

Objectives:

- Solve logarithmic equations.
- Solve exponential equations.
- Solve equations involving exponential models.

Solving Logarithmic Equations:

There are two methods for solving logarithmic equations:

1. Use the One-to-One Property of Logarithms:

$$\text{If } \underline{\log_a M} = \underline{\log_a N}, \text{ then } M = N$$

2. If an equation contains more than one logarithmic term on one side, use the properties of logarithms to rewrite the equation as a single logarithm. Solve by changing to exponential form, or by using the One-to-One Property.

Lesson 9.5 - Exponential & Logarithmic Equations

Examples: Solve.

A) $2 \log_7 x = \log_7 16$

$$\log_7(x^2) = \log_7(16)$$

$$x^2 = 16$$

$$\sqrt{x^2} = \pm \sqrt{16}$$

$$x = \pm 4$$

$x = 4$	$x = -4$
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Check $x=4$

$$2 \log_7(4) = \log_7 16$$

pos ✓

Check $x=-4$

$$2 \log_7(-4) = \log_7(16)$$

neg
X

Lesson 9.5 - Exponential & Logarithmic Equations

Examples: Solve.

B) $\log_4(x + 6) + \log_4 x = 2$

$$\log_4((x+6)x) = 2$$

$$\log_4(x^2 + 6x) = 2$$

$$4^2 = x^2 + 6x$$

$$\begin{array}{r} 16 = x^2 + 6x \\ -16 \quad \quad -16 \\ \hline \end{array}$$

$$0 = x^2 + 6x - 16$$

$$0 = (x-2)(x+8)$$

$$x-2=0$$

$$x+8=0$$

$$x=2$$

~~$$x=-8$$~~

Check $x=2$

$$\log_4(2+6) + \log_4(2) = 2 \quad \checkmark$$

check $x=-8$

$$\log_4(-8+6) + \log_4(-8) = 2$$

- \uparrow
neg

X

Lesson 9.5 - Exponential & Logarithmic Equations

Solving Exponential Equations:

We already talked about solving by common bases, but if you can't rewrite the terms with common bases, you will have to solve using logarithms.

Step 1: Isolate the exponential expression.

Step 2: Rewrite the expression in logarithmic form
(remember that $x = a^y \leftrightarrow \log_a x = y$)

Step 3: Solve and verify your solution.

Lesson 9.5 - Exponential & Logarithmic Equations

Examples: Solve.

$$c) 2^x = 7$$

$$\log_2(7) = x$$

use change of base

$$\frac{\log(7)}{\log(2)} = x$$

$$x = 2.807$$

Lesson 9.5 - Exponential & Logarithmic Equations

Examples: Solve.

$$D) \frac{2e^{x+5}}{2} = \frac{5}{2}$$

$$e^{x+5} = \frac{5}{2}$$

$$\log_e \left(\frac{5}{2} \right) = x+5$$

$$\ln \left(\frac{5}{2} \right) = x+5$$

$$\ln \left(\frac{5}{2} \right) - 5 = x$$

$$x = -4.084$$

Lesson 9.5 - Exponential & Logarithmic Equations

Solving Equations Involving Exponential Models:

We've already *evaluated* exponential models at given values, now we will *solve* the equations using the methods we just learned.

Lesson 9.5 - Exponential & Logarithmic Equations

Examples: Solve.

E) The radioactive half-life for an element measures its rate of decay. The half-life of Plutonium-239 is 24,360 years. The amount A (in grams) of Plutonium-239 after t years is given

by the formula $A(t) = 1 \cdot \left(\frac{1}{2}\right)^{t/24,360}$ Suppose we begin with a 1-gram sample.

a) How long will it take before 0.6 grams of the plutonium-239 is left?

$$\left(\frac{1}{2}\right)^{t/24,360} = 0.6$$
$$\log_{\left(\frac{1}{2}\right)}(0.6) = \frac{t}{24,360} \quad (24,360)$$
$$t = 24,360 \cdot \log_{\left(\frac{1}{2}\right)}(0.6)$$

$$t = 24360 \cdot \log_{\left(\frac{1}{2}\right)}(0.6)$$

$$t = 24360 \cdot \frac{\log(0.6)}{\log\left(\frac{1}{2}\right)} \quad (\text{change of base})$$

$$t = 17,952.482 \text{ yrs}$$

Lesson 9.5 - Exponential & Logarithmic Equations

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

Examples: Solve.

F) Suppose you deposit \$2500 into an IRA today. If the deposit earns 5% interest compounded quarterly, how long will it be before the account is worth

a) \$4,000?

$$\frac{4000}{2500} = \frac{2500 \left(1 + \frac{.05}{4}\right)^{4t}}{2500}$$

$$1.6 = (1.0125)^{4t}$$

↓

$$(1.0125)^{4t} = 1.6$$

$$\frac{\log_{(1.0125)}(1.6)}{4} = \frac{4t}{4}$$

$$\frac{1}{4} \cdot \log_{(1.0125)}(1.6) = t$$

(change of base)

$$\frac{1}{4} \cdot \frac{\log(1.6)}{\log(1.0125)} = t$$

$$t = 9.459 \text{ yrs}$$

Lesson 9.5 - Exponential & Logarithmic Equations

Examples: Solve.

F) Suppose you deposit \$2500 into an IRA today. If the deposit earns 5% interest compounded quarterly, how long will it be before the account is worth

b) \$5,000?

Objectives:

- Solve logarithmic equations.
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- Solve equations involving exponential models.

Can you?

Homework:

Pg. 760: #7, 13, 15, 23, 25,
33, 41, 45, 47, 51, 55, 57, 59,
61, 65 (17 problems)