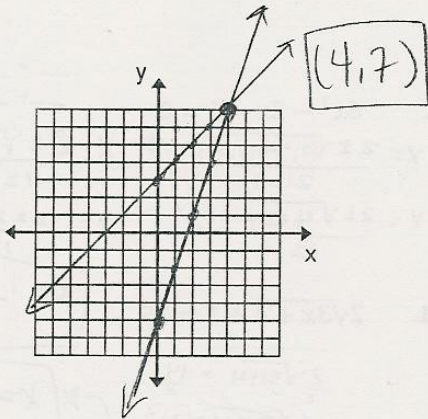


Algebra 2 Semester 1 Final Review (30 points)

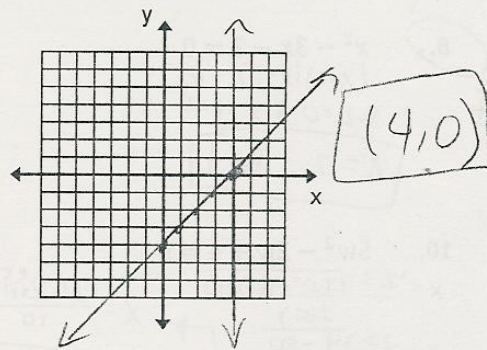
Part 1: No Calculator

Solve the system of equations by graphing:

1. $y = 3x - 5$
 $y = x + 3$

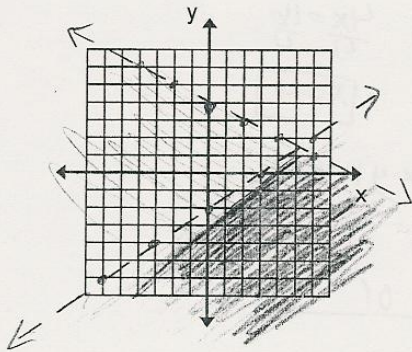


2. $y = x - 4$
 $3x = 12 \quad x = 4$



Graph the system of inequalities.

3. $x + 2y < 8 \quad y < \frac{8-x}{2} \rightarrow y < -\frac{1}{2}x + 4$
 $2x - 3y > 6 \quad -3y > 6 - 2x$
 $y < -2 + \frac{2}{3}x$

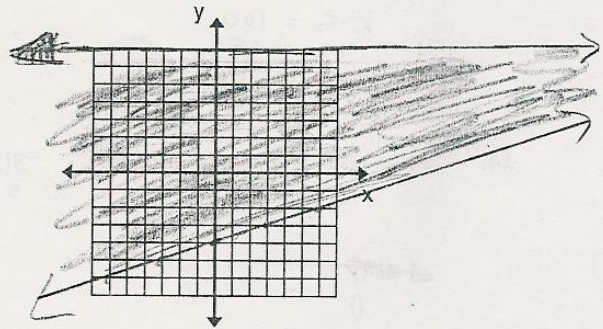


$x + 2y < 8$
Test (0,0)
 $0 < 8$
✓

$2x - 3y > 6$
Test (0,0)
 $0 > 6$
✗

4. $y \geq \frac{1}{3}x - 4$
 $y \leq 7$

Change scale...



$y \geq \frac{1}{3}x - 4$
Test (0,0)
 $0 \geq -4$
✓

Find the zeros of the functions.

5. $f(x) = x^2 - 2x - 24$
 $0 = x^2 - 2x - 24$
 $0 = (x-6)(x+4)$
 $x-6=0 \quad x+4=0$
 $x=6, x=-4$

7. $h(x) = 5x^2 - 40x$ $0 = 5x(x-8)$
 $0 = 5x(x-8)$
 $\frac{5x}{5} = \frac{0}{5} \quad x-8=0$
 $x=0, x=8$

6. $g(x) = 4x^2 - 7x - 15$
 $0 = 4x^2 - 7x - 15$
 $0 = (4x+5)(x-3)$
 $4x+5=0 \quad x-3=0$
 $\frac{4x}{4} = \frac{-5}{4} \quad \frac{x-3}{+3} = \frac{0}{+3}$
 $x = -\frac{5}{4}, x=3$

$\frac{-12}{-12} = \frac{5}{5} = -60$
 $\frac{-12}{-12} + \frac{5}{5} = -7$
 $x=3$

| | |
|--------|--------|
| $4x^2$ | $-12x$ |
| $+5x$ | -15 |

Solve.

8. $x^2 - 3x - 2 = 0$
 $\frac{3 \pm \sqrt{17}}{2}$

9. $6x^2 - 2x - 1 = 0$
 $x = \frac{2 \pm \sqrt{(2)^2 - 4(6)(-1)}}{2(6)}$
 $x = \frac{2 \pm \sqrt{4+24}}{12}$
 $x = \frac{2 \pm \sqrt{28}}{12}$
 $x = \frac{2 \pm 2\sqrt{7}}{12}$
 $x = \frac{1 \pm \sqrt{7}}{6}$

10. $5w^2 - 2w + 4 = 0$
 $x = \frac{2 \pm \sqrt{(2)^2 - 4(5)(4)}}{2(5)}$
 $x = \frac{2 \pm \sqrt{4-80}}{10}$
 $x = \frac{2 \pm \sqrt{-76}}{10}$
 $x = \frac{2 \pm 2i\sqrt{19}}{10}$
 $x = \frac{1 \pm i\sqrt{19}}{5}$

11. $2\sqrt{3x+4} + 1 = 15$
 $2\sqrt{3x+4} = 14$
 $\sqrt{3x+4} = 7$
 $(\sqrt{3x+4})^2 = 7^2$
 $3x+4 = 49$
 $3x = 45$
 $x = 15$

12. $-4 + \sqrt{x-5} = 6$
 $\sqrt{x-5} = 10$
 $(\sqrt{x-5})^2 = (10)^2$
 $x-5 = 100$
 $x = 105$

13. $(\sqrt[3]{4x-8})^3 = (2)^3$
 $4x-8 = 8$
 $4x = 16$
 $x = 4$

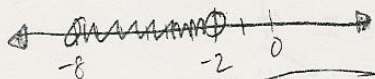
14. Solve and graph the inequality:

$3|x-4| > 12$
 $|x-4| > 4$
 $x-4 > 4 \quad x-4 < -4$
 $x > 8 \quad \text{or} \quad x < 0$



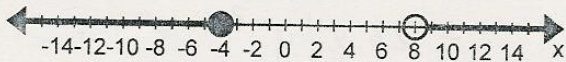
15. Solve and graph the inequality:

$|x+5| - 2 < 1$
 $|x+5| < 3$
 $x+5 < 3 \quad x+5 > -3$
 $x < -2 \quad x > -8$



$-8 < x < -2$

16. Write a compound inequality for the following graph:



$$x \leq -4 \text{ or } x > 8$$

17. Write a compound inequality for the following graph:



$$-3 \leq x < 2$$

Simplify.

$$\begin{aligned} 18. \quad & (3m^2 + 5m - 6) - (7m^2 - 9) \\ &= 3m^2 + 5m - 6 - 7m^2 + 9 \\ &= -4m^2 + 5m + 3 \end{aligned}$$

$$\begin{aligned} 19. \quad & (4x + 7x^3 + 50) + (8x^3 - x + 3x^2) \\ &= 4x + 7x^3 + 50 + 8x^3 - x + 3x^2 \\ &= 15x^3 + 3x^2 + 3x + 50 \end{aligned}$$

$$\begin{aligned} 20. \quad & (4n - 19) - (7n + 1) \\ &= 4n - 19 - 7n - 1 \\ &= -3n - 20 \end{aligned}$$

$$\begin{aligned} 21. \quad & (a + 3)(2a^2 + a - 4) \\ &= 2a^3 + a^2 - 4a + 6a^2 + 3a - 12 \\ &= 2a^3 + 7a^2 - a - 12 \end{aligned}$$

$$\begin{aligned} 22. \quad & (2x + 5)(x + 1)(3x - 2) \\ &= (2x^2 + 2x + 5x + 5)(3x - 2) \\ &= (2x^2 + 7x + 5)(3x - 2) \\ &= 6x^3 - 4x^2 + 21x^2 - 14x + 15x - 10 \\ &= 6x^3 + 17x^2 + x - 10 \end{aligned}$$

$$\begin{aligned} 23. \quad & \left(\frac{-2m}{4}\right)^4 = \frac{(-2m)^4}{(4)^4} = \frac{(-2)^4 m^4}{256} \\ &= \frac{+16m^4}{256} = \frac{+m^4}{16} \end{aligned}$$

$$\begin{aligned} 24. \quad & \left(\frac{2}{-5p^3r^4}\right)^3 = \frac{2^3}{(-5)^3(p^3)^3(r^4)^3} \\ &= \frac{8}{-125p^9r^{12}} \end{aligned}$$

$$25. \quad \left(\frac{4}{n}\right)^{-3} = \left(\frac{n}{4}\right)^3 = \frac{n^3}{64}$$

$$26. (-5x^2y^3)(-2x^{-5}y^{-1})$$

$$= 10x^{-3}y^2$$

$$= \boxed{\frac{10y^2}{x^3}}$$

$$28. (4a^2)(3b^3)(2a^2b)$$

$$= (12a^2b^3)(2a^2b)$$

$$= \boxed{24a^4b^4}$$

$$30. \frac{2x^3y^5z^3}{8x^3y^2z} = \boxed{\frac{1}{4} \frac{y^3z^2}{1}}$$

$$27. (4x^3y^{-4})(7xy^2)$$

$$= 28x^4y^{-2}$$

$$= \boxed{\frac{28x^4}{y^2}}$$

$$29. 5a^5 \cdot 7a^8 \cdot 2a$$

$$= 35a^{13} \cdot 2a$$

$$= \boxed{70a^{14}}$$

$$31. \frac{-6x^{-2}y^4}{24x^8y^{-5}} = \frac{-1x^{2-8}y^{4-(-5)}}{4} = \frac{-1x^{-10}y^9}{4}$$

$$= \boxed{\frac{-y^9}{4x^{10}}}$$

$$32. 5\sqrt{3} + 8\sqrt{3} - 2\sqrt{2} - \sqrt{3}$$

$$= 13\sqrt{3} - 2\sqrt{2} - \sqrt{3}$$

$$= \boxed{12\sqrt{3} - 2\sqrt{2}}$$

$$33. \sqrt{20} + \sqrt{8} - 5\sqrt{5}$$

$$\begin{matrix} \sqrt{4 \cdot 5} & \sqrt{4 \cdot 2} \\ 4 \cdot 5 & 4 \cdot 2 \end{matrix}$$

$$= 2\sqrt{5} + 2\sqrt{2} - 5\sqrt{5}$$

$$= \boxed{-3\sqrt{5} + 2\sqrt{2}}$$

$$34. (5 + \sqrt{6})(5 - \sqrt{2})$$

$$= 25 - 5\sqrt{2} + 5\sqrt{6} - \sqrt{12}$$

$$= \boxed{25 - 5\sqrt{2} + 5\sqrt{6} - 2\sqrt{3}}$$

$$35. (8 - 4\sqrt{2})(2 - \sqrt{2})$$

$$= 16 - 8\sqrt{2} - 8\sqrt{2} + 4\sqrt{4}$$

$$= 16 - 16\sqrt{2} + 4(2)$$

$$= 16 + 8 - 16\sqrt{2}$$

$$= \boxed{24 - 16\sqrt{2}}$$

$$36. (2 + 3i) - (4 - 2i)$$

$$= 2 + 3i - 4 + 2i$$

$$= \boxed{-2 + 5i}$$

$$37. 3(2 + 2i) + (3 - i)$$

$$= 6 + 6i + 3 - i$$

$$= \boxed{9 + 5i}$$

$$\begin{aligned}
 38. \quad & 2(6-i) - (i+4) \\
 & = 12 - 2i - i - 4 \\
 & = 12 - 3i - 4 \\
 & = \boxed{8 - 3i}
 \end{aligned}$$

$$\begin{aligned}
 39. \quad & (3i)(4i)^2(6i) \\
 & = (3i)(16i^2)(6i) \\
 & = (3i)(-16)(6i) \\
 & = (-48i)(6i) \\
 & = -288i^2 \\
 & = \boxed{288}
 \end{aligned}$$

$$\begin{aligned}
 40. \quad & (3-4i)(3+4i) \\
 & = 9 + 12i - 12i - 16i^2 \\
 & = 9 + 16 \\
 & = \boxed{25}
 \end{aligned}$$

$$\begin{aligned}
 41. \quad & (2+i)(4-i) \\
 & = 8 - 2i + 4i - i^2 \\
 & = 8 + 2i + 1 \\
 & = \boxed{9 + 2i}
 \end{aligned}$$

$$\begin{aligned}
 42. \quad & i^{37} = i^1 = \boxed{i} \\
 & \begin{array}{r} 9 \\ 4 \overline{)37} \\ \underline{-36} \\ 1 \end{array}
 \end{aligned}$$

$$\begin{aligned}
 43. \quad & i^{16} = i^0 = \boxed{1} \\
 & \begin{array}{r} 4 \\ 4 \overline{)16} \\ \underline{-16} \\ 0 \end{array}
 \end{aligned}$$

$$\begin{aligned}
 44. \quad & i^{74} = i^2 = \boxed{-1} \\
 & \begin{array}{r} 18 \\ 4 \overline{)74} \\ \underline{-34} \\ 32 \\ \underline{-32} \\ 0 \end{array}
 \end{aligned}$$

Evaluate.

$$45. 64^{\frac{2}{3}} = (4)^2 = \boxed{16}$$

$$46. 8^{-\frac{1}{3}} = \boxed{\frac{1}{2}}$$

$$47. \frac{12}{9^{\frac{2}{3}}} = \frac{12}{(3)^3} = \frac{12}{27} = \boxed{\frac{4}{9}}$$

Write in simplest radical form.

$$48. a^{\frac{3}{2}}b^{\frac{5}{2}}c^{\frac{1}{2}}$$

$$\begin{aligned}
 & = \sqrt[2]{a^3b^5c^1} \\
 & = \boxed{ab^2\sqrt{abc}}
 \end{aligned}$$

$$49. m^{\frac{1}{3}}n^{\frac{5}{6}}c^{\frac{8}{3}} = m^{\frac{2}{6}}n^{\frac{5}{6}}c^{\frac{16}{6}}$$

$$\begin{aligned}
 & = \sqrt[6]{m^2n^5c^{16}} \\
 & = \boxed{c^2\sqrt[6]{m^2n^5c^4}}
 \end{aligned}$$

$$50. (16a^5b^7)^{\frac{1}{3}}$$

$$\begin{aligned}
 & = \sqrt[3]{16a^5b^7} \\
 & = \boxed{2ab^2\sqrt[3]{2a^2b}}
 \end{aligned}$$

Write a quadratic equation which has the given roots.

51. 3 and -8

$$\begin{aligned}
 & x=3 \quad x=-8 \\
 & x-3=0 \quad x+8=0 \\
 & (x-3)(x+8)=0 \\
 & x^2+8x-3x-24=0 \\
 & \boxed{x^2+5x-24=0}
 \end{aligned}$$

52. -4 and -2/3

$$\begin{aligned}
 & x=-4 \quad x=-\frac{2}{3} \\
 & x+4=0 \quad 3x=-2 \\
 & \quad \quad \quad 3x+2=0 \\
 & (x+4)(3x+2)=0 \\
 & 3x^2+2x+12x+8=0 \\
 & \boxed{3x^2+14x+8=0}
 \end{aligned}$$

Part 2: Calculator Section

Find the vertex of each parabola.

53. $y = x^2 - 9x + 9$

$(4.5, -11.25)$

54. $y = \frac{1}{2}x^2 - 3x$

$(3, -4.5)$

55. Solve – show all steps (Linear Programming).

A company has two machines which produce bolts and nuts. The cost to run Machine X for an hour is \$30. During that hour, Machine X produces 200 bolts and 100 nuts. The cost to run Machine Y for an hour is \$20. During that hour, Machine Y produces 100 bolts and 150 nuts. Machine X and Machine Y combined can run no longer than 40 hours. The company has an order to produce at least 4000 bolts and 3000 nuts. The company wants to minimize the cost of running the machines.

$x = \text{Machine X}$
 $y = \text{Machine Y}$

$x \geq 0$
 $y \geq 0$

$f(x,y) = 30x + 20y$

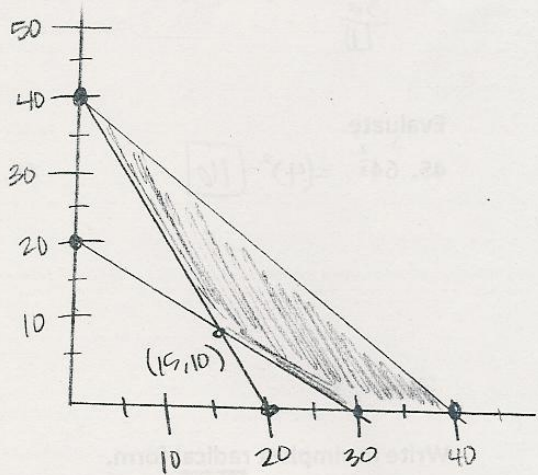
$x + y \leq 40$

$100x + 150y \geq 3000$

$200x + 100y \geq 4000$

| Machine | nuts | bolts | Cost |
|-----------|-------------|-------------|----------|
| X | 100 | 200 | 30 |
| Y | 150 | 100 | 20 |
| ≤ 40 | ≥ 3000 | ≥ 4000 | $f(x,y)$ |

| x-int | y-int |
|--------|--------|
| (40,0) | (0,40) |
| (30,0) | (0,20) |
| (20,0) | (0,40) |



| (x,y) | $f(x,y) = 30x + 20y$ | $f(x,y) =$ |
|---------|-------------------------|------------------|
| (0,40) | $30(0) + 20(40) = 800$ | $f(0,40) = 800$ |
| (15,10) | $30(15) + 20(10) = 650$ | $f(15,10) = 650$ |
| (30,0) | $30(30) + 20(0) = 900$ | $f(30,0) = 900$ |
| (40,0) | $30(40) + 20(0) = 1200$ | $f(40,0) = 1200$ |

The minimum cost is \$650 with running machine X 15 hrs and running machine Y 10 hrs.

56. 15 members of ski and snow board club went on a one-day ski trip. Members can rent skis for \$25.00 per day or snowboards for \$30.00 per day. The club paid \$332 for rental equipment. Write the system of equations that represents the number of members who rented two types of equipment. (You don't have to solve.)

x = ski
y = snowboard

$$\begin{cases} x + y = 15 \\ 25x + 30y = 332 \end{cases}$$

57. All 231 math club students went on a field trip. Some students rode in vans which hold 7 students each and some rode on buses which hold 25 students each. If they used 15 vehicles total, write the system of equations that represents the number of each type of vehicle. (You don't have to solve.)

x = van
y = bus

$$\begin{cases} x + y = 15 \\ 7x + 25y = 231 \end{cases}$$

58. The Photo Shop charges \$1.60 to develop a roll of film plus 11¢ for each print. Photos R Us charges \$1.20 to develop a roll plus 12¢ per print. At how many prints is it cheaper to use The Photo Shop?

| | | |
|------------|---------------|-----------------------------------|
| Photo Shop | $1.60 + .11x$ | $1.60 + .11x < 1.20 + .12x$ |
| Photo R Us | $1.20 + .12x$ | $-1.20 - .11x \quad -1.20 - .11x$ |

$$\frac{.40}{.01} < \frac{.01x}{.01}$$

$$40 < x$$

More than 40 prints

59. The cell phone company A charges a \$15.00 monthly fee plus 5 cents for every text. Company B charges a \$35.00 monthly fee plus 3 cents for every text. At how many texts would it be better to use company B?

Company A $15.00 + .05x$
Company B $35.00 + .03x$

$$15.00 + .05x > 35.00 + .03x$$

$$-15.00 - .03x \quad -15.00 - .03x$$

$$\frac{.02x}{.02} > \frac{20}{.02}$$

$$x > 1000$$

More than 1000 texts

60. A ball is thrown in the air. The height of the ball is given by the equation $h(t) = -16t^2 + 30t$, where t is the time in seconds. How long will it take for the ball to hit the ground, $h(t) = 0$, rounded to the nearest hundredth of a second?

$$0 = -16t^2 + 30t$$

$$0 = -2t(8t - 15)$$

$$\frac{-2t}{-2} = 0 \quad \frac{8t - 15}{+15} = 0$$

$$t = 0 \quad \frac{8t}{8} = \frac{15}{8}$$

$$t = \frac{15}{8}$$

$$t = 1.875 \text{ seconds}$$

$$1.88 \text{ sec}$$

61. A cannonball is fired from a cannon. Its height above the ground is given by the equation $h(t) = 85t - 16t^2$. How long after the cannonball is fired does it hit the ground? Round to the nearest tenth of a second.

$$0 = 85t - 16t^2$$

$$0 = t(85 - 16t)$$

$$t = 0 \quad \frac{85 - 16t}{-16} = 0$$

$$\frac{-16t}{-16} = \frac{-85}{-16}$$

$$t = 5.3125 \text{ seconds}$$

$$5.3 \text{ seconds}$$

62

Determine the number of solutions for each of the following systems of equations. (You don't have to solve, just find the answer using the graph.)

a. $y = 2x^2 + 4x - 5$
 $y = 0.5x - 3$

2 solutions
(crosses twice)

b. $y = 1$
 $y = x^2 + 3$

no solutions
(doesn't cross)

c. $2x + 3y = 12$
 $x = 5$

one solution
(crosses once)