Lesson 13: Linear Programming

By the end of the lesson, you will be able to:

~ Solve a linear programming problem.
~ Find a maximum or minimum of a linear programming problem.

Lesson 13: Linear Programming

~Review~

Graph the following system of inequalities (x and y int):

$x - 3y \geq -9$

x-int: $(-9, 0)$

y-int: $(0, 3)$

Test: $(0,0)$

$0 - 0 \geq -9$

$0 \geq -9$ ✓

$4x - y \leq 4$

x-int: $(1, 0)$

y-int: $(0, -4)$

Test: $(0,0)$

$0 - 0 \leq 4$

$0 \leq 4$ ✓

$x + 2y \geq -2$

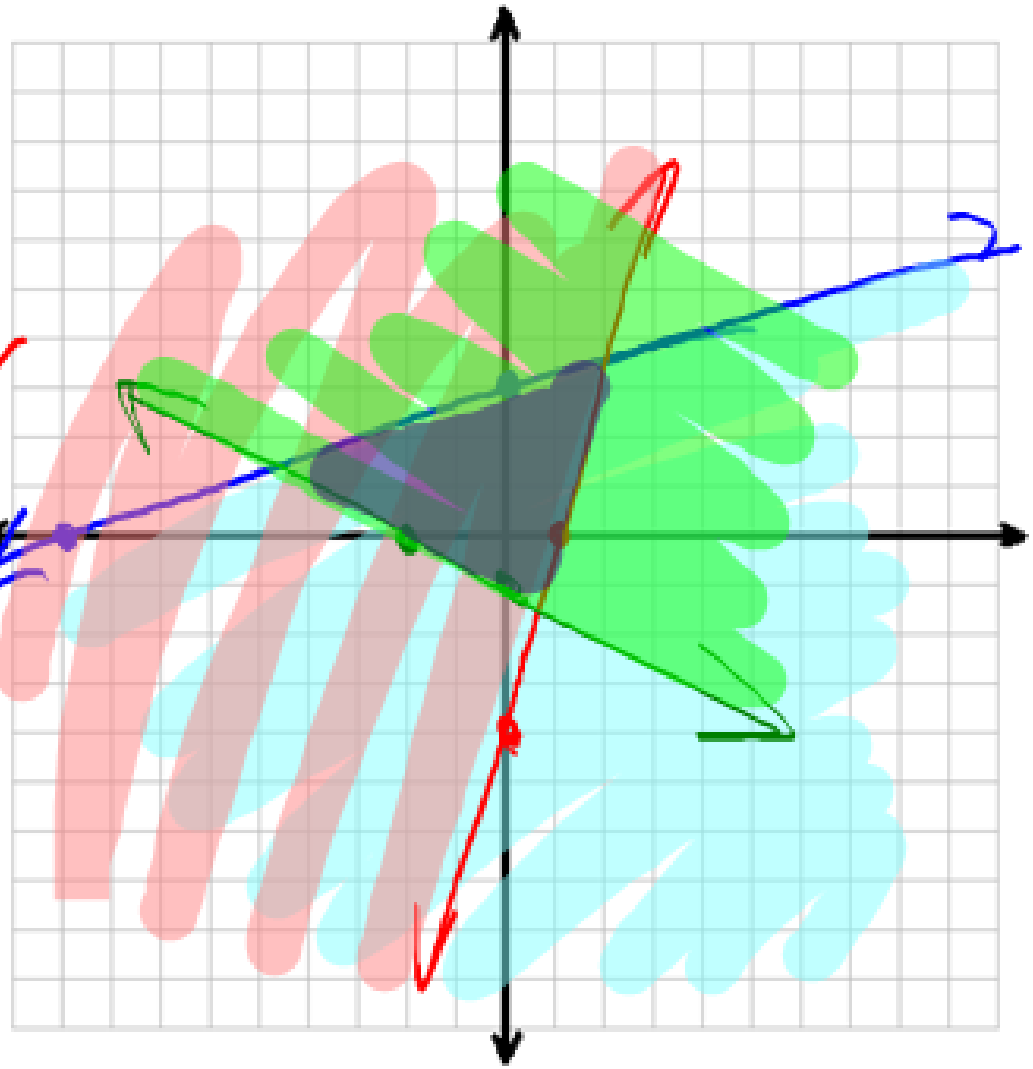
x-int: $(-2, 0)$

y-int: $(0, -1)$

Test: $(0,0)$

$0 + 0 \geq -2$

$0 \geq -2$ ✓



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$x - 3y \geq -9$ $\quad$ $4x - y \leq 4$ $\quad$ $x + 2y \geq -2$

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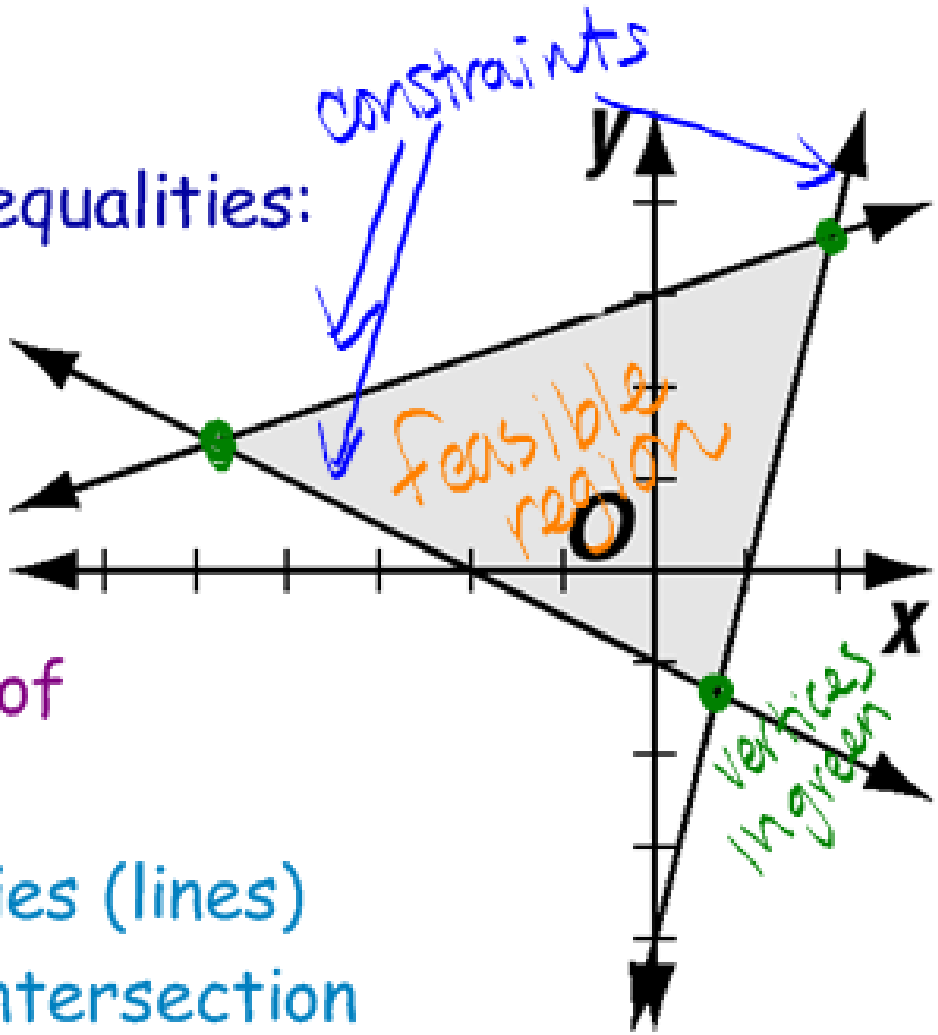
Using the same systems of inequalities:

$$x - 3y \geq -9$$

$$4x - y \leq 4$$

$$x + 2y \geq -2$$



There are 3 parts to systems of inequalities:

1. Constraints: the inequalities (lines)
2. Vertices: The points of intersection
3. Feasible Region: The shaded region

~Label each part on the given graph~

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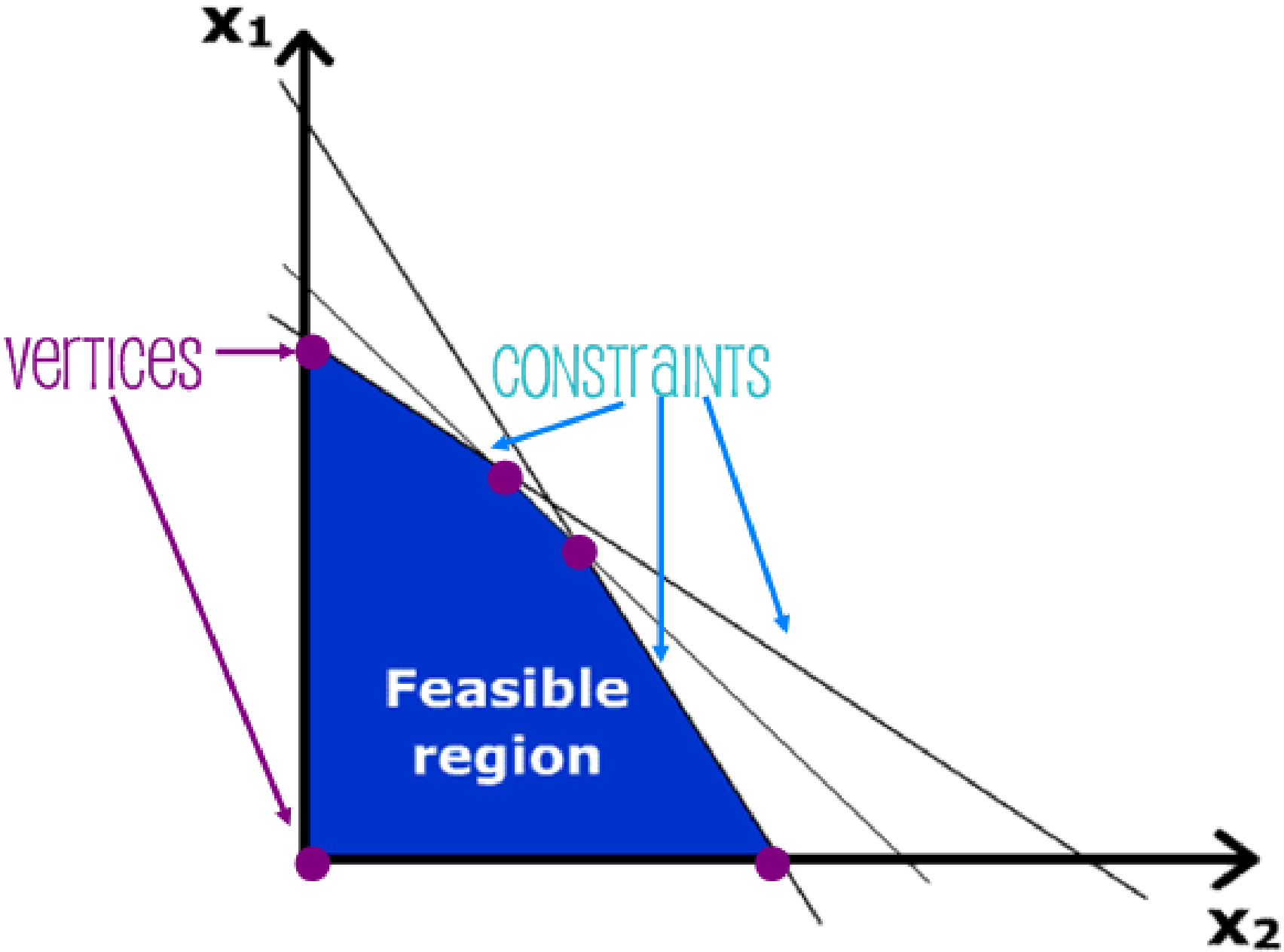

Linear Programming

is a process of finding a maximum or minimum of a function by using vertices of the polygon formed by the graph of the constraints.

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~New notation~

$f(x)=y$

example: $y=x+2$ can also be written as $f(x)=x+2$

$y = 50+2$

$y = 52$

So if we were wanting to find out what y is when x=50, we can re-write this as $f(50)=50+2$. therefore, $f(50)=52$ or when x=50, y=52.

$f(50)=52$

~So we know that (50, 52) is a solution to y=x+2~

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$f(x) = y$

Example:

if f(x)=6x-4, what is f(2)?

$f(2) = 6(2) - 4$

$f(2) = 12 - 4$

$\boxed{f(2) = 8}$

What is f(10)?

$f(10) = 6(10) - 4$

$f(10) = 60 - 4$

$\boxed{f(10) = 56}$

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~SIMILARLY~

If $f(x,y)=2x+y$

how would we find $f(3,4)$?

$f(3,4) = 2(3) + 4$

$= 6 + 4$

$f(3,4) = 10$

We would plug 3 in for x and 4 in for y, so we would get:

$f(3,4)=2(3)+4$

Example:

if $f(x,y)=5x-4y$, what is $f(2,1)$?

$f(2,1) = 5(2)-4(1)$

$= 10 - 4$

$f(2,1) = 6$

What is $f(6,3)$?

$f(6,3) = 5(6)-4(3)$

$= 30 - 12$

$f(6,3) = 18$

Finding Maximums and Minimums of Linear Programming:

Follow these four steps:

1. Graph the inequalities. (& shade)
2. Find the vertices of the feasible region. (points)
3. Use a chart to find the max & min values of the function.

points $f(x,y) = 2x - 3y$

(X,Y)	FUNCTION EQUATION	F(X,Y)
(2, 1)	$2(2) - 3(1) =$	$f(2,1) = 1$
( )		
( )		

4. The point which has the biggest f(x,y) is the max. The point that has the smallest f(x,y) is the min.

Example 1: Find the Max and Min for the polygonal region. Use the following equation: $f(x, y) = 2x - 3y$

$x \geq 1$

$y \geq 2$

$x + 2y \leq 9$

Test: (0,0)

$0 + 0 \leq 9$

$0 \leq 9$

x-int: (9,0)

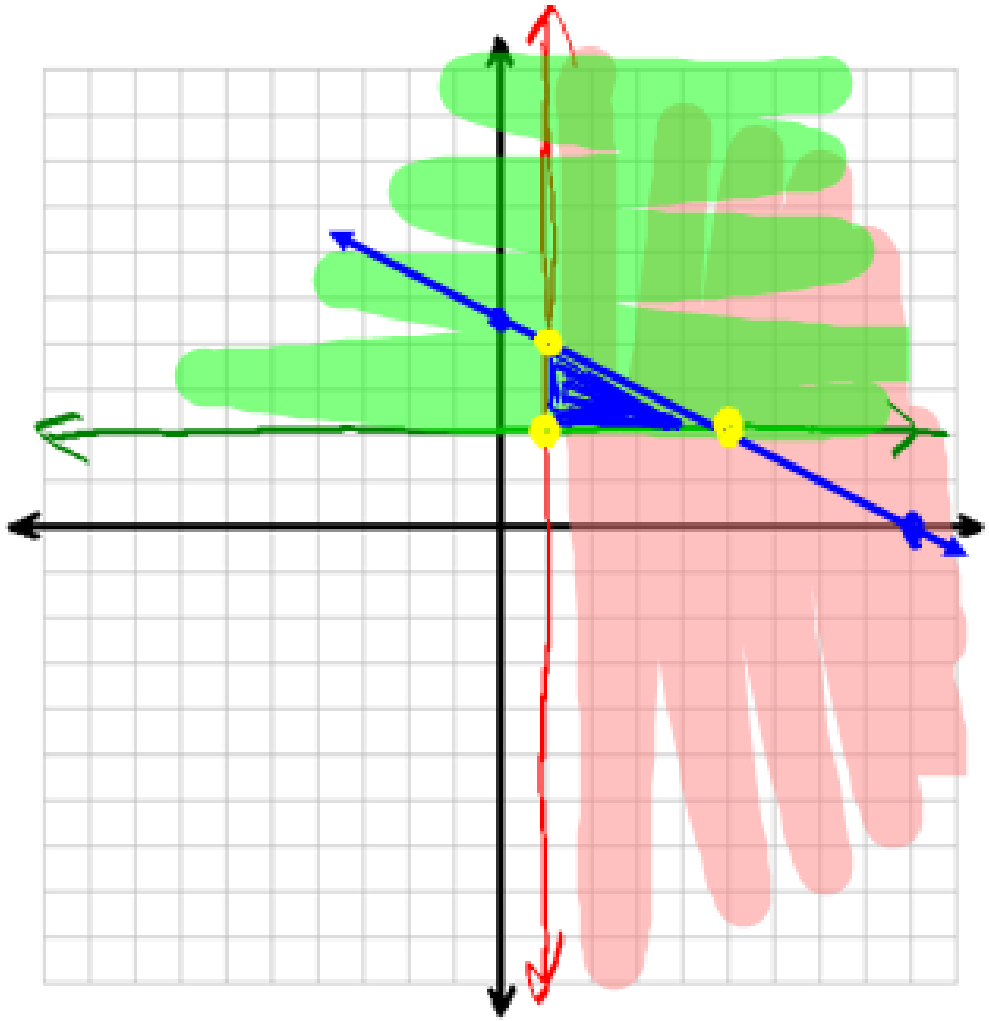
y-int: (0, 9/2)

What are the vertices?

(1,2)

(5,2)

(1,4)



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Work for Example 1: (x and y int so you can graph)

$x \geq 1$ $y \geq 2$ $x + 2y \leq 9$

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Work for Example 1: (Put vertices in chart so you can find the Maximum and Minimum.) $f(x, y) = 2x - 3y$

(X,Y)	2X-3Y	F(X,Y)	
(1,2)	2(1)-3(2) =	f(1,2) = -4	
(5,2)	2(5)-3(2)=	f(5,2) = 4	Max
(1,4)	2(1) - 3(4) =	f(1,4) = -10	Min

The Maximum is: 4 at (5,2)

The Minimum is: -10 at (1,4)

Example 2: Find the Max and Min for the polygonal region. Use the following equation: $f(x,y) = -2y + 3x$

$f(x,y) = 3x - 2y$

$y \geq 2$

$1 \leq x \leq 5$

$y \leq x + 3$

x-int $(-3,0)$ y-int $(0,3)$

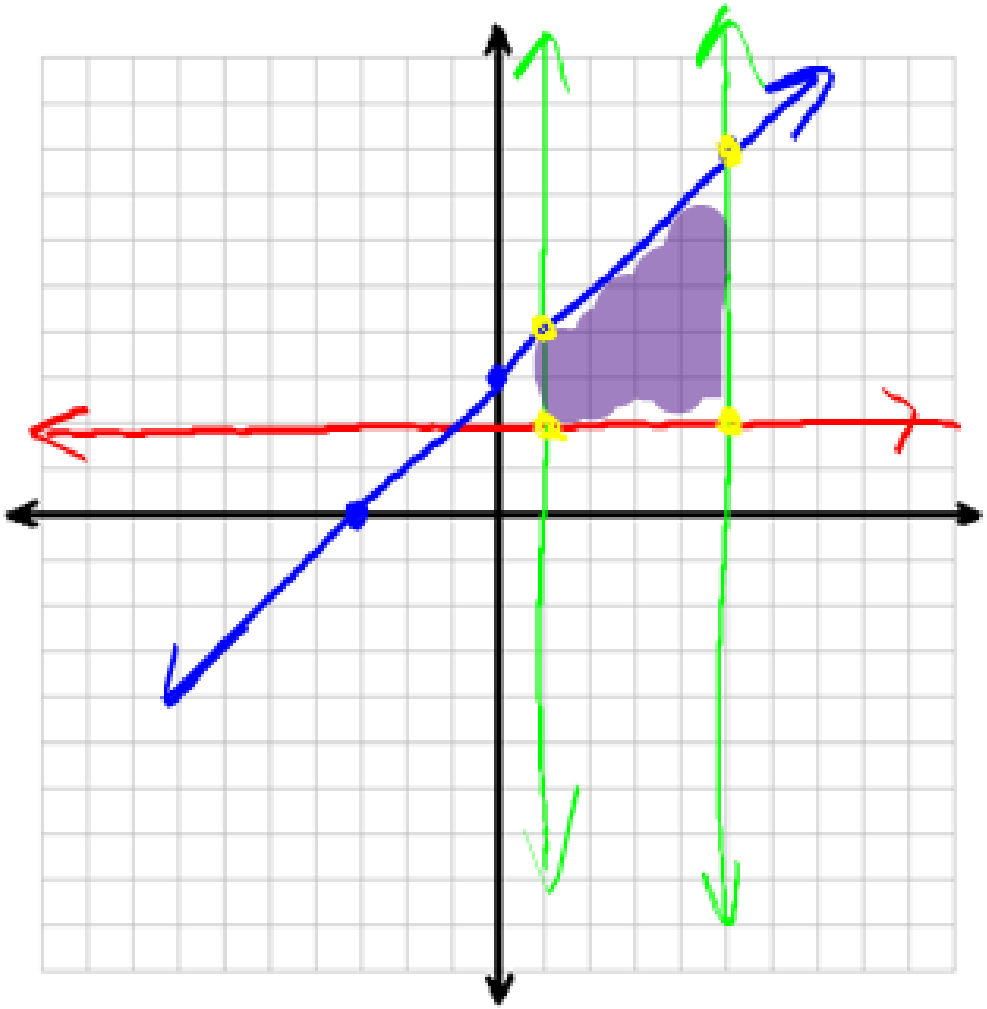
What are the vertices?

$(1,2)$

$(1,4)$

$(5,2)$

$(5,8)$



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Work for Example 2: (x and y int so you can graph)

$y \geq 2$ $\quad$ $1 \leq x \leq 5$ $\quad$ $y \leq x + 3$

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Work for Example 2: (Put vertices in chart so you can find the Maximum and Minimum.) $f(x,y) = -2y + 3x$

$3x - 2y$

(X,Y)	3X-2Y	F(X,Y)	
(1,2)	$3(1)-2(2) = -1$	$f(1,2) = -1$	
(1,4)	$3(1)-2(4) = -5$	$f(1,4) = -5$	min
(5,2)	$3(5)-2(2) = 11$	$f(5,2) = 11$	MAX
(5,8)	$3(5)-2(8) = -1$	$f(5,8) = -1$	

The Maximum is: 11 at (5,2)

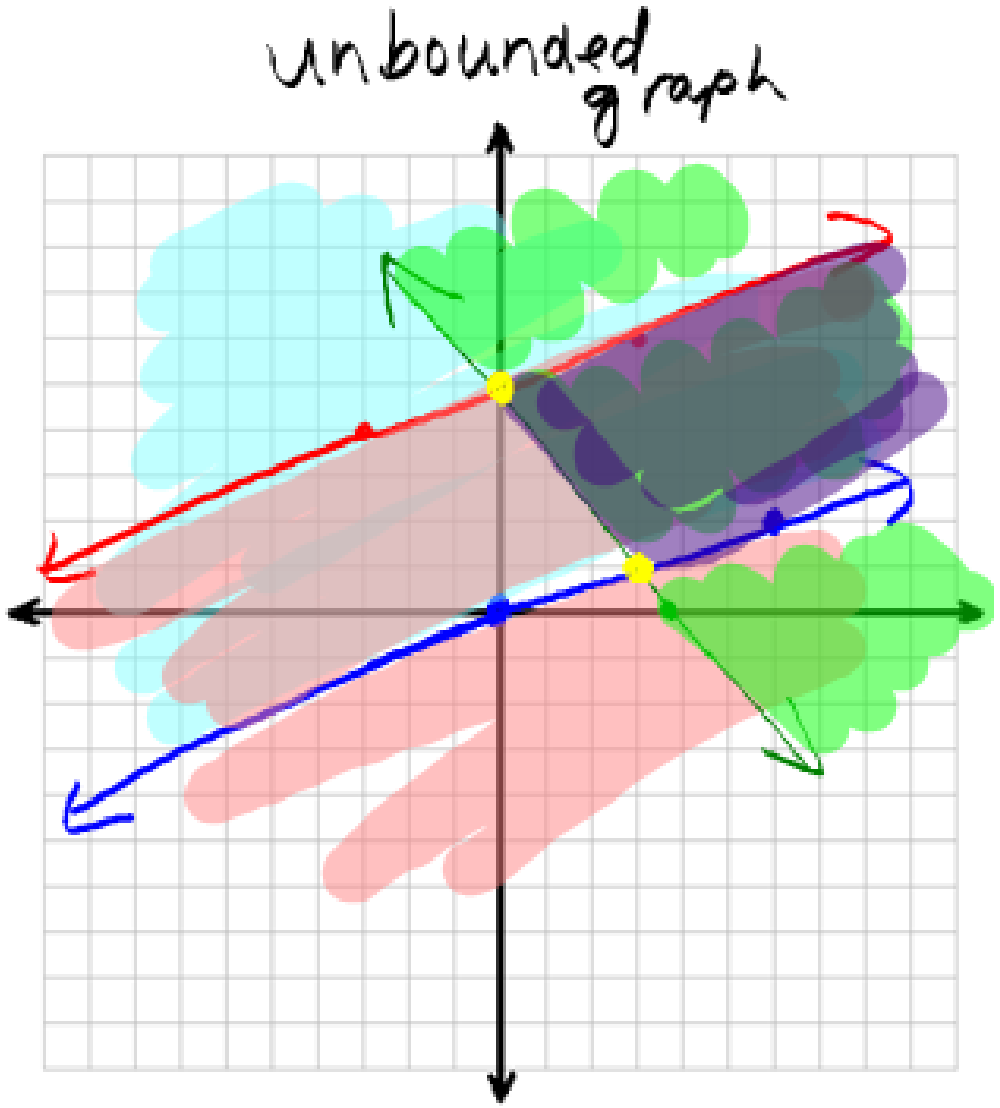
The Minimum is: -5 at (1,4)

Example 3: Find the Max and Min for the polygonal region. Use the following equation: $f(x, y) = 5x + 2y$

$x - 3y \leq 0$ Test: (1,1) $1 - 3 \leq 0$, $-2 \leq 0$ ✓

$x - 3y \geq -15$ Test: (0,0) $0 - 0 \geq -15$, $0 \geq -15$ ✓

$4x + 3y \geq 15$ Test (0,0) $0 \geq 15$ ✗



What are the vertices?

(0,5)

(3,1)

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Work for Example 3: (x and y int so you can graph)

$x - 3y \leq 0$

x-int: (0,0)

y-int: (0,0)

Pick x=6

$6 - 3y \leq 0$

$-6 \quad -6$

$\frac{-3y}{-3} \leq \frac{-6}{-3}$

$y \geq 2$

(6,2)

$x - 3y \geq -15$

x-int: (−15, 0)

y-int: (0, 5)

$\frac{-3y}{-3} \geq \frac{-x-15}{-3}$

$y \geq \frac{1}{3}x + 5$

$4x + 3y \geq 15$

x-int: $(\frac{15}{4}, 0)$ (3.75, 0)

y-int: (0, 5)

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Work for Example 3: (Put vertices in chart so you can find the Maximum and Minimum.) $f(x, y) = 5x + 2y$

(x, y)	$5x + 2y$	$f(x, y) =$	
$(0, 5)$	$5(0) + 2(5) =$	$f(0, 5) = 10$	min
$(3, 1)$	$5(3) + 2(1) =$	$f(3, 1) = 17$	

The Maximum is: unbounded

The Minimum is: 10 at (0, 5)

Example 4: Find the Max and Min for the polygonal region. Use the following equation: $f(x, y) = x - 2y$

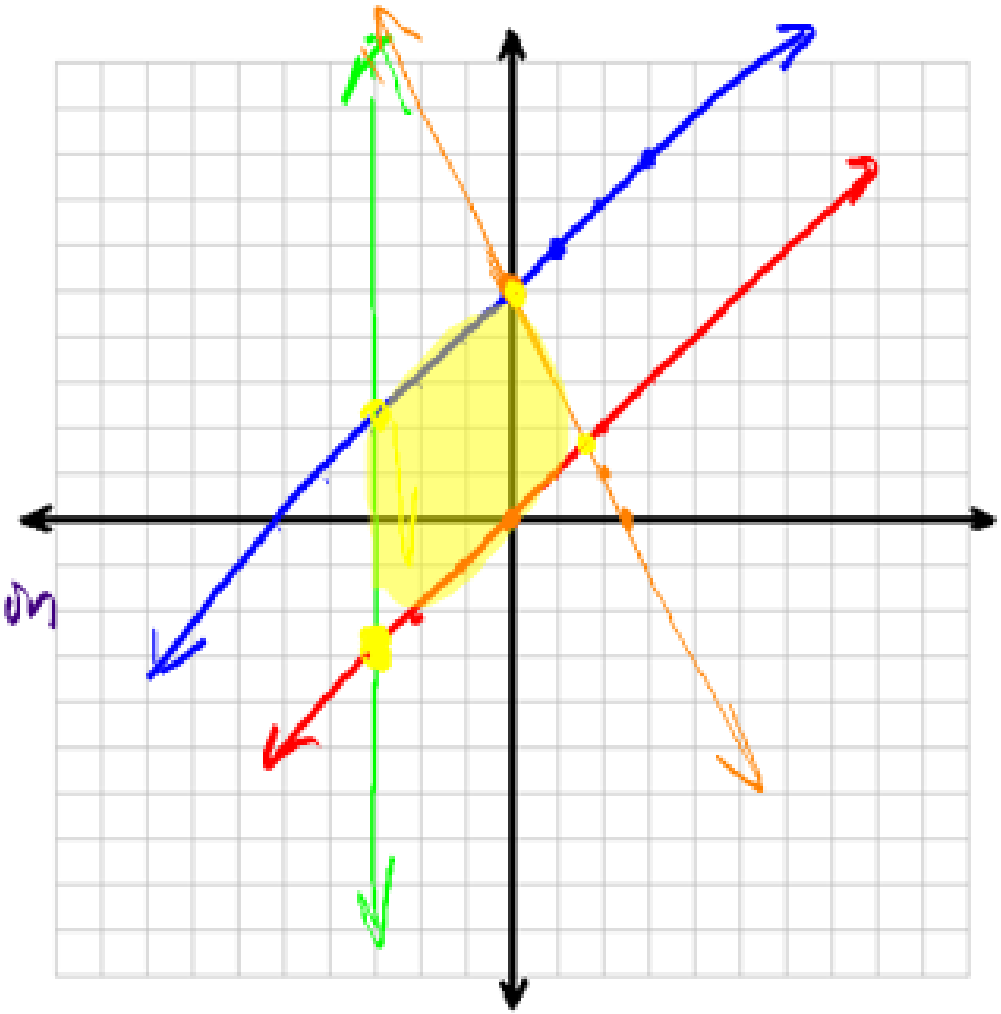
$y \leq x + 5$

$y \geq x$

$x \geq -3$

$y + 2x \leq 5$

x-int $\left(\frac{5}{2}, 0\right)$ y-int $(0, 5)$



What are the vertices?

$(3, 2)$

$(-3, -3)$

$(0, 5)$

$\left(\frac{5}{3}, \frac{5}{3}\right)$ or $(1.67, 1.67)$

find intersection on calc

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Work for Example 4: (x and y int so you can graph)

$y \leq x + 5$ $\quad$ $y \geq x$ $\quad$ $x \geq -3$ $\quad$ $y + 2x \leq 5$

$y \leq -2x + 5$

graph $y = x$

$y = -2x + 5$

and find interseetion on calculator

Round to 2 decimals! $\quad$ (1.67, 1.67)

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Work for Example 4: (Put vertices in chart so you can find the Maximum and Minimum.) $f(x,y) = x - 2y$

(x,y)	$x - 2y$	$f(x,y)$
$(-3,2)$	$-3 - 2(2) = -7$	$f(-3,2) = -7$
$(-3,-3)$	$-3 - 2(-3) = 3$	$f(-3,-3) = 3$ max
$(0,5)$	$0 - 2(5) = -10$	$f(0,5) = -10$ min
$(1.67,1.67)$	$1.67 - 2(1.67) = -1.67$	$f(1.67,1.67) = -1.67$

The Maximum is: 3 at $(-3,-3)$

The Minimum is: -10 at $(0,5)$

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By the end of the lesson, you will be able to:

~ Solve a linear programming problem.
~ Find a maximum or minimum of a linear programming problem.

Can you?

Homework:

Test Review 3 worksheet

Due next time (test day)

~Assignment #13~

Due day after test

You may use a calculator to find the vertices.

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