

Lesson 32 (6.7): Solving Quadratic Inequalities

By the end of the lesson, we will be able to:

- ~ Solve quadratic inequalities. $\frac{a}{b} < \frac{c}{d}$ $\frac{a}{b} > \frac{c}{d}$
- ~ Show the solution of a quadratic inequality on a line graph.
- ~ Write the solution of a quadratic inequality.

Lesson 32 (6.7): Solving Quadratic Inequalities

Let's start by reviewing how to solve a quadratic equation.

1. We can solve by factoring.
2. We can solve by completing the square.
3. We can solve by using the quadratic formula.

Review - List the steps to solve quadratic equations by factoring.

Steps for solving quadratic equations by factoring:

1. Move all terms to one side of the equation so the equation is set to zero.
2. Factor the polynomial.
3. Set each factor equal to zero.
4. Solve each new equation.

Examples:

a.) $x^2 - 3x - 28 = 0$ b.) $x^2 + 4x = 12$

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$$\begin{array}{r} \underline{6} \cdot \underline{-2} = 12 \\ \underline{6} + \underline{-2} = 4 \end{array}$$

Examples:

a.) $x^2 - 3x - 28 = 0$

b.) $x^2 + 4x = 12$

$$\underline{\quad -12 \quad -12}$$

$$x^2 + 4x - 12 = 0$$

$$\underbrace{x^2 + 6x} \quad \underbrace{-2x - 12} = 0$$

$$x(x+6) - 2(x+6) = 0$$

$$(x+6)(x-2) = 0$$

$$x+6=0$$

$$x-2=0$$

$$x = -6$$

$$x = 2$$

Review - List the steps to solve quadratic equations by using the quadratic formula.

Steps to solve using the Quadratic Formula:

1. Set the equation equal to zero. (*MUST* be set to zero, not another number.)
2. Identify the values of ***a***, ***b***, and ***c*** from the equation.
3. Substitute ***a***, ***b***, and ***c*** into the quadratic formula.
4. Simplify the expression using the order of operations and rules for simplifying radicals.
5. If the simplified expression has a radical or *i*, then write it as *one* expression with \pm .
If there is no radical or *i*, then split into *two* expressions (+ and -) and evaluate each.

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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The QUADRATIC FORMULA.

This is a formula that allows you to solve any quadratic equation:

If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Memorize this!!!

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Let's do an example.

Solve the following quadratic equation by using the quadratic formula.

$$\begin{array}{r} x^2 - 7x = 18 \\ \underline{-18 \quad -18} \\ x^2 - 7x - 18 = 0 \end{array}$$

$$a = 1$$

$$b = -7$$

$$c = -18$$

$$X = \frac{7 \pm \sqrt{(-7)^2 - 4(1)(-18)}}{2(1)}$$

$$X = \frac{7 \pm \sqrt{49 + 72}}{2}$$

$$X = \frac{7 \pm \sqrt{121}}{2}$$

$$X = \frac{7 \pm 11}{2}$$

$$\rightarrow \begin{array}{l} X = 9 \\ X = -2 \end{array}$$

Now we can start on solving quadratic inequalities.

Steps to solve quadratic inequalities:

1. Set the inequality to zero, if necessary.
2. Solve the related equation (Factor) to find the critical points.
3. Graph the critical points on a number line.
4. Test a value from each region in the inequality.
(Plug in, see if it's true or false.)
5. Use the graph to write the solution set for the inequality (compound inequality).

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Example 1:

Solve the inequality $x^2 - 3x - 10 > 0$

$$x^2 - 3x - 10 = 0 \quad \begin{array}{l} \underline{-5} \cdot \underline{2} = -10 \\ \underline{-5} + \underline{2} = -3 \end{array}$$

$$(x-5)(x+2) = 0$$

$$x-5 = 0$$

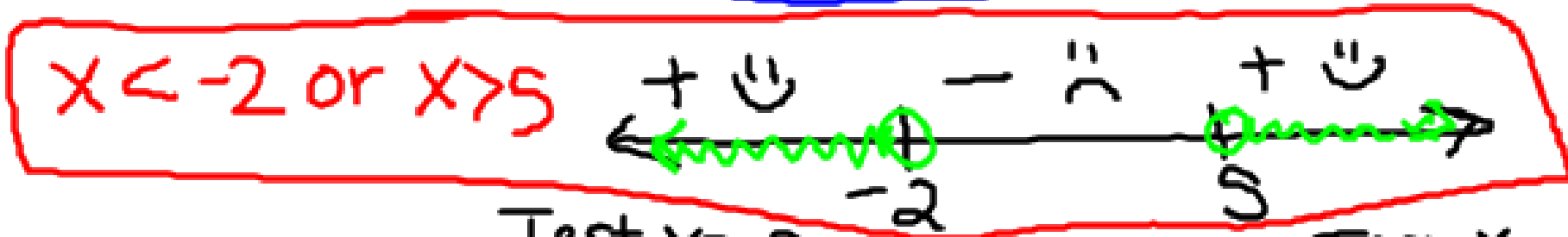
$$x+2 = 0$$

$$x = 5$$

$$x = -2$$

Critical Values

POS. #



Test $x = -5$

$$\begin{array}{l} (-5-5)(-5+2) > 0 \\ (-)(-) > 0 \\ + > 0 \quad \text{☺} \end{array}$$

Test $x = 0$

$$\begin{array}{l} (-)(+) > 0 \\ - > 0 \\ x \quad \text{☹} \end{array}$$

Test $x = 6$

$$\begin{array}{l} (+)(+) > 0 \\ + > 0 \\ \text{☺} \end{array}$$

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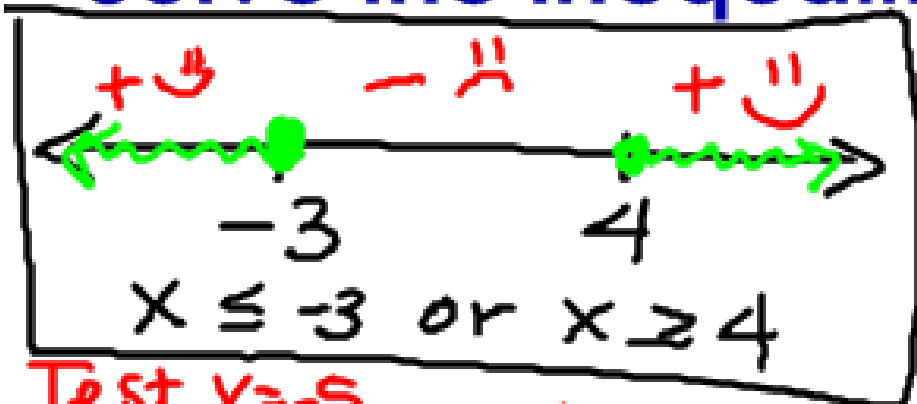
$$\begin{aligned} \underline{-4} \cdot \underline{3} &= -12 \\ \underline{-4} + \underline{3} &= -1 \end{aligned}$$

Example 2:

Solve the inequality

$$x^2 - x - 12 \geq 0$$

POS
#



$$(x-4)(x+3) = 0$$

$$x-4=0 \quad x+3=0$$

$$x=4 \quad x=-3$$

Test $x=-5$

$$(-)(-) \geq 0$$

$$+ \geq 0$$

✓ ☺

Test $x=0$

$$(-)(+) \geq 0$$

$$- \geq 0$$

$$x \neq$$

Test $x=5$

$$(+)(+) \geq 0$$

$$+ \geq 0$$

✓ ☺

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$$\begin{aligned} \underline{7} \cdot \underline{2} &= 14 \\ \underline{7} + \underline{2} &= 9 \end{aligned}$$

Example 3:

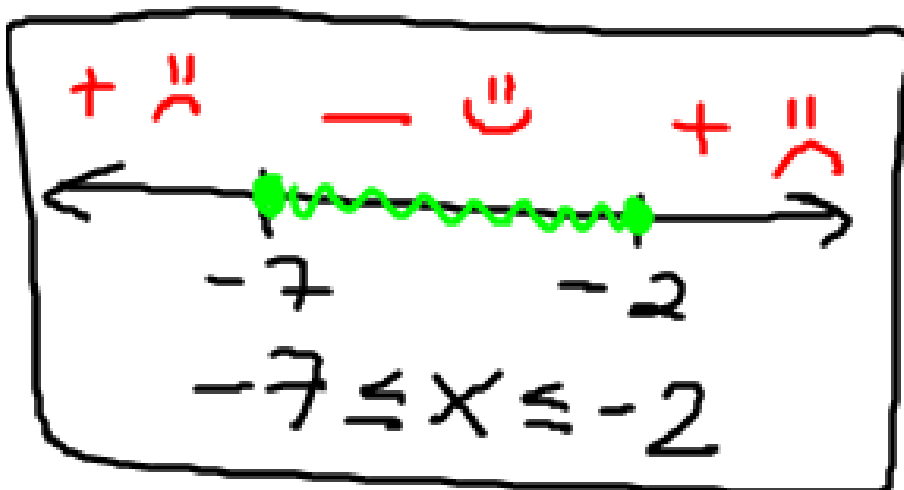
Solve the inequality

$$x^2 + 9x + 14 \leq 0$$

neg

$$(x+7)(x+2) = 0$$

$$\begin{aligned} x+7 &= 0 & x+2 &= 0 \\ x &= -7 & x &= -2 \end{aligned}$$



$$\begin{aligned} \underline{\text{Test } x = -8} \\ (-)(-) \leq 0 \\ + \leq 0 \\ x \text{ smiley} \end{aligned}$$

$$\begin{aligned} \underline{\text{Test } x = -4} \\ (+)(-) \leq 0 \\ - \leq 0 \\ x \text{ frowny} \end{aligned}$$

$$\begin{aligned} \underline{\text{Test } x = 0} \\ (+)(+) \leq 0 \\ x \text{ smiley} \end{aligned}$$

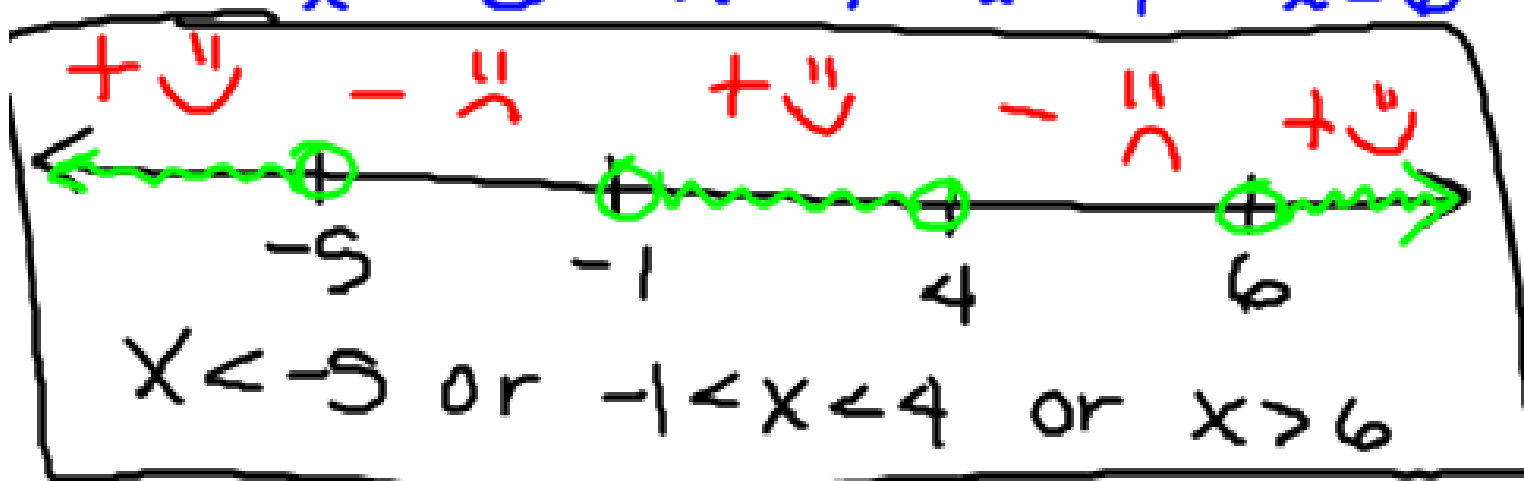
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Example 4:

Solve the inequality

$$(x + 5)(x + 1)(x - 4)(x - 6) > 0$$

$$x = -5 \quad x = -1 \quad x = 4 \quad x = 6$$



POS

Test $x = 7$
 $(+)(+)(+)(+) > 0$
 $+ > 0$
☺

Test $x = -10$
 $(-)(-)(-)(-) > 0$
 $+ > 0$
☺

Test $x = -2$
 $(+)(-)(-)(-) > 0$
 $- > 0$
☹

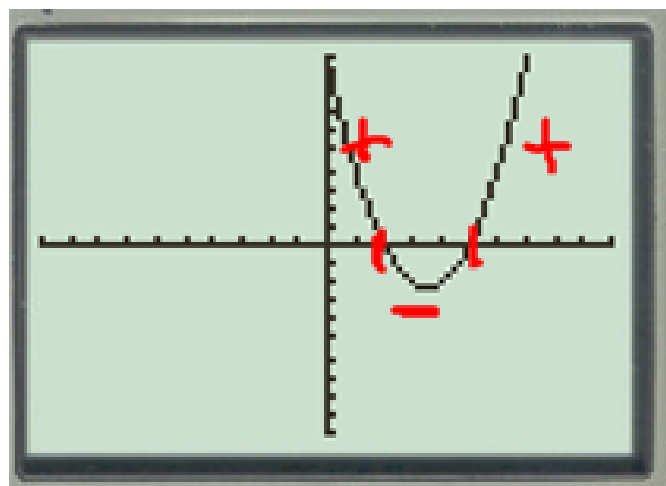
Test $x = 0$
 $(+)(+)(-)(-) > 0$
 $+ > 0$
☺

Test $x = 5$
 $(+)(+)(+)(-) > 0$
 $- > 0$
☹

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How to use your calculator to solve:

- Step 1: Enter equation into "y=". (Must be set to 0.)
- Step 2: Graph.
- Step 3: Find the "zeros".
- Step 4: Put these points on your number line as your critical values.
- Step 5: Test your critical points. (Where is it positive? Where is it negative?)
- Step 6: Write your answers as an inequality.



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Example 5: Use your calculator

Solve the inequality

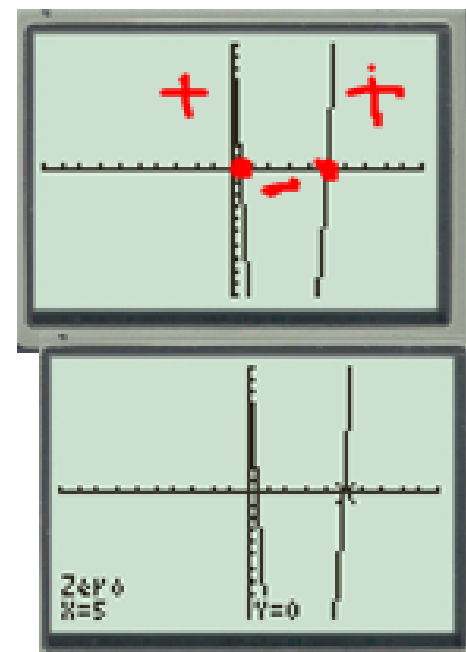
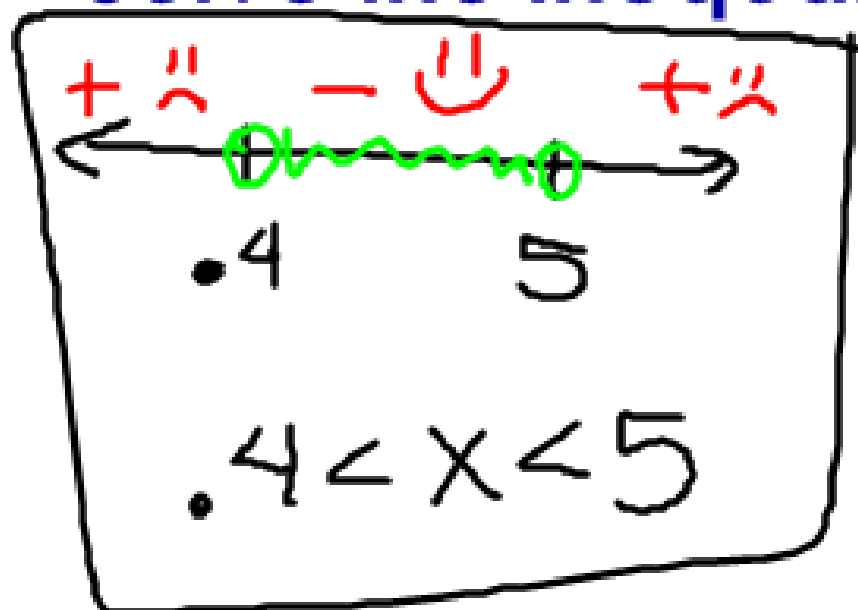
$$5x^2 + 10 < 27x$$

$$-27x \quad -27x$$

$$5x^2 - 27x + 10 < 0 \text{ neg}$$

$$x = .4$$

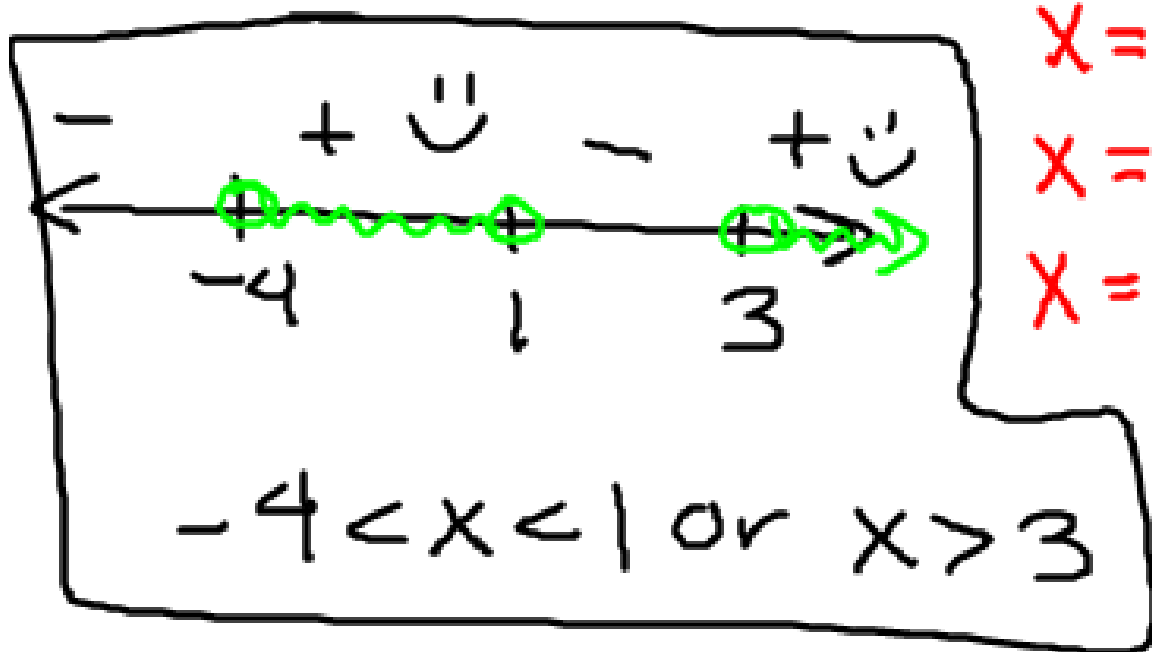
$$x = 5$$



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Example 6: Use your calculator

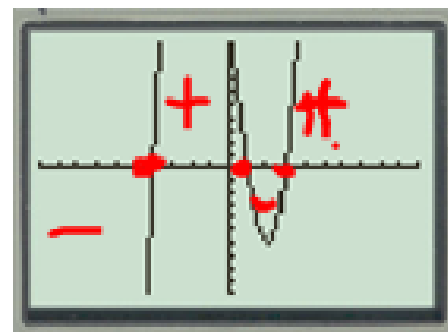
Solve the inequality $x^3 - 13x + 12 > 0$



$$x = -4$$

$$x = 1$$

$$x = 3$$



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By the end of the lesson, we will be able to:

- ~ Solve quadratic inequalities.
- ~ Show the solution of a quadratic inequality on a line graph.
- ~ Write the solution of a quadratic inequality.

Can you?

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Homework:

Assignment 32



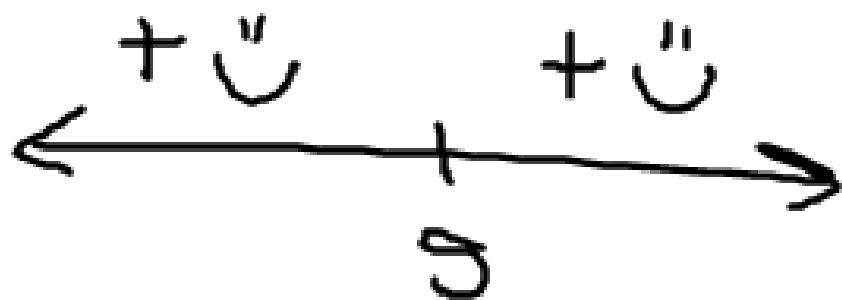
$$b^2 \geq 10b - 25$$

$$b^2 - 10b + 25 \geq 0$$

$$(b - 5)(b - 5) \geq 0$$

$$b = 5$$

$$b = 5$$



If ...
none of it
worked,
it would be
no solution!