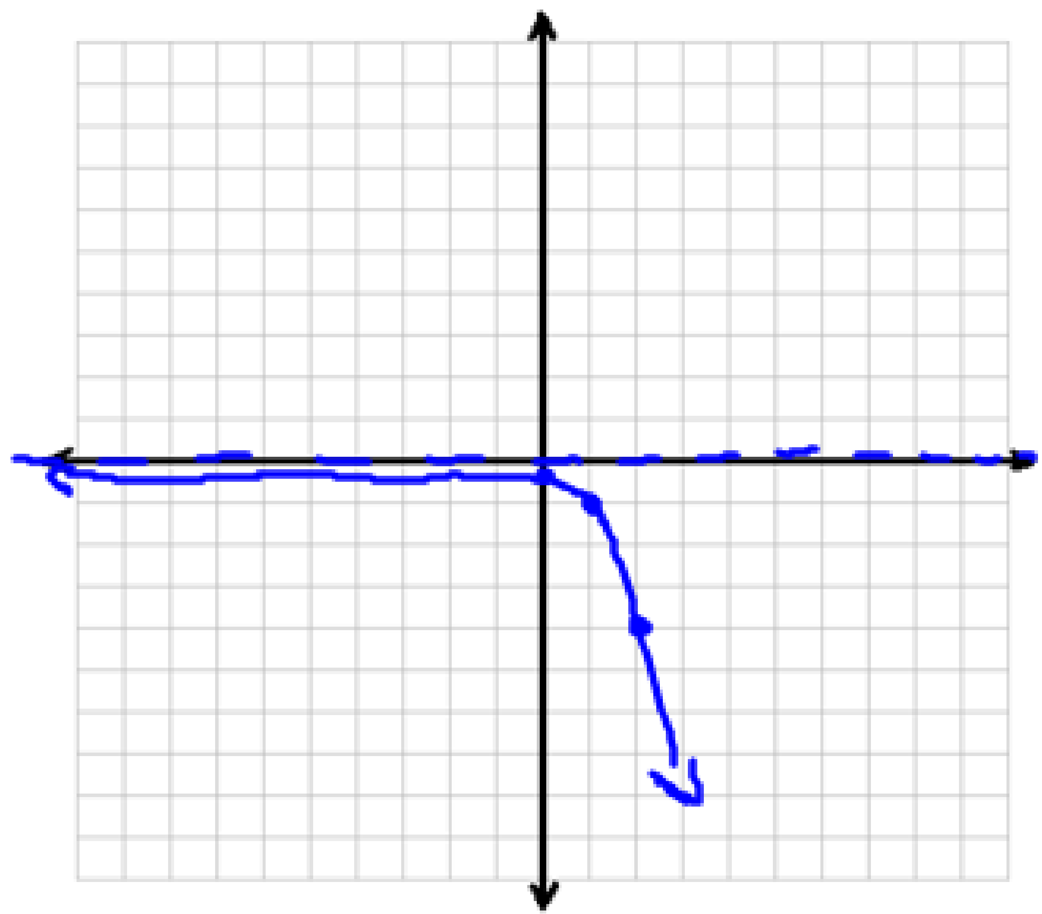
Domain: $\mathbb{R}$ or $(-\infty, \infty)$

Range: $\{y \mid y < 0\}$ or $(-\infty, 0)$

$(-1, \frac{1}{4}) \xrightarrow{h=1} (0, \frac{1}{4}) \xrightarrow{-(y)} (0, -\frac{1}{4})$

$(0, 1) \rightarrow (1, 1) \rightarrow (1, -1)$

$(1, 4) \rightarrow (2, 4) \rightarrow (2, -4)$



Solving Exponential Equations of the Form $a^u = a^v$

Step 1: Use the Laws of Exponents to write both sides of the equation with the same base (a).

Step 2: Set the exponents on each side equal to each other (ignore your bases now that they are the same).

Step 3: Solve the equation from Step 2.

Step 4: Verify your solution(s).

Examples: Solve the following equations.

H) $5^{x-4} = 5^{-1}$

$x - 4 = -1$

$+4 \quad +4$

$\boxed{x = 3}$

Check:

$5^{3-4} = 5^{-1}$

$5^{-1} = 5^{-1}$ ✓

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Examples: Solve the following equations.

I) $3^{x+2} = 81$

$3^{x+2} = 3^4$

$x + 2 = 4$

$\quad -2 \quad -2$

$\boxed{x = 2}$

Examples: Solve the following equations.

$3^{-1} = \frac{1}{3}$ + $3^{-5} = \frac{1}{243}$

J) $\left(\frac{1}{3}\right)^x = \frac{1}{243}$

$3^{-1(x)} = 3^{-5}$

$\frac{-x}{-1} = \frac{-5}{-1}$

$\boxed{x = 5}$

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Examples: Solve the following equations.

$2^2 = 4 \qquad 2^5 = 32$

K) $4^x = 32^{2x-1}$

$2^{2(x)} = 2^{5(2x-1)}$

$2x = 5(2x-1)$

$2x = 10x - 5$

$-10x \quad -10x$

$-8x = -5$

$\frac{-8x}{-8} = \frac{-5}{-8}$

$\boxed{x = 5/8}$

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Modeling Exponential Equations:

Exponential functions are used to describe exponential growth (like interest accrual) and exponential decay (like the half-life of uranium). In Algebra 2, we learned a general equation to be used in these models: $y = ne^{kt}$, or $I = Pe^{rt}$. Our use of these formulas relied on the identity that $e \approx 2.718$. In this section, we are going to expand our definition of the number e, which means our exponential formula will look just a little bit different.

e is defined as the number that the expression $\left(1+\frac{1}{n}\right)^n$ approaches as n gets larger and larger. This value is approximately 2.718.

Therefore, when we look at a general exponential equation, we will replace the e with this value. That's why the formulas you will see in this section are slightly different than what you've seen before.

Examples:

L) From experience, the manager of a crisis helpline knows that between the hours of 3:00 a.m. and 5:00 a.m., calls occur at the rate of 4.2 calls per hour (0.07 calls per hour). The following formula can be used to determine the number of calls that will occur within t minutes of 3 a.m.

$$F(t) = 1 - e^{-0.07t}$$

t

a) Determine the likelihood that a person will call within 5 minutes of 3:00 a.m. $f(5) = 1 - e^{-.07(5)} = .2953$

Calc: $1 - e$^(−.07 * 5)

$\boxed{29.53\%}$

b) Determine the likelihood that a person will call within 20 minutes of 3:00 a.m.

SKIP....

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Examples:

M) The radioactive half-life for an element measures its rate of decay. The half-life of Plutonium-239 is 24,360 years. The amount A (in grams) of Plutonium-239 after t years is given by the formula

$$A(t) = 1 \cdot \left(\frac{1}{2}\right)^{t/24{,}360}$$

Suppose we begin with a 1-gram sample.

t

a) How much Plutonium-239 is left after 350 years?

$$A(350) = 1 \cdot \left(\frac{1}{2}\right)^{350/24{,}360} = \boxed{.99901 \text{ grams}}$$

Calc: (1/2)^(350/24,360)

b) How much is left after 25,000 years?

$A = Pe^{rt}$

Compound Interest: $A = P\left(1 + \frac{r}{n}\right)^{nt}$ *

(start)

where P is the principal amount, r is the annual rate of interest compounded n times a year, and t is the time in years that the principal is invested.

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$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Examples:

N) Suppose you deposit \$2000 into an IRA today. Determine the future value A of the deposit if it earns 6% interest compounded *quarterly* after:

$$A(t) = 2000\left(1 + \frac{.06}{4}\right)^{4t}$$

a) 1 year

$$A(1) = 2000\left(1 + \frac{.06}{4}\right)^{4(1)} = \boxed{\$2122.73}$$

b) 10 years

$$A(10) = 2000\left(1 + \frac{.06}{4}\right)^{4(10)} = \boxed{\$3628.04}$$

c) How much would you have if the interest was compounded *monthly* for 10 years?

$$A(10) = 2000\left(1 + \frac{.06}{12}\right)^{12(10)} = \$3638.79$$

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- Define the Number e.

Can you?

Homework:

PG. 725: # 11, 15, 23-30 all, 31, 35, 39, 41, 47, 49, 53, 59, 61, 67, 71, 75, 81, 87, 89, 93, 95, 97

(29 problems) Skip #85