

By the end of the lesson, we will be able to:

$x^2 = 0$
~ Solve quadratic equations by completing the square.

Look for a pattern...

Multiply:

$$\begin{aligned} \text{a.) } (x + 9)^2 \\ &= (x + 9)(x + 9) \\ &= x^2 + 9x + 9x + 81 \\ &= x^2 + 18x + 81 \end{aligned}$$

$$\begin{aligned} \text{b.) } (x + 12)^2 \\ &= (x + 12)(x + 12) \\ &= x^2 + 24x + 144 \end{aligned}$$

Look for a pattern...

Multiply:

$$\text{c.) } (x + 2)^2$$

$$= (x + 2)(x + 2)$$

$$= x^2 + 4x + 4$$

$$\text{d.) } (x + 5)^2$$

$$= x^2 + 10x + 25$$

Have you noticed a pattern between the middle term and the last term?

$$C = \left(\frac{b}{2}\right)^2$$

Have you noticed a pattern between the middle term and the last term?

$$ax^2 + bx + c, \text{ where } c = \left(\frac{b}{2}\right)^2$$

Lesson 30 (6.3): Completing the Square

Find the missing part (k).

Then put in $(x + \frac{b}{a})^2$ form.

$$b = -6$$

a.) $x^2 + 4x + k$

$$k = \left(\frac{b}{a}\right)^2$$

$$k = \left(\frac{4}{2}\right)^2$$

$$= (2)^2$$

$$k = 4$$

$$(x + 2)^2$$

b.) $x^2 - 6x + k$

$$k = \left(\frac{-6}{2}\right)^2$$

$$= (-3)^2$$


$$k = 9$$

$$(x - 3)^2$$

Lesson 30 (6.3): Completing the Square

Steps for Solving Quadratics by Completing the Square:

$$ax^2 + bx + c = 0$$

Step 0:	Divide everything by " <u>a</u> " if " <u>a</u> " is something other than 1.
Step 1:	Move the constant to the right side of the equation.  <u>no x's</u>
Step 2:	Identify "b". Divide "b" by 2. $\left(\frac{b}{2}\right)$
Step 3:	Square $\left(\frac{b}{2}\right)$. Add $\left(\frac{b}{2}\right)^2$ to <u>both</u> sides of the equation.

Lesson 30 (6.3): Completing the Square

Steps for Solving Quadratics by Completing the Square:

Step 4:	Factor the left side. Hint: It will look like $(x + \frac{b}{2})^2 = \underline{\hspace{2cm}}$.
Step 5:	Combine terms on right side. (You are adding the numbers together).
Step 6:	Solve for x. Hint: Start by taking the square root of both sides. Remember to put \pm with the square root.

Lesson 30 (6.3): Completing the Square

Solve the equation by completing the square:

Example 1: $x^2 - 6x = 40$

$$x^2 - 6x + 9 = 40 + 9$$

$$(x - 3)^2 = 49$$

$$\sqrt{(x - 3)^2} = \pm \sqrt{49}$$

$$x - 3 = \pm 7$$

$$\begin{array}{r} +3 \quad +3 \\ \hline \end{array}$$

$$x = 3 \pm 7$$

\rightarrow

$$x = 3 + 7$$

$$\boxed{x = 10}$$

$$x = 3 - 7$$

$$\boxed{x = -4}$$

$$b = -6$$

$$\frac{b}{2} = -3$$

$$\left(\frac{b}{2}\right)^2 = 9$$

Lesson 30 (6.3): Completing the Square

Solve the equation by completing the square:

Example 2: $x^2 + 7x - 17 = 0$

$$\begin{array}{r} 117 \\ 3 \overline{) 39} \\ 3 \overline{) 13} \end{array}$$

$$\begin{aligned} b &= 7 \\ \frac{b}{2} &= \frac{7}{2} \\ \left(\frac{b}{2}\right)^2 &= \frac{49}{4} \end{aligned}$$

$$x^2 + 7x + \frac{49}{4} = 17 + \frac{49}{4}$$

$$\left(x + \frac{7}{2}\right)^2 = \frac{68}{4} + \frac{49}{4}$$

$$\sqrt{\left(x + \frac{7}{2}\right)^2} = \pm \sqrt{\frac{117}{4}}$$

$$x + \frac{7}{2} = \pm \frac{\sqrt{117}}{2}$$

$$x + \frac{7}{2} = \pm \frac{3\sqrt{13}}{2}$$

$$X + \frac{7}{2} = \frac{\pm 3\sqrt{13}}{2}$$
$$-\frac{7}{2} \quad -\frac{7}{2}$$

$$X = \frac{-7 \pm 3\sqrt{13}}{2}$$

or

$$X = \frac{-7 \pm 3\sqrt{13}}{2}$$

Lesson 30 (6.3): Completing the Square

Solve the equation by completing the square:

Example 3: $x^2 + 8x + 20 = 0$

$\quad\quad\quad -20 \quad -20$

$$x^2 + 8x + \underline{16} = -20 + 16$$

$$\sqrt{(x+4)^2} = \pm \sqrt{-4}$$

$$x+4 = \pm 2i$$

$\quad -4 \quad -4$

$$x = -4 \pm 2i$$

$$b = 8$$

$$\frac{b}{2} = 4$$

$$\left(\frac{b}{2}\right)^2 = 16$$

Lesson 30 (6.3): Completing the Square

Solve the equation by completing the square:

Example 4: $\frac{2x^2}{2} + \frac{8x}{2} + \frac{22}{2} = \frac{0}{2}$

$$\frac{x^2 + 4x + 11}{-11} = 0$$

$$x^2 + 4x + 4 = -11 + 4$$

$$\sqrt{(x+2)^2} = \pm \sqrt{-7}$$

$$\frac{x+2}{-2} = \pm i\sqrt{7}$$

$$x = -2 \pm i\sqrt{7}$$

$$\begin{aligned} b &= 4 \\ \frac{b}{2} &= 2 \\ \left(\frac{b}{2}\right)^2 &= 4 \end{aligned}$$

Lesson 30 (6.3): Completing the Square

Solve the equation by completing the square:

Example 5: $x^2 + 6x + 9 = 0$

$$\begin{aligned} b &= 6 \\ \frac{b}{2} &= 3 \\ \left(\frac{b}{2}\right)^2 &= 9 \end{aligned}$$

$$x^2 + 6x \underline{+9} = -9 + 9$$

$$\sqrt{(x+3)^2} = \pm\sqrt{0}$$

$$x+3 = \pm 0$$

$$\begin{array}{cc} -3 & -3 \end{array}$$

$$x = -3$$

$$x = 3$$

Lesson 30 (6.3): Completing the Square

Solve the equation by completing the square:

Example 6: $\frac{ax^2}{a} + \frac{bx}{a} + \frac{c}{a} = \frac{0}{a}$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$b = \frac{b}{a}$$

$$\frac{b}{2} = \frac{b}{2a}$$

$$\left(\frac{b}{2}\right)^2 = \frac{b^2}{4a^2}$$

$$x^2 + \frac{b}{a}x + \frac{\frac{b^2}{4a^2}}{\frac{4a^2}{4a^2}} = \frac{-\frac{c \cdot 4a}{4a^2} + \frac{b^2}{4a^2}}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac}{4a^2} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$\sqrt{\left(X + \frac{b}{2a}\right)^2} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$X + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$-\frac{b}{2a} = \frac{b}{2a}$$

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Lesson 30 (6.3): Completing the Square

By the end of the lesson, we will be able to:

~ Solve quadratic equations by completing the square.

Can you?



Lesson 30 (6.3): Completing the Square

Homework:

Assignment 30



Additional examples: (with
A=something other than 1.

$$4x^2 - 5x - 21 = 0$$

$$2x^2 - 7x + 12 = 0$$