Lesson 31 (6.4): Solve by Quadratic Formula

By the end of the lesson, you will be able to:

~ Solve by using the Quadratic Formula
~ Find the Discriminant
~ Identify type of roots

Lesson 31 (6.4): Solve by Quadratic Formula

Let's complete the square:

1. $ax^2 + bx + c = 0$

2. $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

   Divide by "a".

3. $x^2 + \frac{b}{a}x = -\frac{c}{a}$

   Subtract $\frac{c}{a}$.

4. $x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$

   Add $\left(\frac{b}{2a}\right)^2$ *to both sides to complete the square.*

Lesson 31 (6.4): Solve by Quadratic Formula

5. $x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$

Factor the left side. Also get a common denominator on the right side.

$$(x + \frac{b}{2a})^2 = -\frac{4ac}{4a^2} + \frac{b^2}{4a^2}$$

$$(x + \frac{b}{2a})^2 = \frac{b^2 - 4ac}{4a^2}$$

6. $\sqrt{(x + \frac{b}{2a})^2} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$

Square root both sides.

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

7. $x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

Subract $\frac{b}{2a}$ *from both sides.*

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Lesson 31 (6.4): Solve by Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The QUADRATIC FORMULA.

This is a formula that allows you to solve <u>any</u> quadratic equation:

If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Memorize this!!!

Lesson 31 (6.4): Solve by Quadratic Formula

Steps to solve using the Quadratic Formula:

1. Set the equation equal to zero. (*MUST* be set to zero, not another number.)
2. Identify the values of ***a***, ***b***, and ***c*** from the equation.
3. Substitute ***a***, ***b***, and ***c*** into the quadratic formula.
4. Simplify the expression using the order of operations and rules for simplifying radicals.
5. If the simplified expression has a radical or i, then write it as *one* expression with $\pm$.
   If there is no radical or i, then split into *two* expressions (+ and −) and evaluate each.

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Let's do an example:

$a = 1$

$b = -7$

$c = -18$

$$x^2 - 7x = 18$$

$$-18 \quad -18$$

$$x^2 - 7x - 18 = 0$$

$$x = \frac{+7 \pm \sqrt{(-7)^2 - 4(1)(-18)}}{2(1)}$$

$$x = \frac{7 \pm \sqrt{49 + 72}}{2}$$

$$x = \frac{7 \pm \sqrt{121}}{2}$$

$$x = \frac{7 \pm 11}{2}$$

$$x = \frac{7+11}{2} \qquad x = \frac{7-11}{2}$$

$$x = 9, \quad x = -2$$

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The **Discriminant:**

The Discriminant is the part under the radical: $b^2 - 4ac$

If the Discriminant is:

1. **Zero (0), then there is one real, rational root.**
2. **Perfect Square (pos), then there are two real, rational roots.**
3. **Positive, then there are two real, irrational roots.**
4. **Negative, then there are two complex roots. (There will be i's in the roots.)**

Lesson 31 (6.4): Solve by Quadratic Formula (D)

Examples: a.) Find the Discriminant.
b.) What kind of root(s)?
c.) Solve by the Quad. Formula.

$a=1$
$b=-8$
$c=16$

1. $x^2 - 8x + 16 = 0$

D: $(-8)^2 - 4(1)(16)$

$= 64 - 64$

D = 0 ; one rational root

$$x = \frac{8 \pm \sqrt{0}}{2(1)}$$

$$x = \frac{8}{2} \rightarrow \boxed{x=4}$$

Lesson 31 (6.4): Solve by Quadratic Formula

Examples: a.) Find the Discriminant.
b.) What kind of root(s)?
c.) Solve by the Quad. Formula.

$a = -3$
$b = 4$
$c = -4$

2. $-3x^2 + 4x - 4 = 0$

$D = (4)^2 - 4(-3)(-4)$

$= 16 - 48$

$D = -32$; two Complex roots

$x = \dfrac{-4 \pm \sqrt{-32}}{2(-3)}$

$x = \dfrac{-4 \pm 4i\sqrt{2}}{-6}$

$x = \dfrac{-2 \pm 2i\sqrt{2}}{-3}$

$x = \dfrac{2 \pm 2i\sqrt{2}}{3}$

Lesson 31 (6.4): Solve by Quadratic Formula

Examples: a.) Find the Discriminant.
b.) What kind of root(s)?
c.) Solve by the Quad. Formula.

$a = 1$
$b = -5$
$c = -50$

3. $x^2 - 5x - 50 = 0$

$D = (-5)^2 - 4(1)(-50)$

$= 25 + 200$

$D = 225$; two rational roots

$x = \frac{5 \pm \sqrt{225}}{2(1)}$

$x = \frac{5 \pm 15}{2}$

$x = \frac{5+15}{2}$

$x = \frac{5-15}{2}$

$x = 10, x = -5$

Lesson 31 (6.4): Solve by Quadratic Formula

Examples: a.) Find the Discriminant.
b.) What kind of root(s)?
c.) Solve by the Quad. Formula.

$a = 2$
$b = -9$
$c = 8$

4. $2x^2 - 9x + 8 = 0$

$D = (-9)^2 - 4(2)(8)$

$= 81 - 64$

$D = 17$; two irrational roots

$x = \frac{9 \pm \sqrt{17}}{2(2)}$

$x = \frac{9 \pm \sqrt{17}}{4}$

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Can you?

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