Lesson 10 (Sect. 3-2):
Solving Systems Algebraically

By the end of the lesson you will be able to:

~ Solve systems of inequalities using the substitution method

~ Solve systems of inequalities using the elimination method

LESSON 10 (SECT. 3-2): SOLVING SYSTEMS OF EQUATIONS ALGEBRAICALLY

Sometimes for systems of equations it may be easier to solve using algebraic methods rather than graphing.

~ALGEBRAIC METHODS~

TO SOLVE SYSTEMS OF EQUATIONS ALGEBRAICALLY, THERE ARE TWO METHODS, THE SUBSTITUTION METHOD & THE ELIMINATION METHOD.

Solving Systems of Equations by Substitution

Step 1:	Solve one of the equations for one variable. Solve for y in this example.	System: $-x + y = 1$ $3x + 2y = -3$ Step 1: $y = x + 1$ $(first\ equation)$
Step 2:	SUBSTITUTE the expression found in Step 1 into the **OTHER** (second) equation.	Step 2: $3x + 2(x + 1) = -3$
Step 3:	Solve the linear equation in one variable found in Step 2.	Step 3: $3x + 2x + 2 = -3$ $5x + 2 = -3$ $5x = -5$ $x = -1$
Step 4:	Substitute the value of the variable into the expression found in Step 1 to find the value of the other variable.	Step 4: $y = (-1) + 1$ (Step 1) $y = 0$
Step 5:	Write answer in point form!!! Check your answer.	Step 5: $(-1, 0)$ Check: (You plug -1 in for x and 0 in for y.... make sure the equations are EQUAL!)

SOLVING BY SUBSTITUTION

example:

Solve the system of equations by using substitution

$$2X + Y = 4$$
$$3X + 2Y = 1$$

$2x + y = 4$

$-2x \quad -2x$

$y = -2x + 4$

$3x + 2(-2x+4) = 1$

$3x - 4x + 8 = 1$

$-x + 8 = 1$

$-8 \quad -8$

$\frac{-x}{-1} = \frac{-7}{-1} \rightarrow x = 7$

$y = -2(7) + 4$

$y = -14 + 4$

$y = -10$

$\boxed{(7, -10)}$

SOLVING BY SUBSTITUTION

EXAMPLE:

SOLVE THE SYSTEM OF EQUATIONS BY USING SUBSTITUTION

$X - 9 = 3Y \longrightarrow x = \underbrace{3y + 9}$

$X + 2Y = 1$

$3y + 9 + 2y = 1$

$5y + 9 = 1$

$\quad\; -9 \quad -9$

$\frac{5y}{5} = \frac{-8}{5}$

$y = -8/5$

$x = \frac{3}{1}\left(\frac{-8}{5}\right) + 9$

$x = \frac{-24}{5} + \frac{9 \cdot 5}{1 \cdot 5}$

$x = \frac{-24}{5} + \frac{45}{5}$

$x = \frac{21}{5}$

$\boxed{\left(\frac{21}{5}, \frac{-8}{5}\right)}$

SOLVING BY SUBSTITUTION

example:

Solve the system of equations by using substitution

$X + 3Y = 8 \rightarrow x = -3y + 8$

$\frac{1}{3}X + Y = 9$

$\frac{1}{3}\left(-\frac{3}{1}y + \frac{8}{1}\right) + y = 9$

$\frac{1}{\cancel{3}} \cdot \frac{-\cancel{3}}{1}y + \frac{1}{3} \cdot \frac{8}{1} + y = 9$

$-y + \frac{8}{3} + y = 9$

$\frac{8}{3} \neq 9 \quad \times \times$

$\boxed{\text{No solution}}$

Solving Systems of Equations by Elimination

To use the elimination method effectively, first compare the coefficients of the variables. The elimination method is most effective when the coefficients for one variable are the **same**, or when they have **opposite signs**. This will allow you to easily eliminate one variable to solve for the other one.

Some examples when the elimination method is most effictive:

1. $3a - 2b = -3$
   $3a + b = 3$

2. $m + n = 6$
   $m - n = 5$

3. $x - 2y = 1$
   $+ \quad 3x + 2y = 19$
   $4x = 20$

4. $4p + 5q = 7$
   $3p - 2q = 34$

Solving Systems of Equations by Elimination

Step 0:	Align the equations with like terms in columns (if necessary).	System: $-3x + y = 1$ $2x + 2y = 10$
Step 1:	Multiply both sides of one or both equations by a number so that there are opposite coefficients for one of the variables. (Ex.: 3x and -3x have opposite coefficients- they equal zero when added.)	Step 1: $-3x + y = 1\ (Mult. by -2)$ $2x + 2y = 10$ $\mathbf{6x - 2y = -2}$ $+\ \underline{2x + 2y = 10}$
Step 2:	Add equations together (like terms) to eliminate the variable.	Step 2: $\mathbf{6x - 2y = -2}$ $+\ \underline{2x + 2y = 10}$ $8x \quad = 8$
Step 3:	Solve the resulting equation for the variable that is left.	Step 3: $8x = 8$ $x = 1$
Step 4:	Substitute the value of the variable found in Step 3 into one of the <u>ORIGINAL</u> equations to find the value of the remaining variable.	Step 4: $-3(\mathbf{1}) + y = 1$ $-3 + y = 1$ $y = 4$
Step 5:	Write answer in <u>point form!!!</u> Check your answer.	Step 5: (1,4) Check: (You plug 1 in for x and 4 in for y…. make sure the equations are EQUAL!)

SOLVING SYSTEMS OF EQUATIONS BY ELIMINATION

EXAMPLE: SOLVE BY USING THE ELIMINATION METHOD

$(3a - 2B = -3) \cdot -1 \rightarrow$

$3a + B = 3$

$$\begin{array}{r} -3a + 2b = 3 \\ +\quad 3a + b = 3 \\ \hline 3b = 6 \end{array}$$

$$\frac{3b}{3} = \frac{6}{3}$$

$b = 2$

$$\begin{array}{r} 3a + 2 = 3 \\ -2 \quad -2 \\ \hline 3a = 1 \end{array}$$

$$\frac{3a}{3} = \frac{1}{3}$$

$a = 1/3$

$$\boxed{\left(\frac{1}{3}, 2\right)}$$

* do alphabetical order for point.

Solving by Elimination

Example:

Solve the system of equations by using elimination

$M + N = 6$
$M - N = 5$

$$\frac{2m}{2} = \frac{11}{2}$$

$$m = \frac{11}{2}$$

$$\frac{11}{2} + n = 6$$
$$-\frac{11}{2} \qquad -\frac{11}{2}$$

$$n = \frac{6}{1}\cdot\frac{2}{2} - \frac{11}{2}$$

$$n = \frac{12}{2} - \frac{11}{2}$$

$$n = \frac{1}{2}$$

$$\boxed{\left(\frac{11}{2}, \frac{1}{2}\right)}$$

Solving by Elimination

$\frac{4}{3} \cdot \frac{9}{1} = 12$

example:

Solve the system of equations by using elimination

$\left(\frac{1}{4}x + \frac{1}{3}y = 1\right)12 \rightarrow 3x + 4y = 12$

$\left(\frac{1}{3}x - \frac{4}{9}y = \frac{4}{3}\right)9 \rightarrow 3x - 4y = 12$

$\frac{6x}{6} = \frac{24}{6}$

$x = 4$

*plug into original

$\frac{1}{4}\left(\frac{4}{1}\right) + \frac{1}{3}y = 1$

$1 + \frac{1}{3}y = 1$

$-1 \qquad -1$

$\frac{1}{3}y = 0$

$y = 0$

$\boxed{(4, 0)}$

$$\begin{array}{r} 34 \\ \times 5 \\ \hline 170 \end{array}$$

Solving by Elimination

Example:

Solve the system of equations by using elimination

$(4P + 5Q = 7)\,2 \rightarrow 8P + 10Q = 14$

$(3P - 2Q = 34)\,5 \rightarrow 15P - 10Q = 170$

$$\frac{23P}{23} = \frac{184}{23}$$

$P = 8$

*plug into original eq.

$4(8) + 5Q = 7$

$32 + 5Q = 7$

$-32 \qquad -32$

$$\frac{5Q}{5} = \frac{-25}{5}$$

$Q = -5$

$\boxed{(8, -5)}$

Lesson 10 (Sect. 3-2):
Solving Systems Algebraically

By the end of the lesson you will be able to:

~ Solve systems of inequalities using the substitution method

~ Solve systems of inequalities using the elimination method

Can you?

Print Lesson 11 notes

~Assignment #10~

Due at the beginning of class

Remember: the "New" part is NO CALCULATOR.

Read instructions to know when you can use a calculator.