Lesson 9.4 : Properties of Logarithms

Objectives:

- Understand the properties of Logarithms
- Write a logarithmic expression as a sum or difference of logarithms.
- Write a logarithmic expression as a single logarithm.
- Evaluate logarithms with bases other than 10 and e.

Inverse Properties of Logarithms:

1. If a and M are positive real numbers, with $a \neq 1$, then $a^{\log_a M} = M$
2. If a is a positive real number, $a \neq 1$, and r is any real number, then $\log_a a^r = r$

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Examples: Evaluate each logarithm.

A) $3^{\log_3 17}$

17

B) $0.5^{\log_{0.5} 11}$

11

C) $\log_2 2^6$

6

D) $\ln e^7$

7

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The Product Rule of Logarithms:

If M, N, and a are positive real numbers, with $a \neq 1$, then $\log_a(MN) = \log_a M + \log_a N$

Examples: Write each expression as the sum of logs.

E) $\log_3(6 \cdot 5)$

$= \log_3(6) + \log_3(5)$

F) $\ln(2k)$

$= \ln(2) + \ln(k)$

The Quotient Rule of Logarithms:

If M, N, and a are positive real numbers, with $a \neq 1$, then

$$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$$

Examples: Write each as the difference of logs.

G) $\log_3\left(\frac{7}{5}\right)$

$= \log_3(7) - \log_3(5)$

H) $\ln\left(\frac{7}{2}\right)$

$= \ln(7) - \ln(2)$

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Examples: Write each expression as the sum or difference of logarithms.

I) $\log_4\left(\frac{3x}{y}\right)$

$= \log_4(3) + \log_4(x) - \log_4(y)$

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Examples: Write each expression as the sum or difference of logarithms.

Extra example: $\log_3\left(\frac{x}{yz}\right)$

$= \log_3(x) - \left(\log_3(y) + \log_3(z)\right)$

$= \log_3(x) - \log_3(y) - \log_3(z)$

The Power Rule of Logarithms:

If M and a are positive real numbers, with $a \neq 1$, and r is any real number, then

$$\log_a M^r = r \log_a M$$

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Example: Express all powers as factors.

J) $\log_7 2^4$

$4 \log_7(2)$

K) $\ln x^{\sqrt{2}}$

$\sqrt{2} \ln(x)$

Examples: Use the rules of Logarithms to expand the logarithm (write as a sum or difference, with all exponents written as factors).

L) $\log_3(9x^2y^4)$

$\log_3(9) + \log_3(x^2) + \log_3(y^4)$

$= \log_3(9) + 2\log_3(x) + 4\log_3(y)$

$= \boxed{2 + 2\log_3(x) + 4\log_3(y)}$

M) $\log\left(\frac{100x}{\sqrt{y}}\right)$

$= \log_{10}(100) + \log(x) - \log(\sqrt{y})$

$= 2 + \log(x) - \log(y^{1/2})$

$= \boxed{2 + \log(x) - \frac{1}{2}\log(y)}$

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Examples: Use the rules of Logarithms to condense the expression into a single logarithm. Simplify if possible.

N) $\log_4 32 + \log_4 8$

$= \log_4 (32 \cdot 8)$

$= \log_4 (256) = 4$

$4^? = 256$

O) $\ln(x+3) - \ln x$

$= \ln\left(\frac{x+3}{x}\right)$

Examples: Use the rules of Logarithms to condense the expression into a single logarithm. Simplify if possible.

P) $\frac{1}{2}\log_3(x+2) + 2\log_3 x$

$= \log_3(x+2)^{1/2} + \log_3(x^2)$

$= \log_3\left(\sqrt{x+2}\right) + \log_3(x^2)$

$= \boxed{\log_3\left(x^2\sqrt{x+2}\right)}$

Examples: Use the rules of Logarithms to condense the expression into a single logarithm. Simplify if possible.

Q) $\log_2 9 + 2\log_2 x - \log_2(x-4)$

$= \log_2(9) + \log_2(x^2) - \log_2(x-4)$

$= \log_2\left(\frac{9x^2}{(x-4)}\right)$

Change of Base Formula:

To evaluate logarithms that have bases other than 10 or e on the calculator, you must use the Change of Base formula:

$$\log_a M = \frac{\log M}{\log a} = \frac{\ln M}{\ln a}$$

ex: $\log_3 17 = \frac{\log(17)}{\log(3)}$ or $\frac{\ln(17)}{\ln(3)}$

Examples: Approximate the following expressions.

Round to 3 decimal places. Show your work.

R) $\log_3 11$

$= \frac{\log(11)}{\log(3)}$ or $\frac{\ln(11)}{\ln(3)}$

≈ 2.183

S) $\log_2 9$

$= \frac{\log(9)}{\log(2)}$ or $\frac{\ln(9)}{\ln(2)}$

≈ 3.170

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Can you?

Homework:

Pg. 752: #11, 15, 17, 19, 23, 27, 31, 33, 37, 41, 47, 49, 55, 63, 67, 73, 75, 77, 79, 81, 89

(21 problems)