Lesson 9.3: Logarithmic Functions

Objectives:

- Change logarithmic expressions to exponential expressions and vice versa.
- Evaluate logarithmic functions.
- Graph logarithmic functions.
- Work with natural and common logarithms. ln $\log_{10}$
- Solve logarithmic equations.

Definition:

The logarithmic function to the base a,

$a \geq 2$

where $a > 0$ and $a \neq 1$, is denoted by

$y = \log_a x$ and is defined by:

☆ $$\boxed{y = \log_a x \text{ is equivalent to } x = a^y}$$

$$y = \log_a(x) \longrightarrow a^y = x$$

$$\overset{\curvearrowright}{a^y \; |{=}| \; x \qquad y}$$

$$\log_a x \; |{=}| \; y$$

Lesson 9.3: Logarithmic Functions

Examples: Rewrite each exponential expression as an equivalent expression involving a logarithm.

A) $2^3 = x$

$\log_2(x) = 3$

B) $4^n = 16$

$\log_4(16) = n$

C) $p^2 = 25$

$\log_p(25) = 2$

Examples: Rewrite each logarithmic expression as an equivalent exponential equation.

D) $x = \log_2 16$

$2^x = 16$

E) $-2 = \log_a \frac{1}{4}$

$a^{-2} = \frac{1}{4}$

F) $2 = \log_5 y$

$5^2 = y$

$y = \log_a x$ **is equivalent to** $x = a^y$

Evaluating Logarithmic Functions:

To find the exact value of a logarithmic expression, you must -

Step 1: Rewrite the logarithm in exponential notation (using our identity above).

Step 2: Use the fact that if $a^u = a^v$, then $u = v$. (i.e.: Solve the exponential equation using common bases.)

Lesson 9.3: Logarithmic Functions

Examples: Find the exact value of the following.

G) $\log_5 25 = x$

$5^y = 25$

$5^x = 5^2$

$\boxed{x = 2}$

$\boxed{\log_5(25) = 2}$

H) $\log_2 \frac{1}{8} = \boxed{-3}$

$2^x = \frac{1}{8}$

$2^x = 2^{-3}$

$x = -3$

$\boxed{\log_2(\frac{1}{8}) = -3}$

Examples: Find the value of each of the following, given that $f(x) = \log_2 x$.

(Hint: use substitution)

I) $f(4)$

$f(4) = \log_2(4)$

$\boxed{f(4) = 2}$

$2^? = 4$

J) $f\left(\frac{1}{2}\right)$

$f\left(\frac{1}{2}\right) = \log_2\left(\frac{1}{2}\right)$

$\boxed{f\left(\frac{1}{2}\right) = -1}$

$2^? = \frac{1}{2}$

Examples: Find the value of each of the following, given that $f(x) = \log_2 x$.

(Hint: use substitution)

K) $f(1)$

$f(1) = \log_2(1)$

$\boxed{f(1) = 0}$

$2^? = 1$

Domain and Range of Logarithmic Functions:

The ***RANGE*** of a logarithmic function is always ***ALL REAL NUMBERS***. $(-\infty, \infty)$ Why? Because $y = \log_a x$ is equivalent to $x = a^y$, and because we can use any number as an exponent, that means that y can be any number.

"Sheep"

$\log_3(x)$

The ***DOMAIN*** will be the set of all ***POSITIVE NUMBERS*** greater than the value which makes the argument equal to zero. (The "argument" is the part of the expression after the base.) To find the domain, set the argument >0, then solve for x. That solution is your domain.

Examples: Find the domain of each function.

argument

L) $f(x) = \log_3(3x+1)$

D: $3x+1 > 0$

$-1 \quad -1$

$3x > -1$

$\frac{3x}{3} > \frac{-1}{3}$

$x > -\frac{1}{3}$

$\boxed{D: \{x \mid x > -\frac{1}{3}\}}$

$(-\frac{1}{3}, \infty)$

M) $f(x) = \log_6(x-5)$

$x - 5 > 0$

$+5 \quad +5$

$x > 5$

$\boxed{D: \{x \mid x > 5\} \text{ or } (5, \infty)}$

Graphing Logarithmic Functions:

1. There are no y-intercepts on a log graph.
   The x-intercept is at (1, 0)
2. The graph will always contain three special points: $\left(\frac{1}{a}, -1\right)$, $(1,0), and\ (a,1)$. *

*Note that these are the same points we used on the exponential graphs, except the order of the x and the y is reversed (INVERSE).

Lesson 9.3: Logarithmic Functions $y = \log_a(x-h) + k$

Examples: Graph the following logarithmic functions.

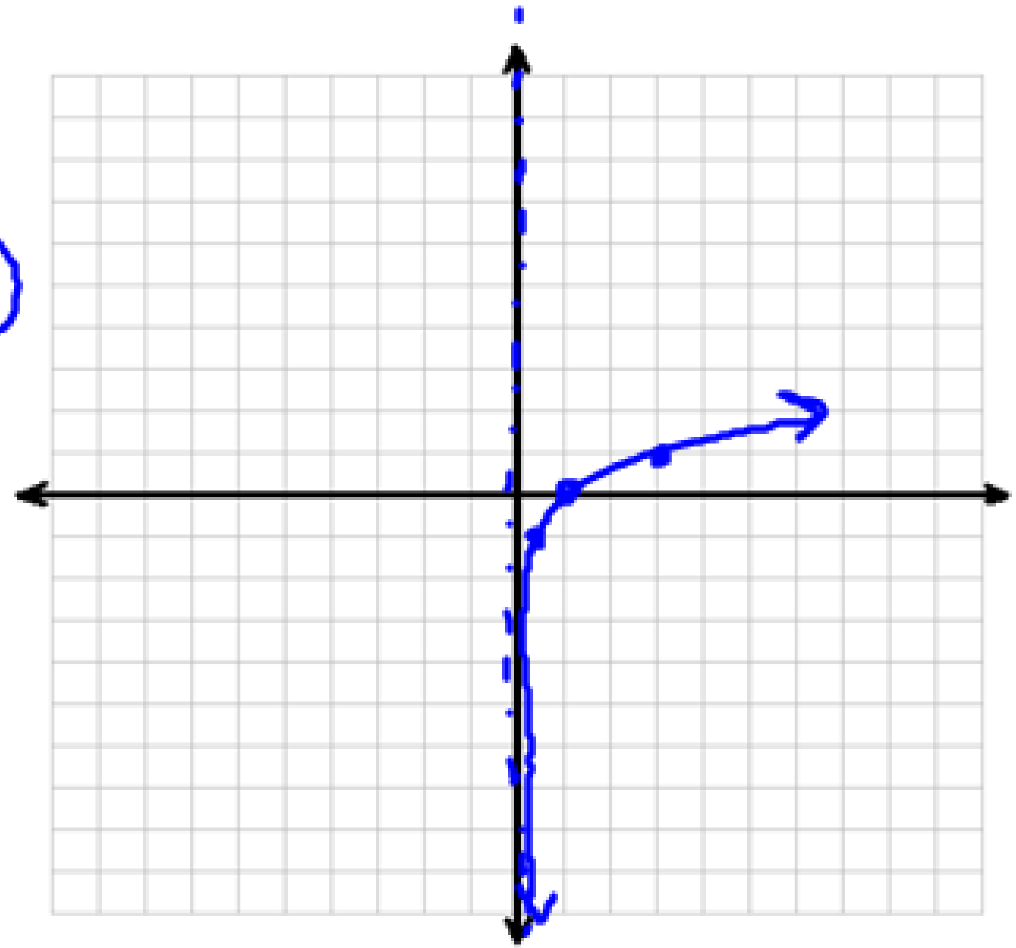
State the special points.

N) $f(x) = \log_3 x$ $a=3$

Points: $\left(\frac{1}{3}, -1\right), (1,0), (3,1)$

Domain: $\{x \mid x>0\}$ or $(0, \infty)$

Range: $\mathbb{R}$ or $(-\infty, \infty)$



Examples: Graph the following logarithmic functions. State the special points.

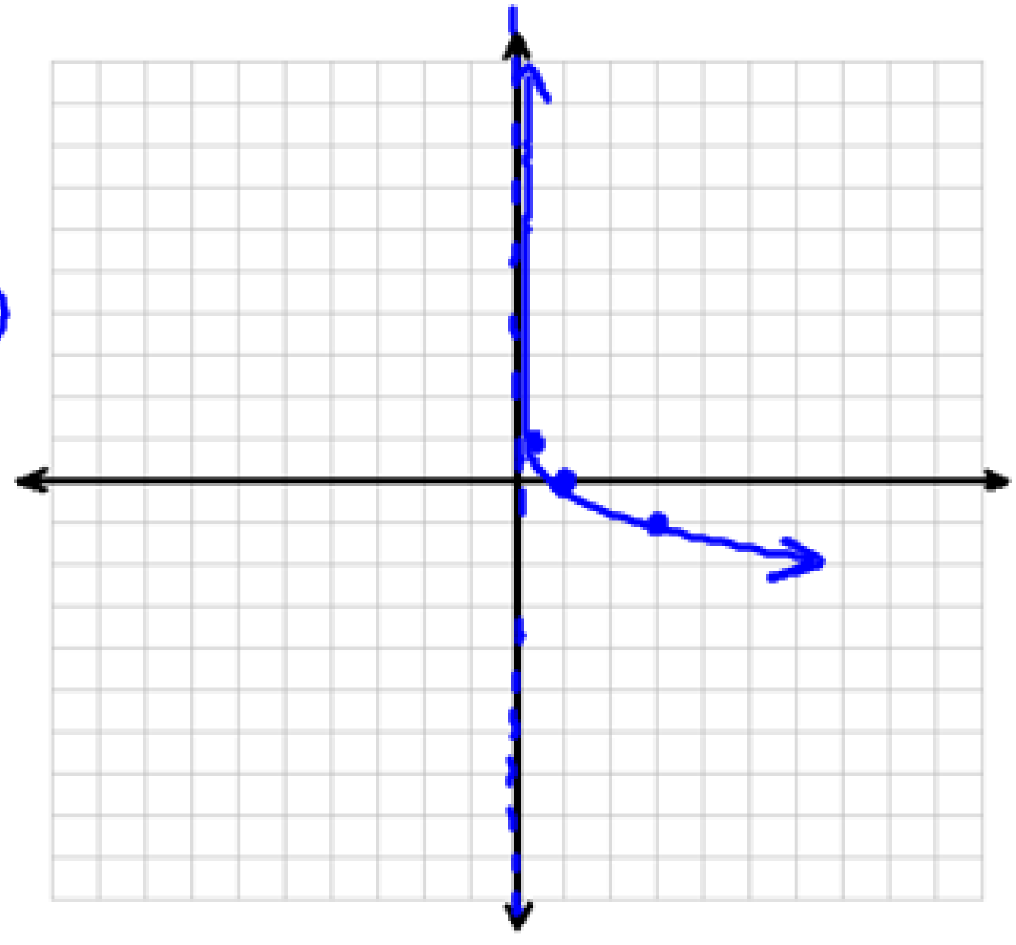
0) $f(x) = \log_{1/3} x$

Points: $(3,-1), (1,0), (\frac{1}{3}, 1)$

Domain: $\{x \mid x > 0\}$ or $(0, \infty)$

Range: $\mathbb{R}$ or $(-\infty, \infty)$

$\frac{1}{1/3} = 3$



Natural and Common Logarithms:

We have two special logarithms that are written differently from the others (their bases aren't written):

- Natural Logarithms have a base of e:

$\log_e(x) \rightarrow \ln x$

$$y = \ln x \text{ if and only if } x = e^y$$

- Common logarithms have a base of 10:

$$y = \log x \text{ if and only if } x = 10^y$$

$\log_{10}(x) \rightarrow \log(x)$

Examples: Use a calculator to evaluate each of the following. Round to 3 decimal places.

P) $\log 25$

$\log(25)$

$= 1.398$

Q) $\ln 25$

$= 3.219$

R) $\ln 0.7$

$= -.357$

Solving Logarithmic Equations:

1. Rewrite the logarithmic equation as an exponential equation.
2. Solve the resulting equation. *Make sure you check your solutions!* The base and the argument must both be positive, so if your solution makes either of them negative, it will be an extraneous solution.

Examples: Solve the equations.

S) $\log_3(4x - 7) = 2$

$3^2 = 4x - 7$

$9 = 4x - 7$

$+7 \quad\quad +7$

$16 = 4x$

$\frac{16}{4} = \frac{4x}{4}$

$\boxed{x = 4}$

Check:

$\log_3(4(4) - 7) = 2$

pos ✓

Examples: Solve the equations.

T) $\log_x 64 = 2$

$x^2 = 64$

don't forget!

$\sqrt{x^2} = \pm\sqrt{64}$

$x = \pm 8$

$\boxed{x = 8 \text{ or } \cancel{x = -8}}$

Check $x = 8$

$\log_{(8)}(64) = 2$ ✓

pos

Check $x = -8$

$\log_{-8}(64) = 2$ ✗

neg

Examples: Solve the equations.

U) $\ln x = 4$

$e^4 = x$

$\boxed{x = e^4}$ ✓

Examples: Solve the equations.

V) $\log(2x-3)=-1$

$10^{-1}=2x-3$

$\frac{1}{10}=2x-3$

$+3 \qquad +3$

$\frac{1}{10}+\frac{30}{10}=2x$

$\frac{\frac{31}{10}}{2}=\frac{2x}{2}$

$x=\frac{31}{10}\cdot\frac{1}{2}$

$\boxed{x=\frac{31}{20}}$ ✓

Objectives:

- Change logarithmic expressions to exponential expressions and vice versa.
- Evaluate logarithmic functions.
- Graph logarithmic functions.
- Work with natural and common logarithms.
- Solve logarithmic equations.

Can you?

Homework:

Page 740: # 9, 11, 15, 19, 21, 25, 27, 31, 33, 35, 37, 39, 41, 43, 47, 49, 55, 59, 61, 65, 69, 71, 75, 83, 87, 93, 95, 99, 101, 111

(30 problems)