**COLLEGE PREP**

**SECTION 1.5 - Compound Inequalities**

**Objectives:**

* Determine the Union or Intersection of two sets.
* Solve compound inequalities involving “and” and “or”.
* Solve compound inequalities problems.

***REFRESHER ON SETS:***

* A **SET** is a collection of “well-defined” objects. “Well-defined” means that there is a rule determining whether or not an object can be in the set.
* **ELEMENTS** are what we call the objects in the set. We use curly braces to enclose the set { }.
* The **ROSTER METHOD** is when we list all the objects in the set.
* **SET-BUILDER NOTATION** is when we denote a set by stating the set’s rule {x|x follows this rule}.
* We **NAME** sets by using capital letters (Example, set A, set B, etc.) When we define rules for the sets, we usually use set A and set B as our generic sets.

***INTERSECTIONS & UNIONS OF SETS:***

Consider the following table:

Let set A be the set of students whose age is less than 25.

 A={Grace, Sophia, Kevin, Jack, George, Teresa}

Let set B be the set of students who are female.

 B={Grace, Sophia, Mary, Nancy, Teresa}

|  |  |  |
| --- | --- | --- |
| **STUDENT** | **Age** | **Gender** |
| Grace | 19 | F |
| Sophia | 23 | F |
| Kevin | 20 | M |
| Robert | 32 | M |
| Jack | 19 | M |
| Mary | 35 | F |
| Nancy | 40 | F |
| George | 22 | M |
| Teresa | 20 | F |

Now: list all the students who are in set A ***and*** set B. A ***and*** B = {Grace, Sophia, Teresa}

 This is called the ***INTERSECTION*** of A and B and is written as follows: $A∩B$.

 The word *and* implies *intersection*.

Now list all the students who are in set A or set B or both.

 A ***or*** B = {Grace, Sophia, Kevin, Jack, Mary, Nancy, George, Teresa}

 This is called the ***UNION*** of A and B, and is written as follows: $A∪B$.

 The word *or* implies *union*.



 This Venn Diagram shows the connection between

 sets. The center overlap is the Intersection.

 Everything within the diagram itself is the union.

**EXAMPLE:** Find the Intersection and Union of the following sets: A={1,3,5, 7,9} B={1, 2, 3, 4, 5}

 Answer: Intersection $A∩B$={1, 3, 5} Union $A∪B$ = {1, 2, 3, 4, 5, 7, 9}

How does this relate to Inequalities? When we graph two inequalities and the graphs overlap, it’s called an INTERSECTION. If we graph two inequalities, and they don’t overlap (they go opposite directions), it’s called a union.

**EXAMPLE:** Find the Intersection and Union of the sets: A={x| x$\leq $ 2}, B ={x | x $\geq $ -1}, C={x | x < -3}

A) Determine $A∩B$. Graph the set and write in set builder notation and in interval notation.

 

 Set-Builder Notation: { x | $-1\leq x\leq 2$} Interval: (-1,2]

B) Determine $B∪C$. Graph the set and write in set builder and interval notation.

 

 Set-builder: { x | $x<-3 or x\geq -1$} Interval: ($-\infty , -3) ∪[-1, \infty )$

***COMPOUND INEQUALITIES:***

 Compound inequalities are just two regular inequalities smashed together into one using “and” or “or”. For example if we have $3x+1>4$ and $2x-3<7$, and we put an “and” in between, we get a compound inequality. **To solve these, we solve each part separately, then we find either the union or intersection of the solution sets.**

**EXAMPLE:** Inequalities with “and”: Solve $3x+2>7 and 4x+1\leq 9$. Graph the solution set.

 $3x+2>7 and 4x+1\leq 9$

 $3x>5 and 4x\leq 8$

 $x>\frac{5}{3} and x\leq 2$ We can write our answers more

 compactly.

  If a<b, and x >a and x <b, we can

 Write it as a < x < b.

 So: $\frac{5}{3}<x\leq 2$ or $\left(\frac{5}{3}, 2\right]$

**EXAMPLE:** Inequalities with “and” Solve $-3<-4x+1<13$, then graph the solution set.

 $-3<-4x+1<13$ Solve all 3 parts at once - start by subtracting 1 from every part.

 $-4<-4x<12$ Divide each part by -4. NOTE: we’re dividing by a negative so flip signs!

 $1>x>-3$

  (-3,1)

***INEQUALITIES INVOLVING “OR”:*** Solve each part separately, and graph each part separately. There is no shortcut way to write the answer, it will always be two pieces with “or” between.

**EXAMPLE:** Solve and graph: $\frac{1}{2}x-1<1 or \frac{2x-1}{3}\geq -1$

 $\frac{1}{2}x-1<1 or \frac{2x-1}{3}\geq -1$

 $\frac{1}{2}x<2 or 2x-1\geq -3$

 $x<4 or 2x \geq -2$

 $ x<4 or x\geq -1$

 

Since the graphs overlap, but it’s an “or”, we are including everything in both sets. So that means that every number is covered, and our final answer is: { x | x is any real number}, or $\left(-\infty , \infty \right)$

**EXAMPLE:** Solve and graph: $5\left(x+2\right)>20 or 4\left(x-4\right)<-20$

 $5x+10>20 or 4x-16<-20$

 $5x>10 or 4x<-4$

 $x>10 or x<-1$ $\left(10, \infty \right) or \left(-\infty , -1\right)$



***STORY PROBLEMS:***

**EXAMPLE:** In your Psychology class, you have scores of 82 and 84 on your two tests. To get a grad of B, the average of the 2 tests and the final exam, which counts as 2 tests, must be greater than or equal to 80, and less than 90. Find the range of the scores that you need on the final exam to get a B.

**Identify:** x must be $\geq $ 80 and x must be < 90. So this is an “and” problem. Why?

**Name the variable:** x is our test score.

**Translate:**  Find the range of the scores. Think a minute -- we need to have an entire expression in the middle, not just x, because we’re averaging a bunch of scores.

 80 is less than or equal to the average of all the test scores, which is less than 90.

 $80\leq \frac{82+84+2x}{4}<90$

**SOLVE:** $320\leq 166+2x<360$

$154\leq 2x<194$

$77\leq x<97$

So: you must score between 77 and 96 on the final exam. Why 96? (it has to be less than 97)

Homework: Pg. 109-112: #2, 4, 5, 7, 9, 11-17 odds, 19, 21-25 odds, 37-43 odds, 79, 83