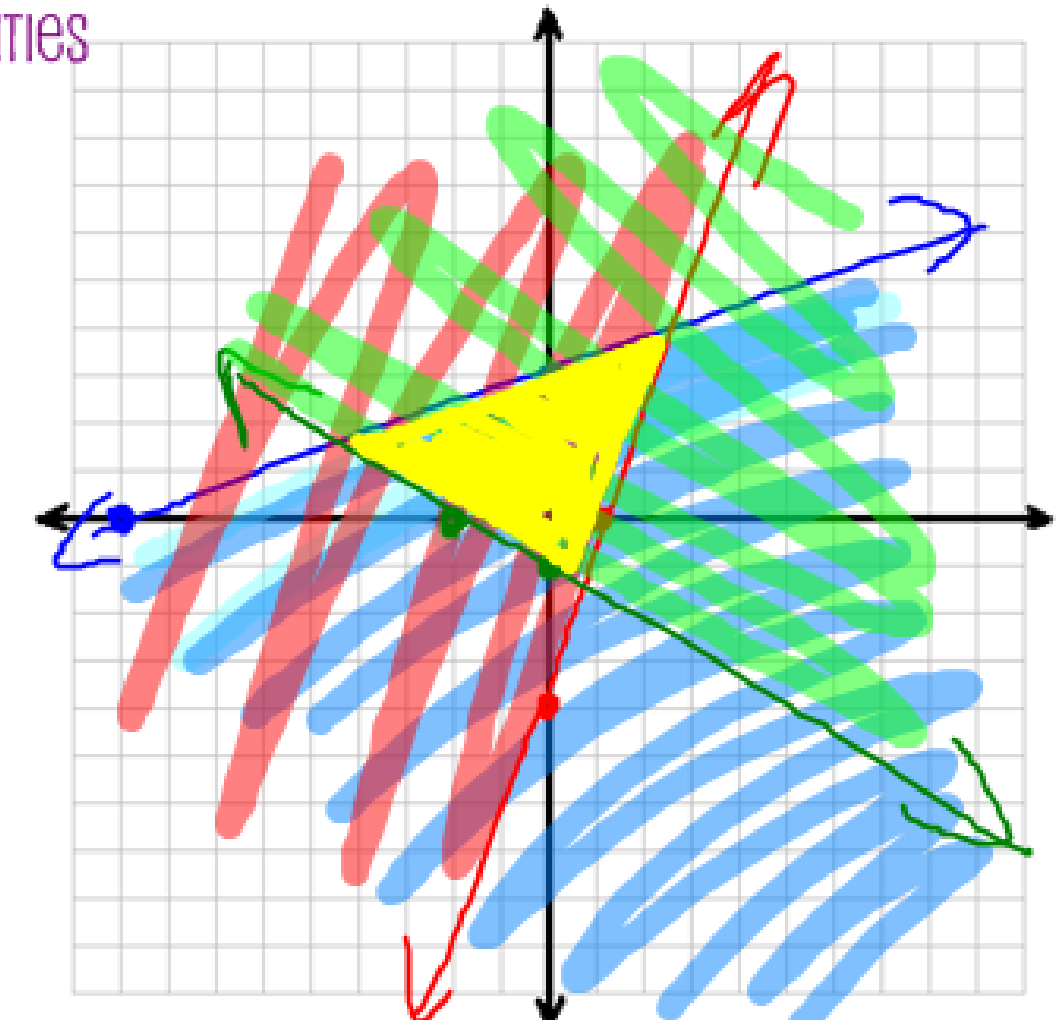
Section 3-5: Linear Programming

~review~

Graph the following system of inequalities

$x - 3y \geq -9$
$4x - y \leq 4$
$x + 2y \geq -2$



$X - 3Y \geq -9$

x-int: $(-9, 0)$

$x \geq -9$

y-int: $(0, 3)$

$\frac{-3y}{-3} \geq \frac{-9}{-3}$

$y \leq 3$

Test $(0,0)$

$0 - 3(0) \geq -9$

$0 \geq -9$ ✓

$4X - Y \leq 4$

x-int: $(1, 0)$

$\frac{4x}{4} \leq \frac{4}{4}$

$x \leq 1$

y-int: $(0, -4)$

$\frac{-y}{-1} \leq \frac{4}{-1}$

$y \geq -4$

Test $(0,0)$

$4(0) - 0 \leq 4$

$0 \leq 4$ ✓

$X + 2Y \geq -2$

x-int: $(-2, 0)$

$x \geq -2$

y-int: $(0, -1)$

$\frac{2y}{2} \geq \frac{-2}{2}$

$y \geq -1$

Test $(0,0)$

$0 + 2(0) \geq -2$

$0 \geq -2$ ✓

Section 3-5: Linear Programming

Using the same systems of inequalities:

$x - 3y \geq -9$

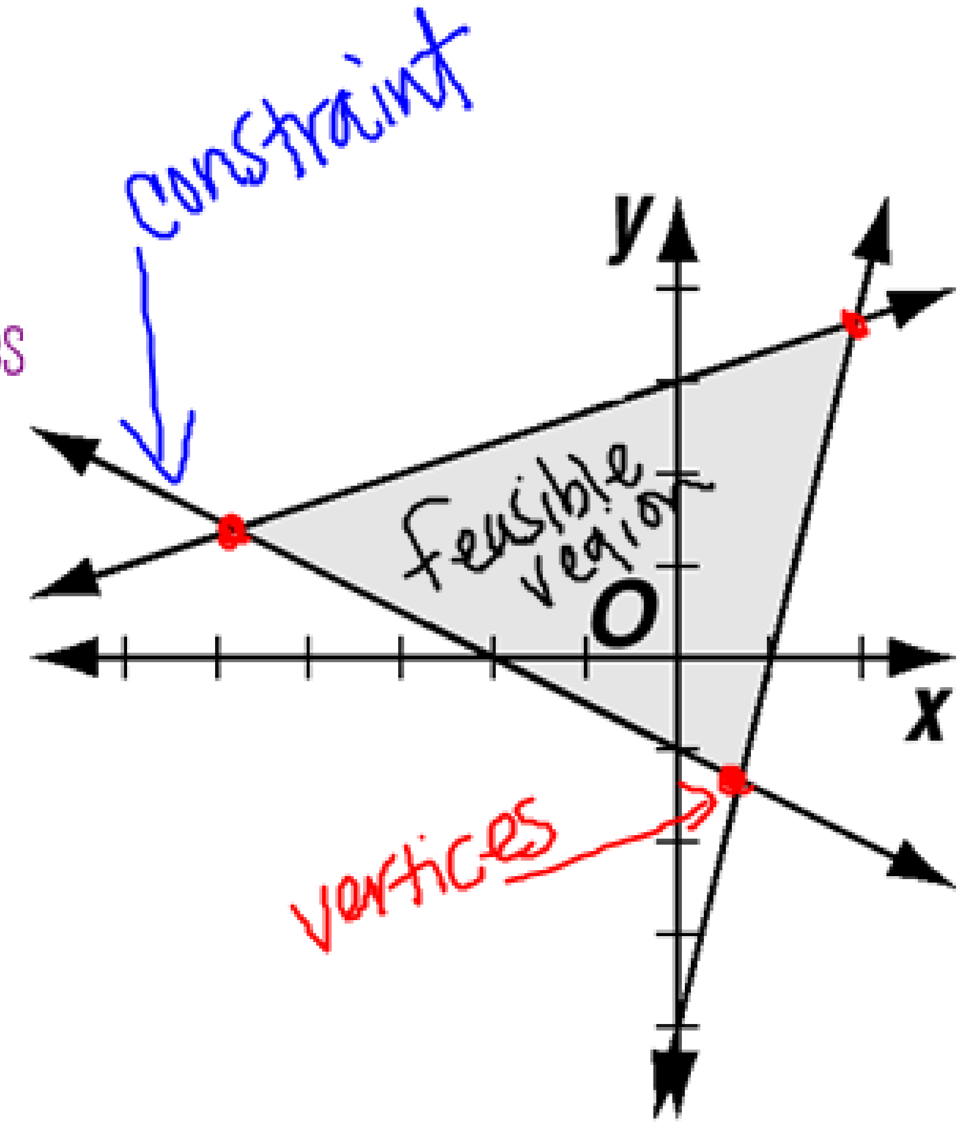
$4x - y \leq 4$

$x + 2y \geq -2$

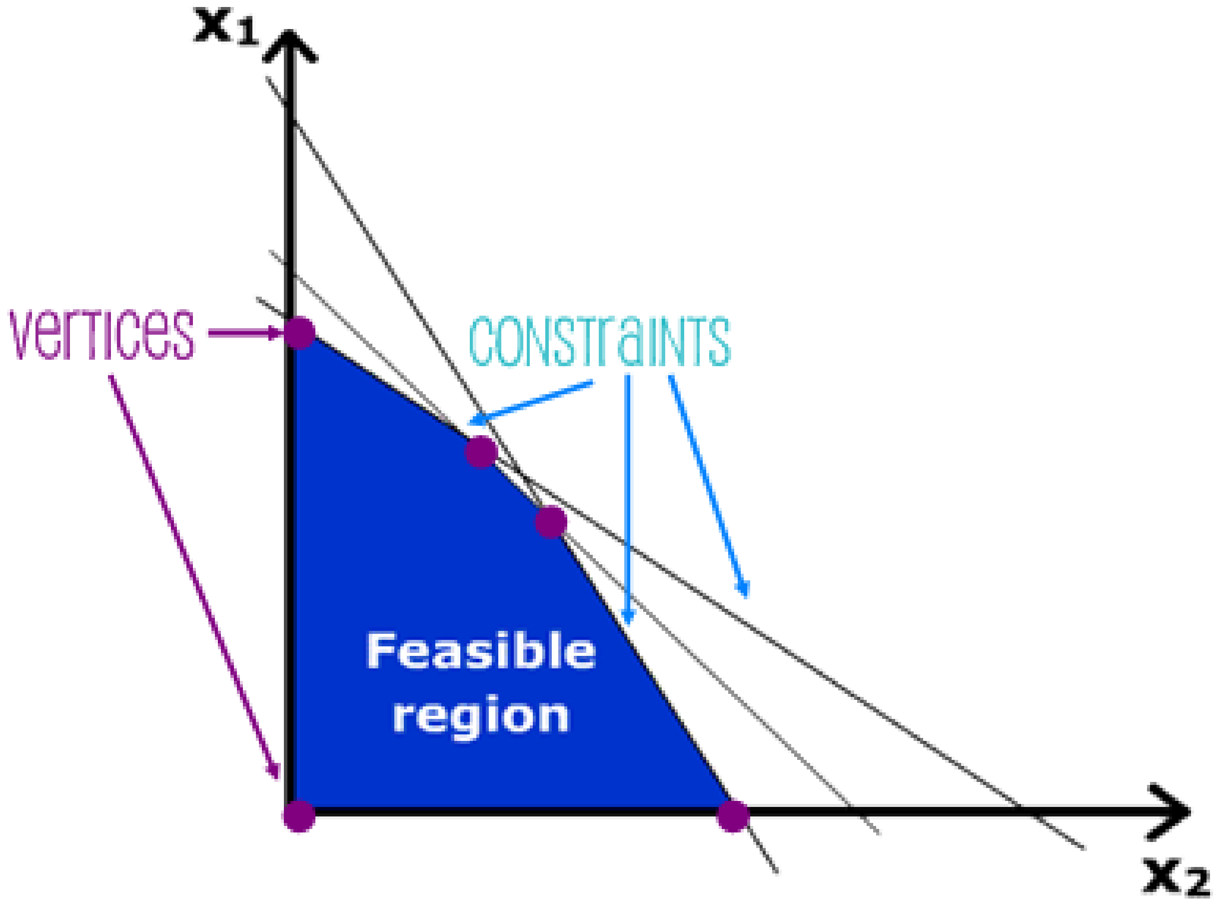
There are 3 parts to systems of inequalities

1. Constraints: the inequalities (lines)
2. Vertices: the points of intersection
3. Feasible region: the shaded region

~Label each part on the given graph~



Linear Programming is a process of finding a *maximum* or *minimum* of a function by using vertices of the polygon formed by the graph of the constraints.



$f(x)$ = f of x

~New Notation~

$F(X)=Y$

example: $Y=X+2$ can also be written as $F(X)=X+2$

So if we were wanting to find out what Y is when $X=50$, we can re-write this as $F(50)=50+2$. Therefore, $F(50)=52$ or when $X=50$, $Y=52$.

~So we know that (50, 52) is a solution to $Y=X+2$~

(2, 8)

example:

If $F(X)=6X-4$, what is $F(2)$?

$f(2)=6(2)-4 \rightarrow f(2)=12-4$

$f(2)=8$

What is $F(10)$?

$f(10)=6(10)-4 \rightarrow f(10)=60-4 \rightarrow f(10)=56$

(10, 56)

$f(x)=6x-4$

Similarly

If $F(X,Y)=2X+Y$

How would we find $F(3,4)$?

x y

$f(3,4)=2(3)+4$

$f(3,4)=6+4 \rightarrow \boxed{f(3,4)=10}$

We would plug 3 in for X and 4 in for Y, so we would get $F(3,4)=2(3)+4$

Example:

If $F(X,Y)=5X-4Y$, what is $F(2,1)$?

$f(2,1)=5(2)-4(1) \rightarrow f(2,1)=10-4$

$\boxed{f(2,1)=6}$

What is $F(6,3)$? $f(6,3)=5(6)-4(3) \rightarrow f(6,3)=30-12$

$\boxed{f(6,3)=18}$

Finding Maximums and Minimums of Linear Programming

To find maximums and minimums in linear programming follow these four steps:

1. Graph the inequalities (shade)
2. Find the vertices of the feasible region (Points)
3. Use a chart to find the maximum & minimum values of the function

(x,y)	Function equation	f(x,y)

4. The point which has the biggest f(x,y) is the max and the point which has the smallest f(x,y) is the min.

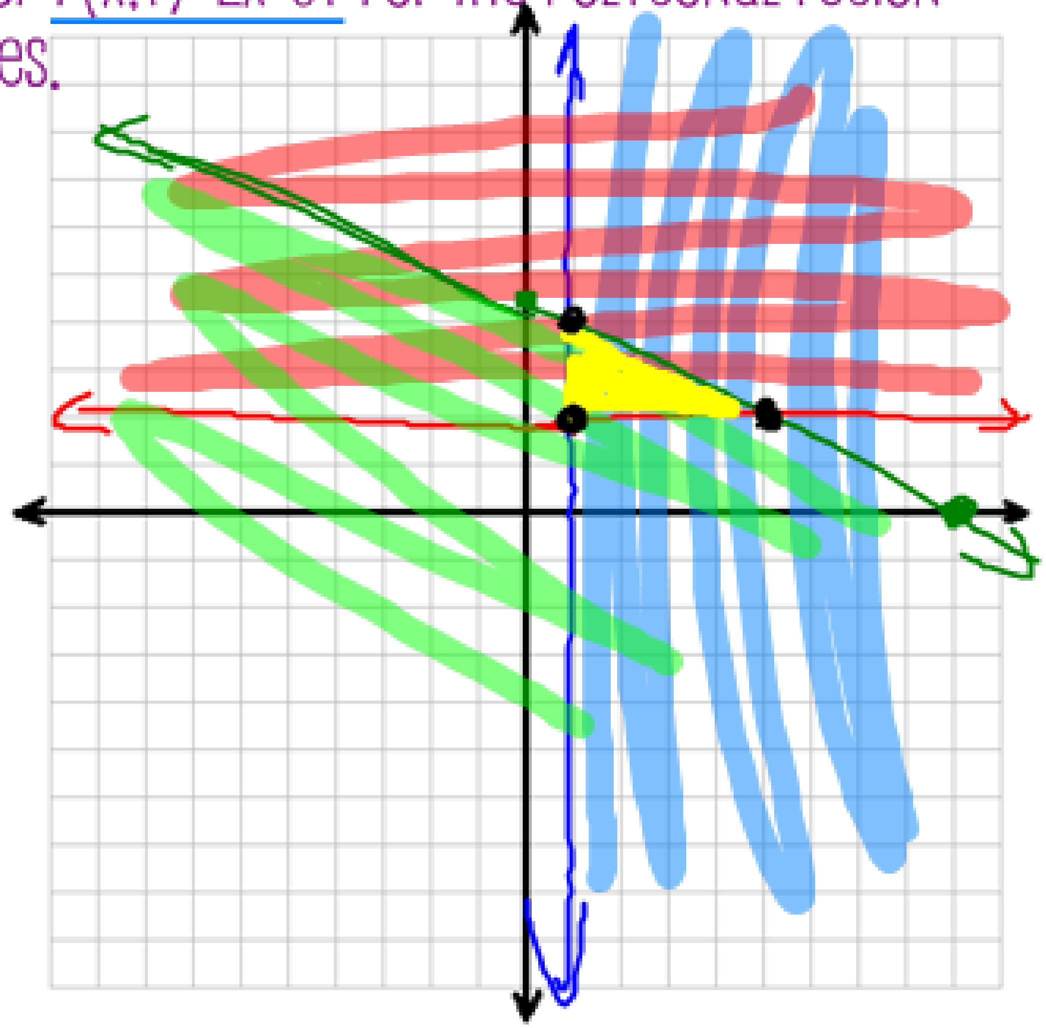
Example of Linear Programming

Find the maximum and minimum values of F(X,Y)=2X-3Y for the polygonal region determined by the system of inequalities.

$X \geq 1$

$Y \geq 2$

$X + 2Y \leq 9$



(X,Y)	2X-3Y	F(X,Y)
(1,2)	2(1)-3(2)	f(1,2)=-4
(5,2)	2(5)-3(2)=10-6	f(5,2)=4
(1,4)	2(1)-3(4)=2-12	f(1,4)=-10

Max is 4 at (5,2).
Min is -10 at (1,4).

$X \geq 1$

$Y \geq 2$

$X + 2Y \leq 9$

X-int: $(9, 0)$

$x \leq 9$

y-int: $(0, 4\frac{1}{2})$

$\frac{2y}{2} \leq \frac{9}{2}$

$y \leq \frac{9}{2}$

$y \leq 4\frac{1}{2}$

Test $(0,0)$

$0 + 2(0) \leq 9$

$0 \leq 9$

Linear Programming

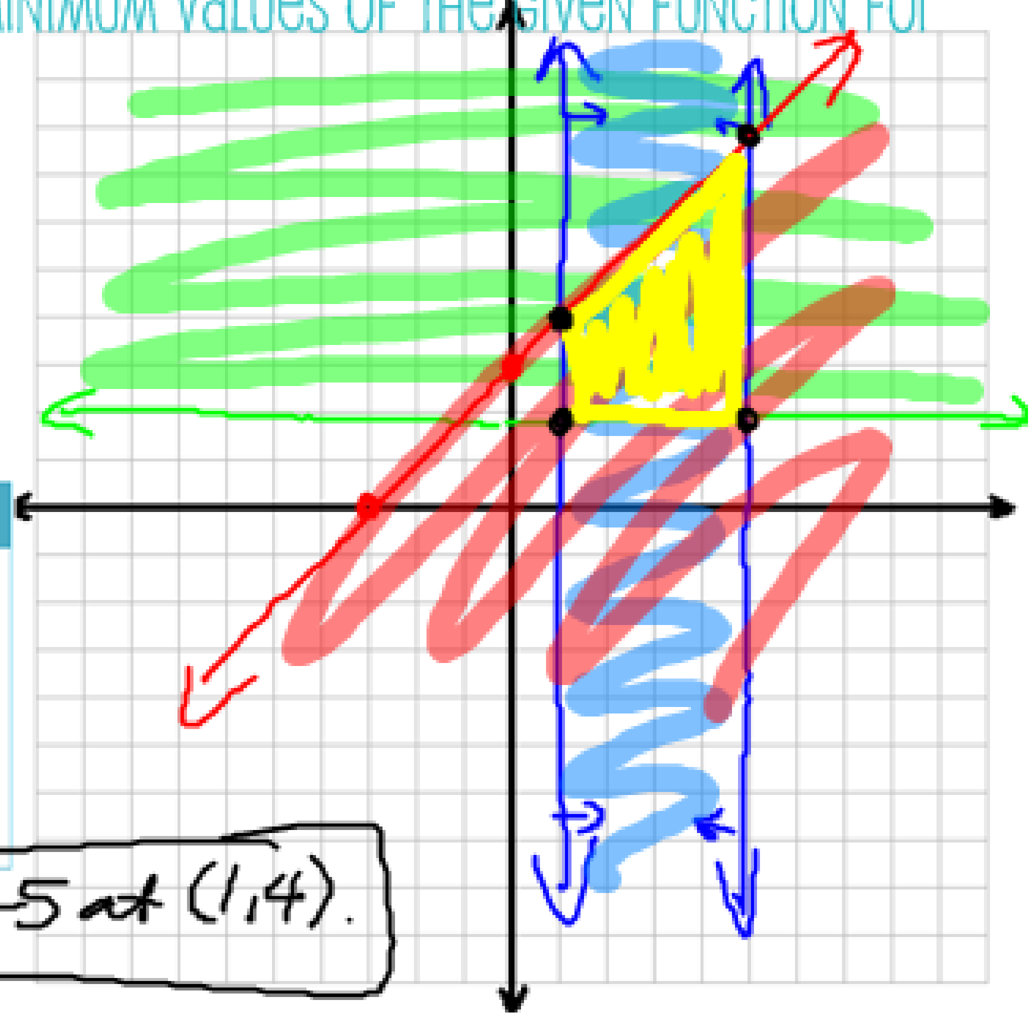
Graph the system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region.

$Y \geq 2$

$1 \leq X \leq 5$ $x \geq 1$ $x \leq 5$

$Y \leq X + 3$

$F(X,Y) = 3X - 2Y$



(X,Y)	3X-2Y	F(X,Y)
(1,2)	3(1)-2(2)=3-4	f(1,2) = -1
(5,2)	3(5)-2(2)=15-4	f(5,2) = 11
(5,8)	3(5)-2(8)=15-16	f(5,8) = -1
(1,4)	3(1)-4(2)=3-8	f(1,4) = -5

Max is 11 at (5,2). Min is -5 at (1,4).

$Y \geq 2$

$1 \leq X \leq 5$

$Y \leq X+3$

x-int: (-3 , 0)

$0 \leq x+3$

$-3 \quad -3$

$-3 \leq x$

y-int: (0 , 3)

$y \leq 3$

Test (0,0)

$0 \leq 0+3$

$0 \leq 3$ ✓

~Assignment #13~

Due next time (review for test)

~Journal #13~

Due the Math Lab after the test