By the end of the lesson, we will be able to:

~ Write Expressions with rational exponents in simplest radical form and vice versa.

~ Evaluate (simplify) expressions in either exponential or radical form.

exponent monster

Lesson #23 (5.7): Rational Exponents

What is a rational number?

* fractions

decimals ~ stop, repeat

~What forms can it have?

$$\frac{a}{b}$$

$\frac{1}{3}, \frac{5}{2}, \ldots$

Rational Exponents

Fraction exponents, called **rational exponents**, are another way to represent roots. For rational exponents, the *numerator* represents the power, and the *denominator* represents the root.

$$\boxed{a^{\frac{1}{m}} = \sqrt[m]{a}}$$

$$5^{\frac{1}{3}} = \sqrt[3]{5}$$

$$\boxed{a^{\frac{n}{m}} = \sqrt[m]{a^n} = \left(\sqrt[m]{a}\right)^n}$$

$$5^{\frac{2}{3}} = \sqrt[3]{5^2} = \left(\sqrt[3]{5}\right)^2$$

Most helpful way!

$$\sqrt[3]{x^{12}} = x^4$$

xxx xxx xxx xxx

Short cut: $\frac{12}{3} = 4$

$$\sqrt[3]{x^{12}} = x^{\frac{12}{3}} = x^4$$

Example 1: * do root FIRST, then power.

a.) $36^{\frac{1}{2}} = \boxed{6}$

$\left(\sqrt[2]{36}\right)^1 = 6^1 = 6$

b.) $64^{\frac{1}{3}} = \boxed{4}$

$\left(\sqrt[3]{64}\right)^1 = 4^1 = 4$

c.) $36^{\frac{3}{2}} = \boxed{216}$

$\left(\sqrt{36}\right)^3 = 6^3 = 216$

d.) $27^{\frac{4}{3}} = \boxed{81}$

$\left(\sqrt[3]{27}\right)^4 = 3^4 = 81$

Example 1:

e.) $(-9)^{\frac{3}{2}} =$ no real root

$\left(\sqrt{-9}\right)^3$

(even index)

f.) $(-27)^{\frac{2}{3}} = \boxed{9}$

$\left(\sqrt[3]{-27}\right)^2 = (-3)^2 = 9$

g.) $49^{-\frac{1}{2}} = \boxed{\frac{1}{7}}$

$= \frac{1}{49^{1/2}} = \frac{1}{\sqrt{49}} = \frac{1}{7}$

h.) $\left(\frac{1}{8}\right)^{-\frac{1}{3}} = \left(\frac{8}{1}\right)^{1/3}$

$= 8^{1/3} = \left(\sqrt[3]{8}\right)^1$

$= \boxed{2}$

Remember?

Rules of Powers

A POWER is an expression in the form of x^n.

Multiplying Powers:

For any real number a and integers m and n,

$$a^m \cdot a^n = a^{m+n}$$

Dividing Powers:

For any real number a, except a=0, and integers m and n,

$$\frac{a^m}{a^n} = a^{m-n}$$

Properties of Powers

Remember?

Suppose m and n are integers and a and b are real numbers. Then the following properties hold.

Power of a Power: $(a^m)^n = a^{mn}$

Power of a Product: $(ab)^m = a^m b^m$

$(a+b)^2 \neq a^2 + b^2$ BAD!

$(a+b)^2 = (a+b)(a+b)$ GOOD!

Properties of Powers

Remember?

Suppose m and n are integers and a and b are real numbers. Then the following properties hold.

Power of a Quotient: $$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Zero Exponents: $$\frac{a^m}{a^m} = a^0 = 1$$

Remember?

Properties of Powers

Negative Exponents: $a^{-n} = \frac{1}{a^n}$ or $\frac{1}{a^{-n}} = a^n$

Power of a Quotient: $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ or $\frac{b^n}{a^n}, a \neq 0, b \neq 0$

All the properties we have used for integer exponents apply to rational exponents.

Add powers

fraction

Example 2: Express using rational exponents.

$$x^{\frac{2}{5}} \cdot x^{\frac{1}{5}} = X^{\frac{2}{5}+\frac{1}{5}} = \boxed{X^{\frac{3}{5}}}$$

All the properties we have used for integer exponents apply to rational exponents.

Example 3: Express using rational exponents.

$$x^{\frac{5}{4}} \cdot x^{\frac{1}{1}} = x^{\frac{5}{4}+\frac{1}{1}\cdot\frac{4}{4}} = x^{\frac{5}{4}+\frac{4}{4}} = \boxed{x^{\frac{9}{4}}}$$

Add exponents

Radical form:

$$\left(\sqrt[4]{x}\right)^{9} = \sqrt[4]{x^{9}}$$

All the properties we have used for integer exponents apply to rational exponents.

Example 4: Express using rational exponents.

$$x^{\frac{2}{3}} \cdot x^{\frac{3}{5}} = x^{\frac{10}{15} + \frac{9}{15}} = \boxed{x^{\frac{19}{15}}}$$

All the properties we have used for integer exponents apply to rational exponents.

Example 5: Express using rational exponents.

$$\left(x^{\frac{2}{5}}\right)^{\frac{3}{2}} = x^{\frac{{}^{1}\cancel{2}}{5}\cdot\frac{3}{\cancel{2}_{1}}} = \boxed{x^{\frac{3}{5}}}$$

mult.
exponents

Example 6: Express using rational exponents.

$$\left(x^{\frac{2}{3}}y^{\frac{7}{2}}\right)^{\frac{6}{1}} = x^{\frac{2}{\cancel{3}_1}\cdot\frac{\cancel{6}^2}{1}} \cdot y^{\frac{7}{\cancel{2}_1}\cdot\frac{\cancel{6}^3}{1}}$$

$$= x^{2\cdot 2}\, y^{7\cdot 3}$$

$$= \boxed{x^4 y^{21}}$$

Example 7: Express using rational exponents.

$$\left(x^{-\frac{1}{4}}\right)^{\frac{-6}{1}} = x^{-\frac{1}{\cancel{4}_2}\cdot\frac{-\cancel{6}^3}{1}} = \boxed{x^{\frac{3}{2}}}$$

Other way

$$\left(\frac{1}{x^{1/4}}\right)^{-6} = \left(\frac{x^{\frac{1}{4}}}{1}\right)^{6} = x^{\frac{1}{4}\cdot\frac{6}{1}} = \boxed{x^{\frac{3}{2}}}$$

Example 8: Express using rational exponents.

$$\frac{x^{\frac{5}{3}}}{x^{\frac{1}{3}}} = x^{\frac{5}{3}-\frac{1}{3}} = \boxed{x^{\frac{4}{3}}}$$

Example 9: Express using rational exponents.

$$\frac{x^{1}}{x^{\frac{3}{4}}} = \quad x^{\frac{1}{1}-\frac{3}{4}} = x^{\frac{4}{4}-\frac{3}{4}} = \boxed{x^{\frac{1}{4}}}$$

Example 10: Express using rational exponents.

$$\left(\frac{16x^3}{y^4}\right)^{\frac{1}{2}} = \frac{16^{\frac{1}{2}}\left(x^{\frac{3}{1}}\right)^{\frac{1}{2}}}{\left(y^{\frac{4}{1}}\right)^{1/2}} = \frac{4x^{\frac{3}{2}}}{y^{\frac{4}{2}}}$$

$$= \boxed{\frac{4x^{\frac{3}{2}}}{y^{2}}}$$

Example 11: Express using rational exponents.

$$\left(\frac{3x^{-\frac{3}{2}}}{y^{-\frac{3}{2}}}\right)^{-2} = \left(\frac{y^{-\frac{3}{2}}}{3x^{-\frac{3}{2}}}\right)^{\frac{2}{1}} = \frac{y^{-\frac{3}{\cancel{2}_1}\cdot\frac{\cancel{2}^1}{1}}}{3^2\, x^{-\frac{3}{\cancel{2}_1}\cdot\frac{\cancel{2}^1}{1}}}$$

$$= \frac{y^{-3}}{9x^{-3}}$$

$$= \boxed{\frac{x^3}{9y^3}}$$

With rational exponents, we are able to simplify radical expressions with different types of roots.

Example 12: Express in simplest radical form

a.) $\sqrt[3]{a} \cdot \sqrt[4]{a} =$

$= a^{\frac{1}{3}} \cdot a^{\frac{1}{4}}$

$= a^{\frac{1}{3}+\frac{1}{4}} = a^{\frac{4}{12}+\frac{3}{12}}$

$= a^{\frac{7}{12}} = \boxed{\sqrt[12]{a^7}}$

b.) $\sqrt{x} \cdot \sqrt[3]{x^4} =$

$x^{\frac{1}{2}} \cdot x^{\frac{4}{3}} = x^{\frac{1\cdot 3}{2\cdot 3}+\frac{4\cdot 2}{3\cdot 2}}$

$= x^{\frac{3}{6}+\frac{8}{6}} = x^{\frac{11}{6}}$

$= \sqrt[6]{x^{11}} = \boxed{x\sqrt[6]{x^5}}$

$x^6 \cdot x^5$

Example 12: Express in simplest radical form

$8^3 = 8 \cdot 8 \cdot 8$

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

c.) $m^{\frac{1}{3}}n^{\frac{3}{4}}p^{\frac{5}{6}} =$

* need common root/denominator

$= m^{\frac{4}{12}} n^{\frac{9}{12}} p^{\frac{10}{12}}$

$= \boxed{\sqrt[12]{m^4 n^9 p^{10}}}$

d.) $8^{\frac{1}{2}}b^{\frac{2}{3}}c^{\frac{7}{6}} =$

$= 8^{\frac{3}{6}} b^{\frac{4}{6}} c^{\frac{7}{6}}$

$= \sqrt[6]{8^3 b^4 c^7}$

$= c\sqrt[6]{2^9 b^4 c}$

$2^6 \cdot 2^3$

$= \boxed{2c\sqrt[6]{8b^4c}}$

Example 12: Express in simplest radical form

e.) $\sqrt[3]{\sqrt{y^3}} =$

$= \left(\sqrt{y^3}\right)^{\frac{1}{3}}$

$= \left(y^{\frac{3}{2}}\right)^{\frac{1}{3}}$

$= y^{\frac{3}{2}\cdot\frac{1}{3}} = y^{\frac{1}{2}}$

$= \boxed{\sqrt{y}}$

f.) $\sqrt[4]{49} = \sqrt[4]{7^2}$

$= 7^{\frac{2}{4}}$

$= 7^{\frac{1}{2}}$

$= \boxed{\sqrt{7}}$

By the end of the lesson, we will be able to:

~ Write Expressions with rational exponents in simplest radical form and vice versa.

~ Evaluate (simplify) expressions in either exponential or radical form.

Can you?

Homework:

Exponent monster worksheet

&

Assignment #23

Quiz: 15 min.

30 problems from

4 problems from

Due at the beginning of next class