**COLLEGE PREP**

**SECTION 1.6 - Absolute Value Equations and Inequalities**

**Objectives:**

* Solve absolute value equations and inequalities
* Solve applied problems including absolute values.

**DEFINITION:** An absolute value is a distance. This means we have both positive and negative answers that correspond to distances left and right from our central value.

 $\left|u\right|=a is the same as u=a AND u=-a$

 Note on two special cases: |u|=0 is equivalent to u=0, so there’s only one solution. If a<0, there will be no solution, because an |u| is always going to revert to a positive value, it’s solutions don’t need to be positive, but the absolute value itself does!

**EXAMPLES:**

A) |x|=3 Answer: x = 3 and x = -3, so {3, -3}

B) |x|= -4 No solution! $∅$

***SOLVING EQUATIONS WITH ABSOLUTE VALUES:***

Step 1: Isolate the absolute value (move anything added or subtracted or multiplied to the absolute value). DO NOT MOVE PIECES WITHIN THE ABSOLUTE VALUE!

Step 2: Check the equation. Will you have one solution or no solution? If not, separate the equation into two pieces – one with a positive answer, and one with a negative answer.

Step 3: Solve each equation.

Step 4: CHECK YOUR ANSWER!

**EXAMPLES:**

C) |2x + 3| - 1 = 6

 |2x + 3| = 7 Isolate.

 2x + 3 = 7 and 2x + 3 = -7

 2x = 4 and 2x = -10

 X = 2 and x = -5 Answer: {2, -5} Go back and check!

D) |3 – x|+2 = 1

 |3 – x| = -1 Answer: $∅$ or { }

***SOLVING EQUATIONS WITH TWO ABSOLUTE VALUES:***

 **Rule:** $If \left|u\right|=\left|v\right|, then u=v and u=-v$

In other words, leave the first equation the same, but make everything in the second absolute value negative.

**EXAMPLE:**

E) Solve |x + 1| = |2x + 3|

 x+1 = 2x+3 and x+1 = -(2x+3)

 -2 = x and x + 1 = -2x-3

 3x = -4

 $x=-\frac{4}{3}$

 Answer: $\left\{-2,-\frac{4}{3}\right\}$

***ABSOLUTE VALUE INEQUALITIES:***

 Follow the same steps as before. There are a couple of things to consider, though:

**Inequalities with <**

 Rule: $If \left|u\right|<a, then -a<u<a$

 $If \left|u\right|\leq a, then -a\leq u\leq a$

 NOTES: |u|<0 has no real solution. $\left|u\right|\leq a$ has one solution, u = 0. If a is a negative, there is no solution.

**EXAMPLE:**

F) Solve and graph: $\left|\frac{1}{2}x+1\right|\leq 2$

 Rewrite: $-2\leq \frac{1}{2}x+1\leq 2$

 $-3\leq \frac{1}{2}x\leq 1$

 $-6\leq x\leq 2$

 Answer: [-6, 2] 

G) Solve and graph |3-2x| + 1 < 6

Inequalities with >

 Rule: $If \left|u\right|>a, then u<-a or u> a$

 $If \left|u\right|\geq a, then u\leq -a or u \geq a$

**EXAMPLE:**

H) Solve and graph |2x + 5| >7

 2x + 5 < -7 OR 2x+5 >7

 2x < -12 Or 2x >2

 x < -6 or x > 1

Answer: $\left(-\infty , -6\right) ∪ \left(1, \infty \right)$ 

**EXAMPLE:**

I) The inequality $\left|x-98.6\right|\geq 1.5$ represents a human body temperature (measured in Farenheit) that is considered “unhealthy.” Solve the inequality and interpret the results.

 Answer: $x\leq 97.1 OR x\geq 100.1$

 SO: a human temperature should range between 97.1 and 100.1 to be healthy.

Homework: Pg. 122: # 57-74 all, 81, 82