**COLLEGE PREP**

**SECTION 1.4 - Linear Inequalities**

**Objectives:**

* Represent inequalities using the real number line and interval notation.
* Understand properties of inequalities.
* Solve linear inequalities and problems involving linear inequalities.

***Definition:*** A **Linear Inequality** in one variable is one that can be written in the form $ax+by>c$, $ax+by<c$, $ax+by\geq c$, or $ax+by\leq c$. > and < are called “strict inequalities” and $\leq or\geq $ are called “nonstrict inequalities”.

We are going to look at different ways of representing the solution sets of inequalities in this section. One way is set builder notation, {x|x<3} for example. However, we also like to use

**INTERVAL NOTATION:** Let *a* and *b* represent two real numbers where *a<b*. *a* is the “left endpoint” and *b* is the “right endpoint”.

* A **closed interval** means that the boundary points are included. This interval includes all real numbers of x for which $a\leq x\leq b$. The notation for a closed interval uses square brackets: $\left[a,b\right]$
* An **open interval** means boundary points are not included, and includes all real numbers of x for which $a<x<b$. The notation for a closed interval uses parentheses: $\left(a,b\right)$. Note: when graphing these, we use open circles. If you cut the open circle in half, it looks like parentheses.
* The **half-open or half-closed intervals** will have one square bracket and one parenthesis. The square bracket encloses the end closest to the $\leq or\geq $, and the parenthesis encloses the end closest to the >or <. So, if $a<x\leq b$ is shown as $\left(a,b\right]$, and $a\leq x<b$ is shown as [a,b).

What if the values of x don’t lie between two boundary points? If this is the case, use $\infty $ if the graph goes right or $-\infty $ if the graph goes left.

**INTERVALS THAT INCLUDE INFINITY:**

* $\left[a,\infty \right)$ is the same as $x\geq a$
* $\left(a,\infty \right)$ is the same as $x>a$
* $\left(-\infty ,a\right]$ is the same as $x\leq a$
* $\left(-\infty ,a\right)$ is the same as $x<a$
* $\left(-\infty ,\infty \right)$ is the same as all real numbers $R$ Note: $\infty $ is always bounded by a parentheses because it’s not a specific value.

**GRAPHING INEQUALITIES WITH INTERVAL NOTATION:**

This is just like graphing the inequalities that we’ve done before, but now, instead of using open and closed circles, we use a parenthesis instead of an open circle, and a square bracket instead of a closed circle.

***EXAMPLES:*** Write each of the inequalities using interval notation, then graph the inequality.

 A) $x>3$ Answer: $\left(3, \infty \right)$ 

 B) $x\leq 4$ Answer: $\left(-\infty , 4\right]$ 

 C) $-1<x\leq 3$ Answer: $\left(-1, 3\right]$ 

 D) $3\leq x\leq 8$ Answer: [3, 8] 

**NOW GO BACKWARDS:**

***EXAMPLES:*** Write each interval in inequality notation involving x, then graph it.

 E) $\left(-4, 1\right]$ Answer: $-4<x\leq 1$ 

 F) $(2, 5)$ Answer: $2<x<5$ 

 G) $\left(-\infty , 3\right)$ Answer: $x<3$ 

**PROPERTIES OF INEQUALITIES:**

The properties of inequalities are important to solving. When we solve a linear inequality, we use the same techniques and operations used in solving regular equations with one big difference – multiplying and dividing by negative numbers require us to change the direction of the inequality signs.

**Addition Properties:** (for all real numbers a, b, and c)

* If $a<b$, then $a+c<b+c$
* If $a>b$, then $a+c>b+c$
* *Adding or subtracting a value to both sides does not switch the inequality*.

**Multiplication Properties:** (for all real numbers a, b, and c)

* If $a<b$, or $a>b$, and $c>0$ (c is a positive number), then $ac<bc$ or $ac>bc$.
* *Multiplying (or dividing – c could be a fraction) by a positive number does not switch the inequality*.
* If $a<b or a>b, and c<0$ (c is a negative number), then $ac>bc or ac<bc$.
* *Multiplying by a negative number requires the inequality to change direction*!

***EXAMPLES:***

 **H)** solve the inequality $2x-5\geq 3$, write the answer in set-builder notation, interval notation, and then graph.

 $2x-5 \geq 3$ Add 5 to both sides. Addition doesn’t change the sign.

 $2x \geq 8$ Divide both sides by 2. 2 is positive, so the sign stays the same.

 $x\geq 8$

 Answers: $\left\{x\geq 8\right\}, \left[8, \infty \right)$ 

**I)** Solve the inequality $3\left(x-1\right)+2x<6x+3$, and graph.

 $3x-3+2x<6x+3$ Distribute the 3

 $5x-3<6x+3$ Simplify

 $-x<6$ Add 3, and subtract 6x from both sides

 $x>-6 $ Multiply both sides by -1, SWITCH THE SIGNS!

 Answers: $\left\{x>-6\right\}, \left(-6, \infty \right)$ 

**J)** Solve and graph $\frac{4+2x}{3}\geq \frac{x-2}{2}$

 $6\left(\frac{4+2x}{3}\right)\geq 6\left(\frac{x-2}{2}\right)$ Multiply both sides by the LCD -- 6

 $2\left(4+2x\right)\geq 3\left(x-2\right)$

 $8+4x\geq 3x-6$ Subtract 8 from both sides, and subtract 3x from both sides.

 $ x\geq -14$

 Answers: $\left\{x\geq -14\right\}, \left[-14, \infty \right)$ 

**TRANSLATING STORY PROBLEMS:**

 Words that translate to inequalities:

 $\geq $ At least, no less than

 $\leq $ No more than, at most

 $>$ Greater than, more than

 $<$ Less than, fewer than

***EXAMPLE:***

 **K)** Computing grades: In order to earn an A in Mrs. Smith’s College Prep course, Mark must maintain an average score of at least 90. On his first four exams, he scored 94, 83, 88, and 92. His final exam is worth two test scores. What score will he need to get on his final test in order to get an A?

 Identify the problem: the AVERAGE of his test scores will need to be $\geq $ 90. Why greater than or equal to? Why not just greater than?

 Name a variable: Let x= final test score

 Translate: $\frac{94+83+88+92+2t}{6}\geq 90$

 Solve: $\frac{357+2t}{6}\geq 90$ simplify

 $375+2t\geq 540$ Multiply both sides by 6

 $2t\geq 183$ subtract 375 from both sides

 $t\geq 91.5$ divide both sides by 2

 Answer: Mark must score a 91.5% or higher on his final exam to maintain an A.

Homework: Pg. 97: #4-8 all, 9-27 odds, 32-36 evens, 61, 67, 74