

Lesson 22 (5.6): Roots (Part 2)

By the end of the lesson, you will be able to:

- ★ Simplify radicals by using distribution and FOIL
- ★ Simplify radicals by rationalizing the denominator
- ★ Finding conjugates to rationalize denominators

LESSON 22 (5.6): Roots (Part 2)

Review: Simplify each

1st: $3\sqrt{5} \cdot 10\sqrt{15}$

$$= 30 \sqrt{5 \cdot 15} = 30 \sqrt{5 \cdot 5 \cdot 3} = \boxed{150\sqrt{3}}$$

2nd: $4\sqrt{10} \cdot 5\sqrt{10} = 20 \sqrt{10 \cdot 10} = \boxed{200}$

LESSON 22 (5.6): Roots (Part 2)

Multiplying using the distributive property

Just like multiplying polynomials, we can distribute and FOIL radical expressions.

EXAMPLES:

$$\text{I. } \sqrt{5}(\sqrt{3} + 2\sqrt{2})$$
$$= \boxed{\sqrt{15} + 2\sqrt{10}}$$

$$\text{2. } 6\sqrt{2}(4 - \sqrt{5})$$
$$= \boxed{24\sqrt{2} - 6\sqrt{10}}$$

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More Examples

a. $(\sqrt{6} + \sqrt{3})(\sqrt{3} + \sqrt{2})$

$$= \sqrt{18} + \sqrt{12} + \sqrt{9} + \sqrt{6}$$

$$\begin{array}{c} 2 \\ \sqrt{2} \\ \diagdown \quad \diagup \\ 9 \\ \diagup \quad \diagdown \\ 3 \quad 3 \end{array}$$

$$= \boxed{3\sqrt{2} + 2\sqrt{3} + 3 + \sqrt{6}}$$

b. $(2\sqrt{3} + 4)(\sqrt{3} + 6\sqrt{5})$

$$= 2\sqrt{9} + 12\sqrt{15} + 4\sqrt{3} + 24\sqrt{5}$$
$$\quad \quad \quad 2 \cdot 3$$

$$= \boxed{6 + 12\sqrt{5} + 4\sqrt{3} + 24\sqrt{5}}$$

LESSON 22 (5.6): Roots (Part 2)

Examples Continued

c. $(4\sqrt{5} + 2\sqrt{7})(4\sqrt{5} - 2\sqrt{7})$

$$= 16\sqrt{25} - \cancel{8\sqrt{35}} + \cancel{8\sqrt{35}} - 4\sqrt{49}$$

$$= 16 \cdot 5 - 4 \cdot 7$$

$$= 80 - 28$$

$$= \boxed{52}$$

d. $(12 + \sqrt{3})(12 - \sqrt{3})$

$$= 144 - 12\sqrt{3} + 12\sqrt{3} - 3$$

$$= 144 - 3$$

$$= \boxed{141}$$

LESSON 22 (5.7): Roots (Part 2)

Dividing radicals by rationalizing the denominator

We can also divide by monomials. However, we don't like square roots (or any roots) in the denominator of a fraction. So we do something called "rationalizing the denominator" to get rid of the root on the bottom.

Lesson 22 (5.6): Roots (Part 2)

Rationalize

We must multiply the numerator and the denominator by the same quantity so that the radicand has an exact root.

(Inside)

(breakout of jail!)

Example 1: What can we multiply by to make the denominator a rational number?

Simplify

$$\frac{\sqrt{b^4}}{\sqrt{a^3}} = \frac{b^2}{a\sqrt{a}} \cdot \frac{\sqrt{a}}{\sqrt{a}} = \frac{b^2\sqrt{a}}{a\sqrt{a}\sqrt{a}} = \boxed{\frac{b^2\sqrt{a}}{a^2}}$$

LESSON 22 (5.6): Roots (Part 2)

Example

What can we multiply by to make the denominator a rational number? (Hint: we are looking for a perfect 5th this time).

$$\sqrt[5]{\frac{3}{4s^2}} = \frac{\sqrt[5]{3}}{\sqrt[5]{4s^2}} = \frac{\sqrt[5]{3}}{\sqrt[5]{2 \cdot 2 \cdot s \cdot s}}$$

$$\frac{\sqrt[5]{2^3 \cdot s^3}}{\sqrt[5]{2 \cdot 2 \cdot s \cdot s \cdot s}} \quad \begin{array}{c} \downarrow \\ 8 \end{array}$$

$$= \frac{\sqrt[5]{3 \cdot 8}}{\sqrt[5]{2^5 s^5}}$$

$$= \boxed{\frac{\sqrt[5]{24s^3}}{2s}}$$

LESSON 22 (5.6): Roots (Part 2)

EXAMPLES

What can we multiply by to make the denominator a rational number?

$$\text{d. } \frac{\sqrt[3]{6}}{2\sqrt{3}} = \frac{3}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{3\sqrt{3}}{\sqrt{9}}$$

$$= \frac{3\sqrt{3}}{3} = \boxed{\sqrt{3}}$$

$$\text{b. } 4\sqrt[4]{\frac{5}{7x}} = \frac{\sqrt[4]{5}}{\sqrt[4]{7x}} \cdot \frac{\sqrt[4]{7^3 x^3}}{\sqrt[4]{7^3 x^3}}$$

$$= \frac{\sqrt[4]{5 \cdot 343 x^3}}{\sqrt[4]{7^4 x^4}}$$

$$= \boxed{\frac{\sqrt[4]{1715 x^3}}{7x}}$$

LESSON 22 (5.6): Roots (Part 2)

Example

What can we multiply by to make the denominator a rational number?

$$\text{C. } \frac{5}{\sqrt[3]{a}} \cdot \frac{\sqrt[3]{a^2}}{\sqrt[3]{a^2}} = \frac{5\sqrt[3]{a^2}}{\sqrt[3]{a^3}} = \boxed{\frac{5\sqrt[3]{a^2}}{a}}$$

LESSON 22 (5.6): Roots (Part 2)

What would happen if we had something like

$\sqrt{6} + \sqrt{3}$ in the denominator? What would we multiply by?

We would need to multiply by the "conjugate" of the binomial.

The CONJUGATE is another binomial that when multiplied by the original binomial, we get a rational number as a result.

The conjugate of $\sqrt{6} + \sqrt{3}$ is $\sqrt{6} - \sqrt{3}$. Test it.

$$(\sqrt{6} + \sqrt{3})(\sqrt{6} - \sqrt{3}) = \sqrt{36} - \sqrt{18} + \sqrt{18} - \sqrt{9}$$
$$= 6 - 3 = \boxed{3}$$

LESSON 22 (5.6): Roots (part 2)

Are these conjugates of each other?

a. $(12 + \sqrt{3})(12 - \sqrt{3})$

yes

b. $(1 - 4\sqrt{5})(1 + 4\sqrt{5})$

$$= 1 + 4\cancel{4\sqrt{5}} - 4\cancel{4\sqrt{5}} - 16\sqrt{25}$$

$$= 1 - 16(5)$$

$$= 1 - 80$$

$$= \boxed{-79}$$

yes

Why is one "+" and the other is "-"?

Cancel middle terms

LESSON 22 (5.6): Roots (Part 2)

Find the conjugate of each.

c. $6 - \sqrt{5}$

$(6 + \sqrt{5})$

d. $\sqrt{3} - 5$

$(\sqrt{3} + 5)$

LESSON 22 (5.6): Roots (Part 2)

Simplify:

What can we multiply by to make the denominator a rational number? (Hint: use conjugates)

$$\begin{aligned} \text{a. } & \frac{(1+2\sqrt{5})(6+\sqrt{5})}{(6-\sqrt{5})(6+\sqrt{5})} \\ &= \frac{6+\sqrt{5}+12\sqrt{5}+2\sqrt{25}}{36+6\sqrt{5}-6\sqrt{5}-\sqrt{25}} \\ &= \frac{6+13\sqrt{5}+10}{36-5} = \boxed{\frac{6+13\sqrt{5}}{31}} \end{aligned}$$

$$\begin{aligned} \text{b. } & \frac{(\sqrt{3}+2)(\sqrt{3}+5)}{(\sqrt{3}-5)(\sqrt{3}+5)} \\ &= \frac{\sqrt{9}+5\sqrt{3}+2\sqrt{3}+10}{\sqrt{9}+5\sqrt{3}-5\sqrt{3}-25} \\ &= \frac{3+10+7\sqrt{3}}{3-25} \\ &= \boxed{\frac{13+7\sqrt{3}}{-22} = \frac{-13-7\sqrt{3}}{22}} \end{aligned}$$

Lesson 22 (5.6): Roots (Part 2)

Simplify:

What can we multiply by to make the denominator a rational number?

$$a. \frac{(5\sqrt{3}+2\sqrt{7})}{4\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}}$$

$$= \frac{5\sqrt{18} + 2\sqrt{42}}{4(6)}$$

$$= \boxed{\frac{15\sqrt{2} + 2\sqrt{42}}{24}}$$

$$b. \frac{5}{2+\sqrt{10}} \cdot \frac{(2-\sqrt{10})}{(2-\sqrt{10})}$$

; ; ;

LESSON 22 (5.6): Roots (Part 2)

Assignment #22:
Due at the beginning of next class