Lesson # 24: Complex Numbers (5.9)

By the end of the lesson, we will be able to:

~ Simplify Imaginary and Complex Numbers.

Lesson # 24: Complex Numbers (5.9)

We already know that the square root of a negative number is not a real number. This is because there is no real number that satisfies the equation $x^2 = -1$.

However, about 400 years ago, a French mathematician named René Descartes suggested that a new number be invented, an imaginary number, represented by the letter i, which is the solution to that equation.

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Definition of i: $\sqrt{-1} = i$ and $i^2 = -1$

Expanding the set of numbers to include *imaginary numbers* allows us to simplify expressions and solve equations that were not possible using only real numbers.

Whenever you see a negative number under a radical, take it out of the radical and write it as an i on the outside.

even root (index)

$$\sqrt{-9} = \sqrt{-1 \cdot 9} = \sqrt{-1} \cdot \sqrt{9} = i(3)$$

$$= 3i$$

Examples: Simplify each expression.

Remember to remove the i's first.

a.) $\sqrt{-81}$ $= i\sqrt{81}$

$= \boxed{9i}$

b.) $\sqrt{-121x^5}$

$= i\sqrt{121x^5}$

$= 11i\sqrt{x^5}$

$= \boxed{11x^2 i\sqrt{x}}$

$= 11x^2\sqrt{x}\, i$

Examples: Simplify each expression.

Remember to remove the i's first.

c.) $\sqrt{-8n^2} = i\sqrt{8n^2}$

2 4

2 2

$= \boxed{2ni\sqrt{2}}$

odd !

d.) $\sqrt[3]{-64x^4}$

$= \boxed{-4x\sqrt[3]{x}}$

Examples: Simplify each expression.

Remember to remove the i's first.

e.) $\sqrt{-\frac{25}{121}} = i\sqrt{\frac{25}{121}}$

$= i\frac{\sqrt{25}}{\sqrt{121}}$

$= \boxed{\frac{5}{11}i}$

f.) $\sqrt{-\frac{252}{4}} = i\sqrt{\frac{252}{4}}$

$= i\frac{\sqrt{252}}{\sqrt{4}} = i\frac{\sqrt{252}}{2}$

$= i\frac{\sqrt{2\cdot 2\cdot 3\cdot 3\cdot 7}}{2} = \frac{\overset{3}{\cancel{6}}i\sqrt{7}}{\underset{1}{\cancel{2}}}$

$= \boxed{3i\sqrt{7}}$

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Examples: Simplify each expression.

Remember to remove the i's first. $i^2 = -1$

g.) $\sqrt{-5} \cdot \sqrt{-20}$

$= i\sqrt{5} \cdot i\sqrt{20}$

$= i^2\sqrt{5 \cdot 20} = i^2\sqrt{100}$

$= 10i^2 = 10(-1)$

$= \boxed{-10}$

h.) $\left(-3\sqrt{-5}\right)^2$

$= \left(-3i\sqrt{5}\right)^2$

$= (-3)^2 i^2 (\sqrt{5})^2$

$= 9(-1)(5)$

$= \boxed{-45}$

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What is the pattern in the powers of i?

$i^0 = 1$	$i^9 = i$
$i^1 = i$	$i^{10} = -$
$i^2 = -1$	$i^{11} = -i$
$i^3 = i^2 \cdot i^1 = -1\,i = \boxed{-i}$	$i^{12} = 1$
$i^4 = i^2 \cdot i^2 = -1 \cdot -1 = \boxed{1}$	$i^{20} = 1$
$i^5 = i^4 \cdot i = 1 \cdot i = \boxed{i}$	$i^{25} = i^1 = \boxed{i}$ $\quad \begin{array}{r} 6 \\ 4\,\overline{)25} \\ -24 \\ \hline \boxed{1} \end{array}$
$i^6 = i^5 \cdot i = i \cdot i = i^2 = \boxed{-1}$	$i^{82} = i^2 = \boxed{-1}$
$i^7 = i^6 \cdot i = -1 \cdot i = \boxed{-i}$	$i^{103} =$
$i^8 = i^4 \cdot i^4 = 1 \cdot 1 = \boxed{1}$	$i^{400} =$

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Since the pattern repeates every 4, we should

- Divide the power on the "i" by 4.
- The remainder will tell us which of the 1st 4 it is like.

Remainder of 0 → like i^0

Remainder of 1 → like i^1

" " 2 → like i^2

" " 3 → like i^3

$$\boxed{\begin{aligned} i^0 &= \\ i^1 &= i \\ i^2 &= -1 \\ i^3 &= \end{aligned}}$$

Memorize!

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A ***complex number*** is an expression of the form $a + bi$, which means it has a real part (a) and an imaginary part (bi). When simplifying expressions with complex numbers is similar to simplifying polynomials, but wherever there is an i^2, change it to -1.

Examples: Simplify each expression.

Remember to remove the i's and $i^2 = -1$.

a.) $3i \cdot 6i^2$

$= 3i \cdot 6(-1)$

$= 3i \cdot (-6)$

$= \boxed{-18i}$

b.) $(2i \cdot 3i)^2$

$= (6i^2)^2$

$= (6(-1))^2 = (-6)^2$

$= \boxed{36}$

Examples: Simplify each expression.

Remember to remove the i's and $i^2 = -1$.

c.) $3(-5-2i)+2(-3+2i)$

$= -15 - 6i - 6 + 4i$

$= \boxed{-21 - 2i}$

Examples: Simplify each expression.

Remember to remove the i's and $i^2 = -1$.

d.) $(7 + 5i) + (-4 - 6i)$

$= 7 + 5i - 4 - 6i$

$= \boxed{3 - i}$

Examples: Simplify each expression.

Remember to remove the i's and $i^2 = -1$.

e.) $i(5i) + i(7 - i)$

$= 5i^2 + 7i - i^2$

$= 5(-1) + 7i - (-1)$

$= -5 + 7i + 1$

$= \boxed{-4 + 7i}$

Examples: Simplify each expression.

Remember to remove the i's and $i^2 = -1$.

FOIL

f.) $(2+3i)(4-5i)$

$= 8 - 10i + 12i - 15\underset{(-1)}{i^2}$

$= 8 + 2i + 15$

$= \boxed{23 + 2i}$

Examples: Simplify each expression.

Remember to remove the i's and $i^2 = -1$.

g.) $(7 - i)(4 + 2i)(5 + 2i)$

$= (28 + 14i - 4i - 2\underset{(-1)}{i^2})(5 + 2i)$

$= (28 + 10i + 2)(5 + 2i)$

$= (30 + 10i)(5 + 2i)$

$= 150 + 60i + 50i + 20\underset{(-1)}{i^2}$

$= \boxed{130 + 110i}$

Examples: Simplify each expression.

Remember to remove the i's and $i^2 = -1$.

h.) $(\sqrt{5} + 2i)^2 = (\sqrt{5} + 2i)(\sqrt{5} + 2i)$

$= 5 + 2i\sqrt{5} + 2i\sqrt{5} + 4i^2$

(-1)

$= \boxed{1 + 4i\sqrt{5}}$

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By the end of the lesson, we will be able to:

~ Simplify Imaginary and Complex Numbers.

Can you?

Homework:

Assignment #24